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Comparing the power of several tests about specific populations

모집단의 특정분포에 따른 여러 검정방법들의 비교분석

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Abstract

The *t*-test, Mann-Whitney test and median test are three tests that can be used to test for the difference in location parameters. We compared powers of the three tests under a variety of population distributions (Cauchy distribution, exponential distribution, log-normal distribution, mixed normal distribution, normal distribution, uniform distribution) through a simulation study. Each test was performed 10,000 times under the same conditions. In every case, equal sizes of 10, 15, 20 and 25 were used. The powers of the tests were estimated, based on the number of times the null hypothesis was rejected as divided by 10,000. The results of the simulation study indicated that the *t*-test was powerful when the underlying distributions were normal, uniform and exponential about sample size of 10. The Mann-Whitney test was powerful when the underlying distributions was mixed normal about 75% $N(0,1)$ and 25% $N(3,36)$, 80% $N(0,1)$ and 20% $N(2,25)$, log-normal, exponential about sample size of 15, 20 and 25. Also, the median test had the largest powers when the Cauchy distribution.

KEY WORDS: Mann-Whitney test, median test, population distribution, power, *t*-test

1. INTRODUCTION

In applied field of statistics, one of the most basic problems is the comparing of the location parameters of two populations. The most common tests of comparing are t-test of parametric method, Mann-Whitney test and median test of non-parametric method. The t-test, Mann-Whitney test and median test are three tests that can be used to test for the difference in location parameters.

A t-test is any statistical hypothesis test in which the test statistic observes the Student' s t-distribution if the hypothesis backs that up. It can be used to affect if two sets of data are considerably different from each other, and is most generally applied when the test statistic would follow a normal distribution if the value of a scaling term in the test statistic were known. When the scaling term is unknown and is substituted an estimate based on the data, the test statistic when under certain conditions follows a Student' s t-distribution.

As an example, in the one-sample t-test

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} \tag{1.1}$$

where $\overline{X}$ is the sample mean from a sample $X_1, X_2, \cdots, X_n$, of size n, s is the ratio of sample standard deviation over population standard deviation, μ_0 is the population mean and freedom is $(n-1)$.

The assumptions underlying a t-test are that

1. X follows a normal distribution with mean μ and variance σ^2.
2. s^2 follows a X^2 distribution with ρ degrees of freedom under the null hypothesis, where ρ is a positive constant and s are independent.

In a special type of t-test, these conditions are result of the population being studied, and of the way in which the data are sampled.

There are also two samples of the comparison method in t-test. Unpaired samples t-test and the other is paired samples t-test. Independent samples, i.e. unpaired t-test is used when two separate sets of independent and identically distributed samples are obtained. Each of the two populations being compared. Also paired samples t-test typically consist of a sample correspond with pairs of similar units, or one group of units that has been tested twice (a "repeated measures" t-test).

In statistics, the Mann-Whitney test is a non-parametric test. Also called the Mann-Whitney-Wilcoxon (MWW), Wilcoxon rank-sum test (WRS), or Wilcoxon-Mann-Whitney test. The null hypothesis that two samples come from the same population against an alternative hypothesis, particularly that a exceptional population tends to have larger values than the other.

Although Mann and Whitney developed the MWW test under the assumption of continuous responses with the alternative hypothesis being that one distribution is stochastically greater than the other, there are many other ways to formulate the null and alternative hypotheses such that the MWW test will give a valid test.

A very general formulation is to assume that:

1. All the observations from both groups are independent of each other,
2. The responses are ordinal (i.e. one can at least say, of any two observations, which is the greater),

It can be applied on unknown distributions contrary to t-test which has to be applied only on normal distributions, and it is nearly as efficient as the t-test on normal distributions.

The median test could be one of the easiest and most useful procedures for testing the null hypothesis that independent random samples came from populations with equal medians (Mood, 1950). It is a non-parametric test that tests the null hypothesis that the medians of the populations from which two or more samples are identical. The data sample are assigned to two groups respectively. One consisting of data whose values are higher than the median value in the two groups combined, and the other consisting of data whose values are at the median or below.

Since the t-test, Mann-Whitney test and the median test can be used to test when the underlying distributions are known, we would like to know which test is the better one to use.

2. SURVEY OF LITERATURE

Most of the initial research efforts in ranked set sampling have concentrated on parametric and non-parametric estimation and testing procedures for the one and two-sample settings. See, for example, Koti and Babu (1996), Ozturk (1999a, b), Kim and Kang (2000), Park (2006) and Kim et al. (2004). Recently, several researchers have expressed interest in the appropriate allocation of order statistics within a ranked set sample. In two sample location problem, Bohn and Wolfe (1992, 1994) proposed Mann-Whitney-Wilcoxon statistic and investigated the properties of test procedures based on ranked-set sampling, for perfect and imperfect judgement, respectively (Kim et al., 2006).

Some comparisons between the t-test, Mann-Whitney test and the median test and among these three tests and other well-known tests have been made. The Mann-Whitney test has been compared to the t-test under specific conditions. These comparisons were made on the basis of each test' s power.

Gibbons, Chakraborti (1990) stated that the Student' s t-test was more powerful than the Mann-Whitney test for any sample size if the population could be assumed normal with equal variances. They compared the powers of for different sample sizes between 4 and 16 under normal distributions. In this case, they concluded that the t-test was more powerful than the Mann-Whitney test, but the power advantage of the t-test was more powerful than the Mann-Whitney test was very small.

Rasmussen (1985) compared the powers of the Mann-Whitney test, the t-test, and the t-test that corrected outlier using the Grubbs-type outlier detection statistic, L_K (Tietjen, Moore, 1972) when the sample sizes and the population distribution were the same as those used by Blair, Higgins (1980). They found that the t-test corrected outlier showed a power advantage over the Mann-Whitney test. He recommended using a parametric test which was corrected for outliers instead of the Mann-Whitney test.

The Kruskal-Wallis test is an extension of the Mann-Whitney test to k populations (Kruskal, Wallis, 1952). The median test may also be extended to k populations (Conover, 1980). Conover (1980) said that the Kruskal-Wallis test was usually more powerful than the median test because the Kruskal-Wallis test statistic was a function of the ranks of the observations in the combined sample, as was true with the Mann-Whitney test, while the median test statistic depended only on the knowledge of whether the observations were below or above the grand median.

Therefore, many statisticians consider the *t*-test as the best parametric two-sample test for the location. And non-parametric methods of Mann-Whitney test in the best for the location. Gibbons (1971) stated that the Mann-Whitney test generally gas greater power than the median test as a test for location.

3. DESIGN OF STUDY

We compared powers of the three tests under a variety of population distributions (Cauchy distribution, exponential distribution, log-normal distribution, mixed normal distribution, normal distribution, uniform distribution) through a simulation study. Each test was performed 10,000 times under the same conditions. In every case, equal sample sizes of 10, 15, 20 and 25 were used. The powers of the tests were estimated, based on the number of times the null hypothesis was rejected divided by 10,000. The power ratio was calculated by dividing the power of each test.

To research the effect of location shifts in Mixed normal distributions, we used the mixture of $N(0,1)$ and $N(2,25)$ and the mixture of $N(0,1)$ and $N(3,36)$. Each of the tests were performed 10,000 times under the same conditions.

The first type of distribution to be considered here was the normal with the following cases considered. When random variable X follows $N(\mu,\sigma^2)$, $E(X)$ is μ and $VAR(X)$ is σ^2. Statistical hypothesis testing form is $N(0,1)$ versus $N(\theta_1,1)$ and based on them was compared as follows:

a. $N(0,1)$ versus $N(0,1)$,

b. $N(0,1)$ versus $N(\theta_1/5,1)$,

c. $N(0,1)$ versus $N(2\theta_1/5,1)$,

d. $N(0,1)$ versus $N(3\theta_1/5,1)$,

e. $N(0,1)$ versus $N(4\theta_1/5,1)$,

f. $N(0,1)$ versus $N(\theta_1,1)$.

The value θ_1 was found by computer simulation. In each case, the significance level of 0.05 was used to obtain the power of the three tests, the equal sample sizes of 10, 15, 20 and 25. The value θ_1 is then divided by 5, and a power comparison of the three tests were made for each of the distributions.

The powers of the three tests were also examined when the populations were a mixture of normal distributions. The Mixed normal populations considered were 75% $N(0,1)$ and 25% $N(3,36)$, 80% $N(0,1)$ and 20% $N(2,25)$. The following cases were considered:

1. mixture of $N(0,1)$ and $N(3,36)$ versus
   mixture of $N(\theta_1,1)$ and $N(3+\theta_1,36)$, and
2. mixture of $N(0,1)$ and $N(2,25)$ versus
   mixture of $N(\theta_2,1)$ and $N(2+\theta_2,25)$.

The values θ_1 and θ_2 were found by computer simulation. In each case, the significance level of 0.05 was used to obtain the power of the three tests. The equal sample sizes of 10, 15, 20 and 25 were used. The value θ_1 and θ_2 was then divided by 5, and a power comparison of the three tests were made for each of the following cases:

1a. mixture of $N(0,1)$ and $N(3,36)$ versus
mixture of $N(0,1)$ and $N(3,36)$,

1b. mixture of $N(0,1)$ and $N(3,36)$ versus
mixture of $N(\theta_1/5,1)$ and $N(3+\theta_1/5,36)$,

1c. mixture of $N(0,1)$ and $N(3,36)$ versus
mixture of $N(2\theta_1/5,1)$ and $N(3+2\theta_1/5,36)$,

1d. mixture of $N(0,1)$ and $N(3,36)$ versus
mixture of $N(3\theta_1/5,1)$ and $N(3+3\theta_1/5,36)$,

1e. mixture of $N(0,1)$ and $N(3,36)$ versus mixture of $N(4\theta_1/5, 1)$ and $N(3+4\theta_1/5, 36)$,

1f. mixture of $N(0, 1)$ and $N(3, 36)$ versus

mixture of $N(\theta_1, 1)$ and $N(3+\theta_1, 36)$,

2a. mixture of $N(0, 1)$ and $N(2, 25)$ versus

mixture of $N(0, 1)$ and $N(2, 25)$,

2b. mixture of $N(0, 1)$ and $N(2, 25)$ versus

mixture of $N(\theta_2/5, 1)$ and $N(2+\theta_2/5, 25)$,

2c. mixture of $N(0, 1)$ and $N(2, 25)$ versus

mixture of $N(2\theta_2/5, 1)$ and $N(2+2\theta_2/5, 25)$,

2d. mixture of $N(0, 1)$ and $N(2, 25)$ versus

mixture of $N(3\theta_2/5, 1)$ and $N(2+3\theta_2/5, 25)$,

2e. mixture of $N(0, 1)$ and $N(2, 25)$ versus

mixture of $N(4\theta_2/5, 1)$ and $N(2+4\theta_2/5, 25)$,

2f. mixture of $N(0, 1)$ and $N(2, 25)$ versus

mixture of $N(\theta_2, 1)$ and $N(2+\theta_2, 25)$.

The next type of distribution to be considered under the equal variance assumption was the Log-normal distribution with following case considered: When random variable X follows $\ln(\mu, \sigma^2)$, $E(X)$ is $e^{\mu+\sigma^2/2}$ and $VAR(X)$ is $(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$. In the form of a hypothesis test it is $\ln(0, 1)$ versus $\ln(\theta_1, 1)$, was compared as follows by using this.

a. $\ln(0, 1)$ versus $\ln(0, 1)$,

b. $\ln(0, 1)$ versus $\ln(\theta_1/5, 1)$,

c. $\ln(0, 1)$ versus $\ln(2\theta_1/5, 1)$,

d. $\ln(0,1)$ versus $\ln(3\theta_1/5,1)$,

e. $\ln(0,1)$ versus $\ln(4\theta_1/5,1)$,

f. $\ln(0,1)$ versus $\ln(\theta_1,1)$.

The value θ_1 was found by computer simulation and the significance level of 0.05 was used to obtain the power of the three tests. In each case, the value θ_1 is then divided by 5, and the equal sample sizes of 10, 15, 20 and 25 were used. We examined the relative power of the three tests in the same way for the equal sample sizes.

The Uniform distribution was next considered under the equal variance assumption. When random variable X follows $U(a,b)$, $E(X)$ is $\frac{1}{2}(a+b)$ and $VAR(X)$ is $\frac{1}{12}(b-a)^2$. Statistical hypothesis testing form is $U(0,1)$ versus $U(\theta_1,1)$.

A power comparison of the three tests were made for each of the following cases:

a. $U(0,1)$ versus $U(0,1)$,

b. $U(0,1)$ versus $U(\theta_1/5,1)$,

c. $U(0,1)$ versus $U(2\theta_1/5,1)$,

d. $U(0,1)$ versus $U(3\theta_1/5,1)$,

e. $U(0,1)$ versus $U(4\theta_1/5,1)$,

f. $U(0,1)$ versus $U(\theta_1,1)$.

The value θ_1 was found by computer simulation. In each case, the significance level of 0.05 was used to obtain the power of the three tests, the equal sample sizes of 10, 15, 20 and 25 and the value θ_1 is then divided by 5.

The next type of distribution to be considered under the equal variance assumption was the Exponential distribution with following case considered. When random variable X follows $\mathrm{Exp}(\lambda)$, $E(X)$ is $\frac{1}{\lambda}$ and $VAR(X)$ is $\frac{1}{\lambda^2}$. Statistical hypothesis testing form is $\mathrm{Exp}(1)$ versus $\mathrm{Exp}(1+\theta_1)$. It compared the following cases :

a. $\mathrm{Exp}(1)$ versus $\mathrm{Exp}(1)$,

b. $\mathrm{Exp}(1)$ versus $\mathrm{Exp}(1+\theta_1/5)$,

c. $\mathrm{Exp}(1)$ versus $\mathrm{Exp}(1+2\theta_1/5)$,

d. $\mathrm{Exp}(1)$ versus $\mathrm{Exp}(1+3\theta_1/5)$,

e. $\mathrm{Exp}(1)$ versus $\mathrm{Exp}(1+4\theta_1/5)$,

f. $\mathrm{Exp}(1)$ versus $\mathrm{Exp}(1+\theta_1)$.

The value θ_1 was found by computer simulation. In each case, the significance level of 0.05 was used to obtain the power of the three tests, the equal sample sizes of 10, 15, 20 and 25. The value θ_1 is then divided by 5, and a power comparison of the three tests were made for each of the distributions.

We also examined the power of the t-test, Mann-Whitney test and median test in the same way for equal sample sizes. The power of the three tests were also examined for the Cauchy distribution. When random variable X follows $Cauchy(x)$, $E(X)$ and $VAR(X)$ is undefined. Hypothesis is a type, such as $Cauchy(0)$ versus $Cauchy(\theta_1)$ and compared it with the next.

a. $Cauchy(0)$ versus $Cauchy(0)$,

b. $Cauchy(0)$ versus $Cauchy(\theta_1/5)$,

c. $Cauchy(0)$ versus $Cauchy(2\theta_1/5)$,

d. $Cauchy(0)$ versus $Cauchy(3\theta_1/5)$,

e. $Cauchy(0)$ versus $Cauchy(4\theta_1/5)$,

f. $Cauchy(0)$ versus $Cauchy(\theta_1)$.

The value θ_1 was found by computer simulation and the significance level of 0.05 was used to obtain the power of the three tests. The equal sample sizes of 10, 15, 20 and 25 were used. In each case, the value θ_1 is then divided by five, and a power comparison of the three tests were made for each of the distributions.

We used the R package program to generate random samples from each distribution. The subroutines RNORM, RLNORM, RUNIF, REXP and RCAUCHY were used to generate random numbers from Normal, Log-normal, Uniform, Exponential and Cauchy distributions, respectively. Two random samples were generated at on time by using the following program.

```
n.generate<-function(p){x<-rnorm(p[1],p[2],p[3])}
u.generate<-function(p){x<-runif(p[1],p[2],p[3])}
e.generate<-function(p){x<-rexp(p[1],p[2])}
l.generate<-function(p){x<-rlnorm(p[1],p[2],p[3])}
c.generate<-function(p){x<-rcauchy(p[1],p[2])}
mix.generate<-function(p){
            n1<-round(p[1]*p[2],0)
            n2<-p[1]-n1
            x<-rnorm(n1,p[3],p[4])
            x<-c(x,rnorm(n2,p[5],p[6]))      }
```

In the above case, the random number was generated in each of the distributions.

Each of the tests statistics was calculated and compared to its respective critical values. In this paper, an alpha value of 0.05 was used. To simulate the samples from each population 10,000 times and to perform the *t*-test, Mann-Whitney test and the median test each time, the R was used. The R programs can be found in APPENDIX B.

4. SIMULATION RESULTS

The goal of this chapter was to examine the results of comparing the powers of the t-test, Mann-Whitney test and median test. The powers were compared under a diversity of population distributions (Normal distribution, mixed normal distribution, log-normal distribution, uniform distribution, exponential distribution, Cauchy distribution). Equal sample sizes of 10, 15, 20 and 25 were used and the alpha value was always 0.05.

These powers were estimated, based on the number of times the null hypothesis was rejected divided by 10,000. In this case, the power ratio was defined for each of the following cases:

1. the power of the t-test divided by the power of the Mann-Whitney test
2. the power of the Mann-Whitney test divided by the power of the median test
3. the power of the t-test divided by the power of the median test

The power ratio greater than 1 indicates each case where the t-test, Mann-Whitney test and t-test are better. The power ratio less than 1 indicates each case where the Mann-Whitney test, median test and median test are better.

4.1 Normal distribution case

Normal distribution was to investigate the results of three tests power in table 1 to 4 and figure 1 to 4. According to the results of Tables 1, 2, 3 and 4, The power ratios were all grater than 1, indicating the power of the t-test was higher. In this case, the power ratio was estimated for the power of the t-test, Mann-Whitney test and median test under the same conditions.

Also, the t-test was found to be more powerful than the Mann-Whitney test and median test for all equal sample sizes of 10, 15, 20 and 25 when the equal variance assumption was true. Therefore, this result indicated that the t-test is more powerful in this case.

On the other hands, Tables 1 to 4 and Figure 1 to 4 show that the power ratios of t-test and Mann-Whitney test are almost same.

4.2 mixed normal distribution case (1) - 75% $N(0,1)$ and 25% $N(3,36)$

We compared the powers of the three tests for equal sample sizes of 10, 15, 20 and 25 from the mixture population 75% $N(0,1)$ and 25% $N(3,36)$.

According to the results of Table 5, 6, 7 and 8, the power ratio of case (6) were all greater than 1 for equal samples of sizes 10, 15, 20 and 25 from the mixture population 75% $N(0,1)$ and 25% $N(3,36)$, indicating the Mann-Whitney test was more powerful than the t-test and median test. All results are given in Table 5 to 8 and Figures 5 to 8.

4.3 mixed normal distribution case (2) - 80% $N(0,1)$ and 20% $N(2,25)$

Tables 9 to 12 and Figure 9 to 12 show the results. We compared the powers of the three tests for equal sample sizes of 10, 15, 20 and 25 from the mixture population 80% $N(0,1)$ and 20% $N(2,25)$.

According to the results of Table 9, the power ratio of case (6) were all greater than 1 for equal samples of sizes 10, 15, 20 and 25 from the mixture population 80% $N(0,1)$ and 20% $N(2,25)$, indicating the Mann-Whitney test was more powerful than the t-test and median test. All results are given in Table 9 to 12 and Figures 9 to 12.

4.4 Log normal distribution case

There are results of three tests power in table 13 to 16 and figure 13 to 16.

According to the results of Tables 13 to 16 and figure 13 to 16, the Mann-Whitney test was found to be more powerful than the t-test and median test for equal sample sizes of 10, 15, 20 and 25 when the equal variance assumption was true.

The power ratio of case (6) was greater than 1, indicating the power of the mann-Whitney test was higher.

4.5 Uniform distribution case

We examined the powers of the three tests when the equal variance assumption was true when the underlying distribution was uniform.

According to the results of Tables 17 to 20 and Figure 17 to 20, the power ratios were all grater than 1, indicating the power of the t-test was higher. Also, the t-test was found to be more powerful than the Mann-Whitney test and median test for all equal sample sizes of 10, 15, 20 and 25 when the equal variance assumption was true. Therefore, this result indicated that the t-test is more powerful in this case.

4.6 Exponential distribution case

When the location shifts were 0.0000 and 2.0423 for equal sample sizes of 10, 15, 20 and 25. Also, The results are given in Table 21 and Figure 21, the Mann-Whitney test was found to be more powerful than the t-test and median test when the equal sample size of 10.

But according to the results of Table 22 to 24 and Figure 22 to 24, the power ratios were all grater than 1. In this case, this result indicated that the t-test is more powerful when the equal sample sizes of 15, 20 and 25..

4.7 Cauchy distribution case

The results are given in Table 25 to 28 and Figure 25 to 28.

The smaller the location difference, the Mann-Whitney test was found to be more powerful. But the larger the location difference, the median test was found to be more powerful than the t-test and Mann-Whitney test when the equal sample sizes of 10, 15, 20 and 25. In this case, location shifts were 0.0000 and 2.2514 for equal sample sizes.

5. CONCLUSIONS AND FUTURE RESEARCH

The t-test, Mann-Whitney test and median test are three tests that can be used to test for the difference in location parameters. The powers of the tests were estimated, based on the number of times the null hypothesis was rejected divided by 10,000. The test was simulated 10,000 times for each situation. The actual α value for the median test always stayed below 0.05 for all cases considered.

In the Tables of the appendix, (1) is the value of location difference, and (2), (3) and (4) are the values of estimated power of t-test, Mann-Whitney test and median test, respectively. (5) is the value of power of the t-test divided by the power of the Mann-Whitney test. The (6) is the value of power of the Mann-Whitney test divided by the power of the median test. The (7) is the value of power of the t-test divided by the power of the median test.

According to the Figures in the appendix, the blue line is the power of the t-test. And the red line is the power of the Mann-Whitney test, the black line is the power of the median test. When the assumption of equal variance was relaxed, the median test was more conservative. Also, according to the results of Figure 1 to 28, the larger the sample sizes, the power grows bigger.

When the assumption of equal variance was true, the results of the simulation study indicated that the t-test was more powerful than the Mann-Whitney test and median test for the cases of normal, uniform and exponential distribution. However, as shown in Table 21, the Mann-Whitney test was more powerful than the t-test and median test for the sample of size 10 of exponential distribution.

Also, the Mann-Whitney test was more powerful than the t-test and median test when the underlying distributions were log-normal, exponential.

When the underlying distribution was Cauchy, the median test was almost always more powerful than the t-test and Mann-Whitney test for the large case of the location difference.

The t-test, Mann-Whitney test and the median test obtained same results for samples of size 10, 15, 20 and 25 with the mixture population of 75% $N(0,1)$ and 25% $N(3,36)$, and with the mixture population of 80% $N(0,1)$ and 20% $N(2,25)$. The Mann-Whitney test had larger powers for samples of size 10, 15, 20 and 25 than the t-test and median test.

Future research is recommended to compare the powers of the t-test with Mann-Whitney test and the median test for the other distribution and various mixtures of populations.

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APPENDIX

A. Estimated Powers of the t-test, Mann-Whitney test and the median test

Table 1. Estimated Powers of the t-test, Mann-Whitney test and the median test for $N(0,1)$ with equal samples of size 10 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0506	0.0455	0.0219	1.1121	2.0776	2.3105
0.2101	0.0725	0.0666	0.0351	1.0886	1.8974	2.0655
0.4203	0.1436	0.1295	0.0651	1.1089	1.9892	2.2058
0.6304	0.2596	0.2324	0.1195	1.1170	1.9448	2.1724
0.8406	0.4212	0.3849	0.2102	1.0943	1.8311	2.0038
1.0507	0.5987	0.5549	0.3324	1.0789	1.6694	1.8011

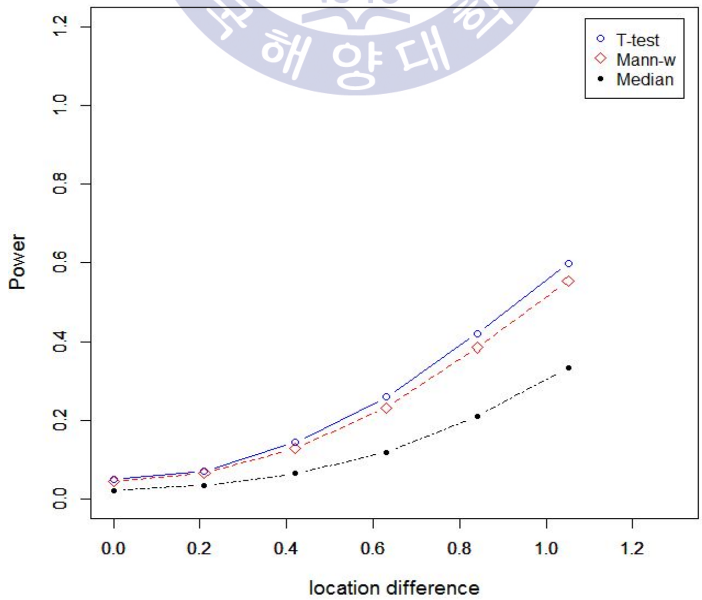

Fig. 1 Normal distribution (n=10)

Table 2. Estimated Powers of the t-test, Mann-Whitney test and the median test for $N(0,1)$ with equal samples of size 15 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0467	0.0420	0.0248	1.1119	1.6935	1.8831
0.2101	0.0804	0.0718	0.0387	1.1198	1.8553	2.0775
0.4203	0.2013	0.1831	0.0996	1.0994	1.8384	2.0211
0.6304	0.3780	0.3468	0.1956	1.0900	1.7730	1.9325
0.8406	0.6001	0.5682	0.3503	1.0561	1.6220	1.7131
1.0507	0.7878	0.7521	0.5158	1.0475	4.4581	1.5273

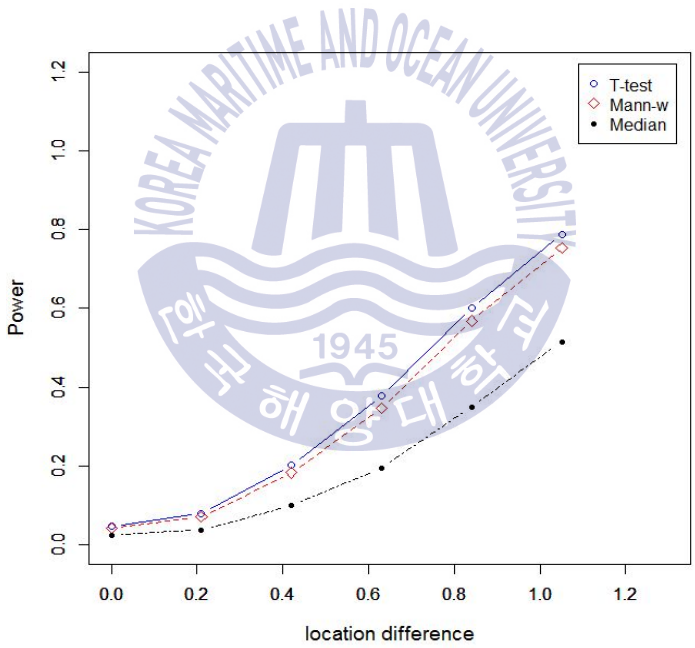

Fig. 2 Normal distribution (n=15)

Table 3. Estimated Powers of the t-test, Mann-Whitney test and the median test for $N(0,1)$ with equal samples of size 20 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0467	0.0420	0.0248	1.1119	1.6935	1.8831
0.2101	0.0804	0.0718	0.0387	1.1198	1.8553	2.0775
0.4203	0.2013	0.1831	0.0996	1.0994	1.8384	2.0211
0.6304	0.3780	0.3468	0.1956	1.0900	1.7730	1.9325
0.8406	0.6001	0.5682	0.3503	1.0561	1.6220	1.7131
1.0507	0.7878	0.7521	0.5158	1.0475	4.4581	1.5273

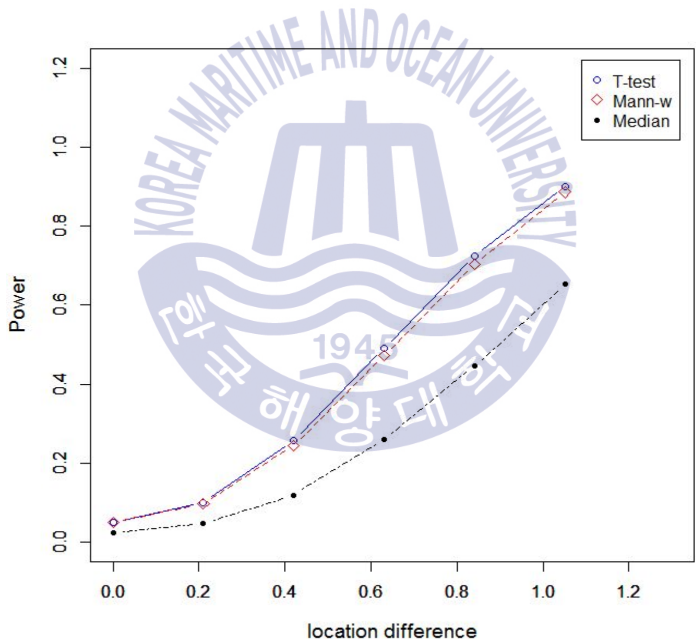

Fig. 3 Normal distribution (n=20)

Table 4. Estimated Powers of the t-test, Mann-Whitney test and the median test for $N(0,1)$ with equal samples of size 25 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0455	0.0471	0.0216	0.9660	2.1806	2.1065
0.2101	0.1100	0.1078	0.0473	1.0204	2.2791	2.3256
0.4203	0.3038	0.2893	0.1400	1.0501	2.0664	2.1700
0.6304	0.5839	0.5581	0.3021	1.0462	1.8474	1.9328
0.8406	0.8278	0.8084	0.5321	1.0240	1.5193	1.5557
1.0507	0.9500	0.9413	0.7413	1.0092	1.2698	1.2815

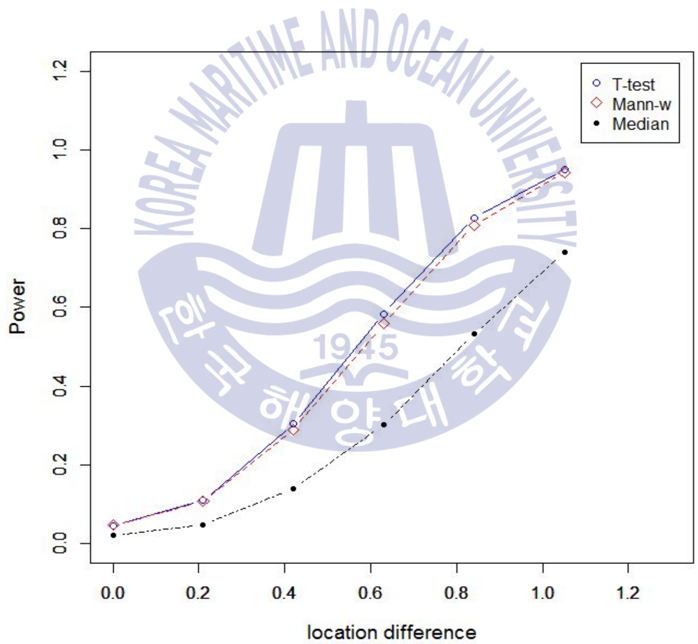

Fig. 4 Normal distribution (n=25)

Table 5. Estimated Powers of the t-test, Mann-Whitney test and the median test for mixture of 75% $N(0,1)$ and 25% $N(3,36)$ with equal samples of size 10 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0203	0.0371	0.0214	0.5472	1.7336	0.9486
0.3504	0.0296	0.0640	0.0414	0.4625	1.5459	0.7150
0.7008	0.0640	0.1477	0.1005	0.4333	1.4697	0.6368
1.0512	0.1108	0.2722	0.2065	0.4071	1.3182	0.5366
1.4017	0.1802	0.4441	0.3595	0.4058	1.2353	0.5013
1.7521	0.2757	0.6038	0.5435	0.4566	1.1109	0.5073

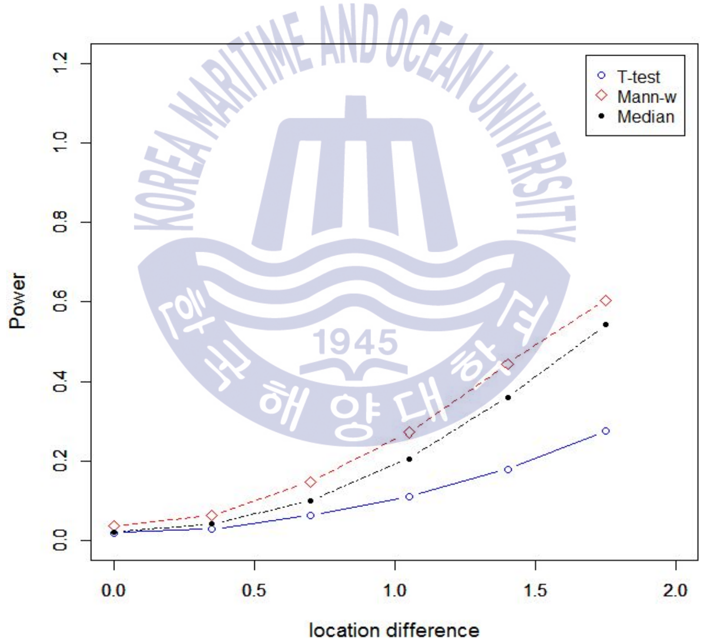

Fig. 5 mixed normal distribution (n=10)
75% $N(0,1)$ and 25% $N(3,36)$

Table 6. Estimated Powers of the t-test, Mann-Whitney test and the median test for mixture of 75% $N(0,1)$ and 25% $N(3,36)$ with equal samples of size 15 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0251	0.0320	0.0197	0.7844	1.6244	1.2741
0.3504	0.0354	0.0676	0.0439	0.5237	1.5399	0.8064
0.7008	0.0689	0.1762	0.1349	0.3910	1.3062	0.5107
1.0512	0.1140	0.3382	0.2788	0.3371	1.2131	0.4089
1.4017	0.1883	0.5362	0.4933	0.3512	1.0870	0.3817
1.7521	0.2778	0.7043	0.6891	0.3944	1.0221	0.4031

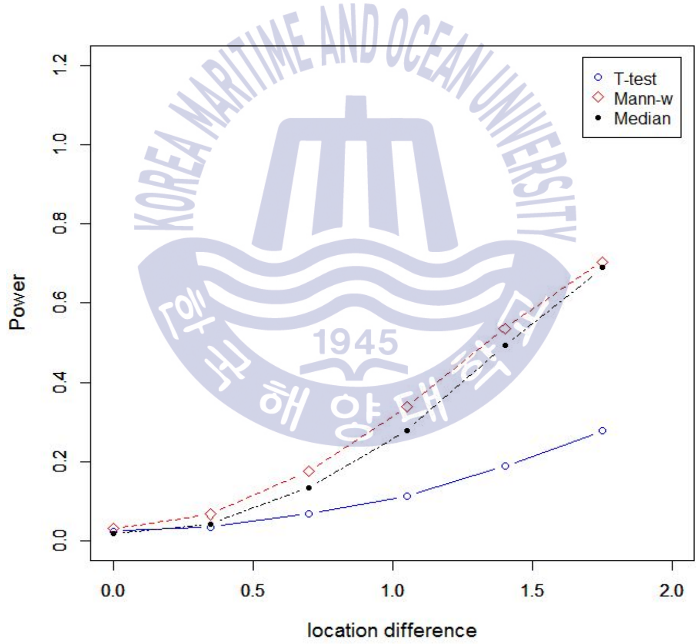

Fig. 6 mixed normal distribution (n=15)

75% $N(0,1)$ and 25% $N(3,36)$

Table 7. Estimated Powers of the t-test, Mann-Whitney test and the median test for mixture of 75% $N(0,1)$ and 25% $N(3,36)$ with equal samples of size 20 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0280	0.0410	0.0213	0.6829	1.9249	1.3146
0.3504	0.0416	0.0913	0.0572	0.4556	1.5962	0.7273
0.7008	0.0840	0.2546	0.1809	0.3299	1.4074	0.4643
1.0512	0.1538	0.4921	0.4010	0.3125	1.2272	0.3835
1.4017	0.2546	0.7163	0.6520	0.3554	1.0986	0.3905
1.7521	0.3830	0.8761	0.8496	0.4372	1.0312	0.4508

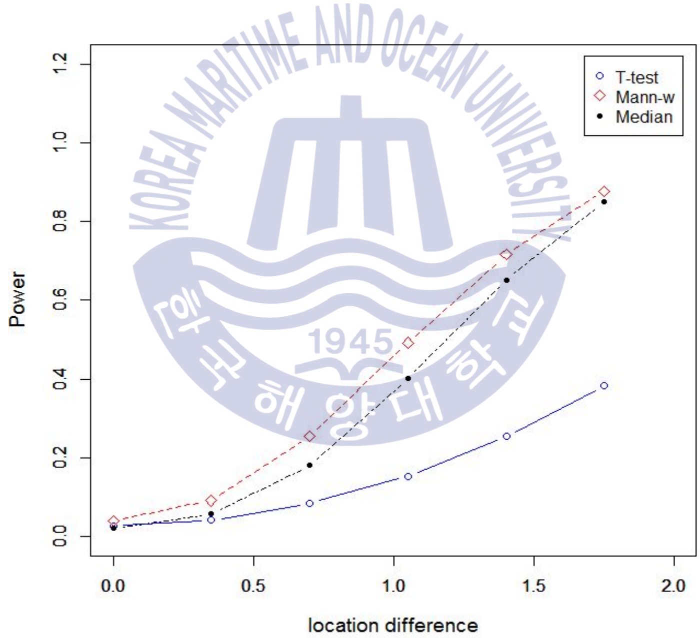

Fig. 7 mixed normal distribution (n=20)
75% $N(0,1)$ and 25% $N(3,36)$

Table 8. Estimated Powers of the t-test, Mann-Whitney test and the median test for mixture of 75% $N(0,1)$ and 25% $N(3,36)$ with equal samples of size 25 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0280	0.0377	0.0170	0.7427	2.2176	1.6471
0.3504	0.0455	0.1069	0.0594	0.4256	1.7997	0.7660
0.7008	0.1042	0.3275	0.2176	0.3182	1.5051	0.4789
1.0512	0.1909	0.6099	0.4846	0.3130	1.2586	0.3939
1.4017	0.3259	0.8480	0.7663	0.3843	1.1066	0.4253
1.7521	0.4688	0.9500	0.9234	0.4935	1.0288	0.5077

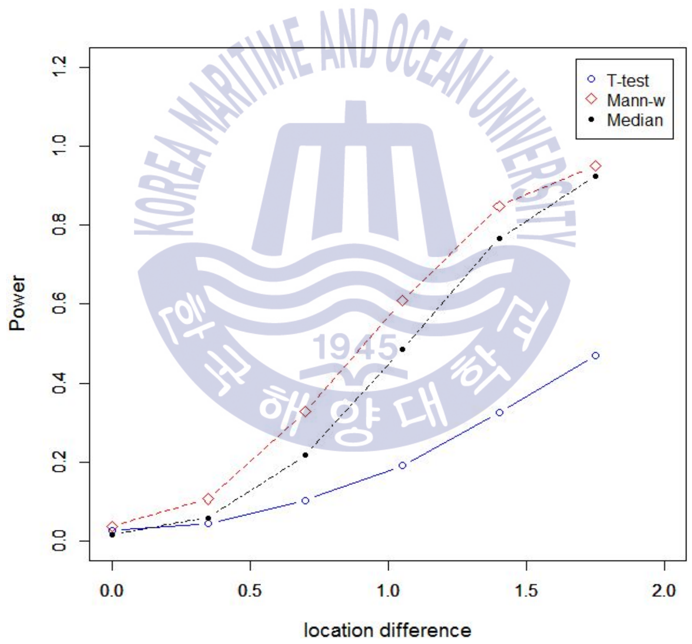

Fig. 8 mixed normal distribution (n=25)
75% $N(0,1)$ and 25% $N(3,36)$

Table 9. Estimated Powers of the t-test, Mann-Whitney test and the median test for mixture of 80% $N(0,1)$ and 20% $N(2,25)$ with equal samples of size 10 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0254	0.0407	0.0217	0.6241	1.8756	1.1705
0.3105	0.0380	0.0608	0.0378	0.6250	1.6085	1.0053
0.6211	0.0743	0.1312	0.0851	0.5663	1.5417	0.8731
0.9316	0.1262	0.2390	0.1683	0.5280	1.4201	0.7499
1.2422	0.2062	0.3921	0.2971	0.5259	1.3198	0.6940
1.5527	0.3082	0.5478	0.4562	0.5626	1.2008	0.6756

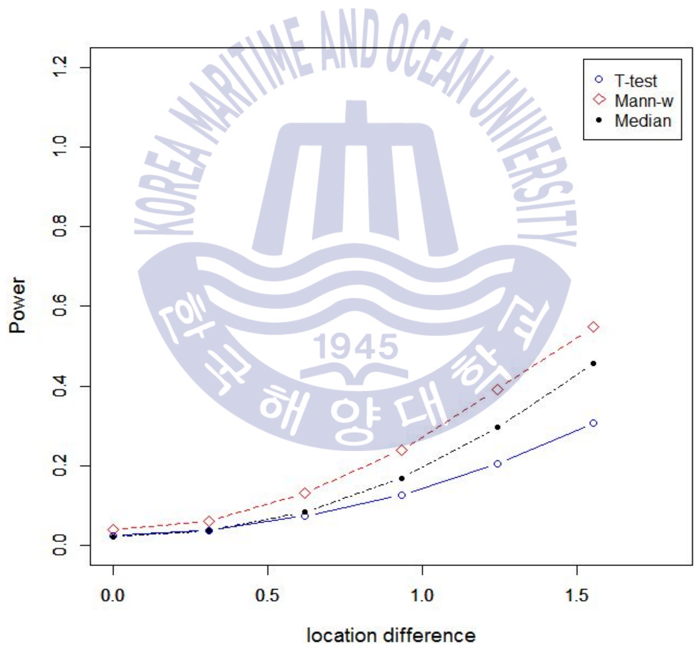

Fig. 9 mixed normal distribution (n=10)
80% $N(0,1)$ and 20% $N(2,25)$

Table 10. Estimated Powers of the t-test, Mann-Whitney test and the median test for mixture of 80% $N(0,1)$ and 20% $N(2,25)$ with equal samples of size 15 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0290	0.0370	0.0224	0.7838	1.6518	1.2946
0.3105	0.0443	0.0735	0.0449	0.6027	1.6370	0.9866
0.6211	0.0939	0.1895	0.1343	0.4955	1.4110	0.6992
0.9316	0.1675	0.3676	0.2730	0.4557	1.3465	0.6136
1.2422	0.2860	0.5846	0.4842	0.4892	1.2074	0.5907
1.5527	0.4046	0.7688	0.6856	0.5263	1.1214	0.5901

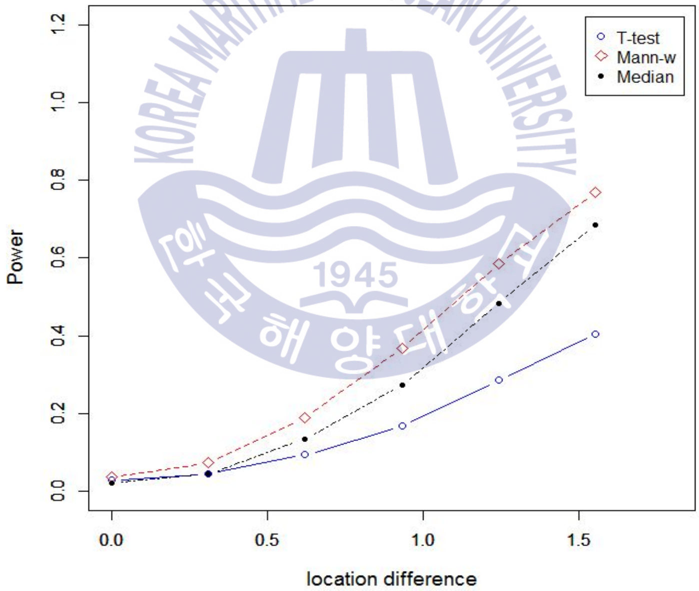

Fig. 10 mixed normal distribution (n=15)

80% $N(0,1)$ and 20% $N(2,25)$

Table 11. Estimated Powers of the t-test, Mann-Whitney test and the median test for mixture of 80% $N(0,1)$ and 20% $N(2,25)$ with equal samples of size 20 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0335	0.0445	0.0230	0.7528	1.9348	1.4565
0.3105	0.0548	0.0986	0.0553	0.5558	1.7830	0.9910
0.6211	0.1167	0.2604	0.1723	0.4482	1.5113	0.6773
0.9316	0.2172	0.5083	0.3771	0.4273	1.3479	0.5760
1.2422	0.3478	0.7377	0.6181	0.4715	1.1935	0.5627
1.5527	0.5088	0.8993	0.8208	0.5658	1.0956	0.6199

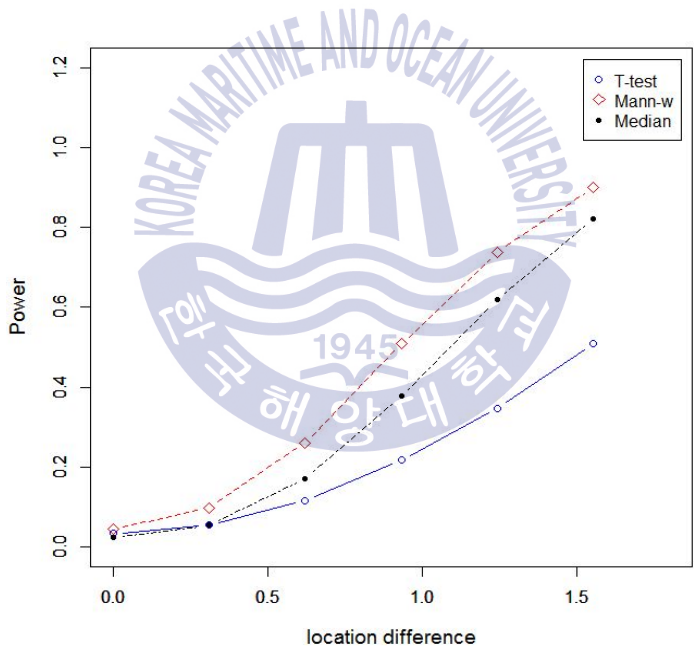

Fig. 11 mixed normal distribution (n=20)

80% $N(0,1)$ and 20% $N(2,25)$

Table 12. Estimated Powers of the t-test, Mann-Whitney test and the median test for mixture of 80% $N(0,1)$ and 20% $N(2,25)$ with equal samples of size 25 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.3105	0.0577	0.1070	0.0553	0.5393	1.9349	1.0434
0.6211	0.1390	0.3229	0.1944	0.4305	1.6924	0.7150
0.9316	0.2588	0.6007	0.4403	0.4308	1.3643	0.5878
1.2422	0.4246	0.8402	0.7170	0.5054	1.1718	0.5922
1.5527	0.5872	0.9500	0.8936	0.6181	1.0631	0.6571

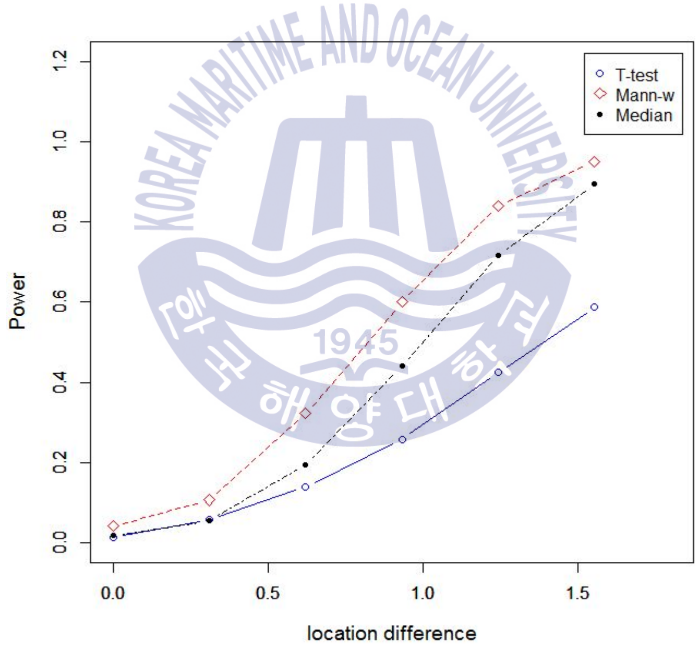

Fig. 12 mixed normal distribution (n=25)

80% $N(0,1)$ and 20% $N(2,25)$

Table 13. Estimated Powers of the t-test, Mann-Whitney test and the median test for $\ln(0,1)$ with equal samples of size 10 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.2145	0.0413	0.0679	0.0354	0.6082	1.9181	1.1667
0.4291	0.0763	0.1331	0.0668	0.5733	1.9925	1.1422
0.6436	0.1506	0.2406	0.1240	0.6259	1.9403	1.2145
0.8581	0.2483	0.4011	0.2193	0.6190	1.8290	1.1322
1.0727	0.3659	0.5721	0.3460	0.6396	1.6535	1.0575

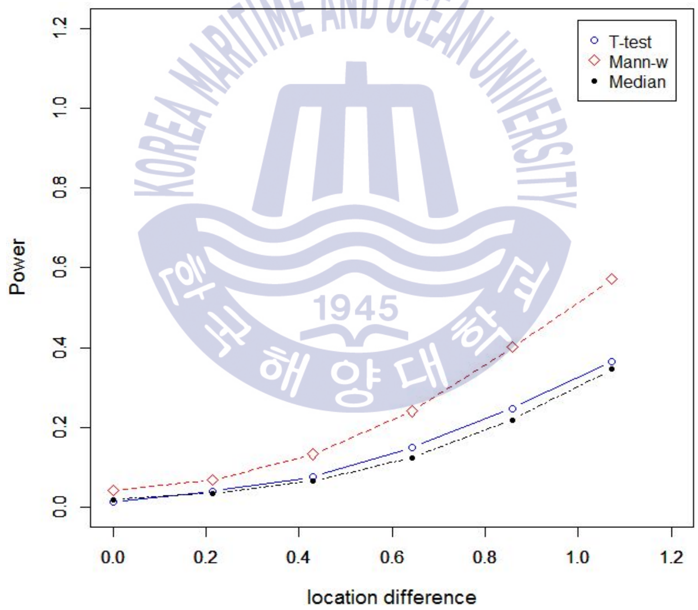

Fig. 13 Log-normal distribution (n=10)

Table 14. Estimated Powers of the t-test, Mann-Whitney test and the median test for $\ln(0,1)$ with equal samples of size 15 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.2145	0.0495	0.0730	0.0393	0.6781	1.8575	1.2595
0.4291	0.1276	0.1887	0.1026	0.6762	1.8392	1.2437
0.6436	0.2488	0.3576	0.2033	0.6957	1.7590	1.2238
0.8581	0.4115	0.5849	0.3650	0.7035	1.6025	1.1274
1.0727	0.5697	0.7720	0.5355	0.7380	1.4416	1.0679

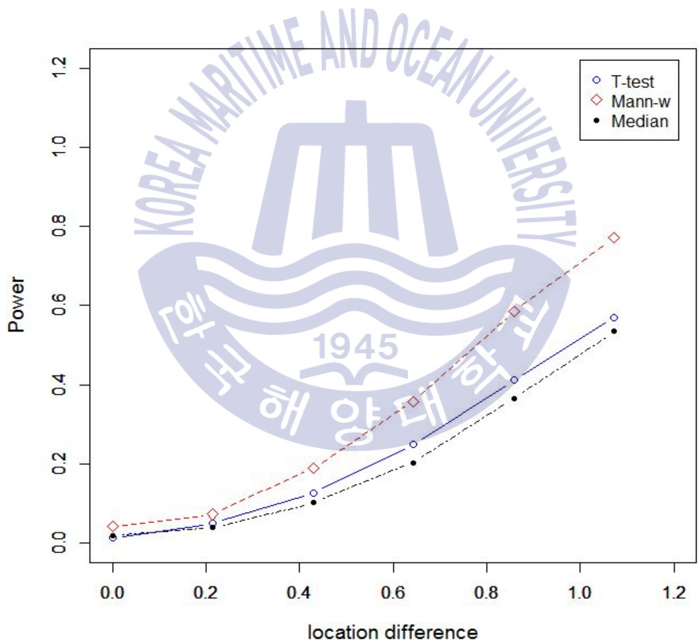

Fig. 14 Log-normal distribution (n=15)

Table 15. Estimated Powers of the t-test, Mann-Whitney test and the median test for $\ln(0,1)$ with equal samples of size 20 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.2145	0.0662	0.0998	0.0490	0.6633	2.0367	1.3510
0.4291	0.1717	0.2544	0.1246	0.6749	2.0417	1.3780
0.6436	0.3393	0.4867	0.2716	0.6971	1.7920	1.2493
0.8581	0.5465	0.7192	0.4632	0.7599	1.5527	1.1798
1.0727	0.7288	0.8965	0.6704	0.8129	1.3373	1.0871

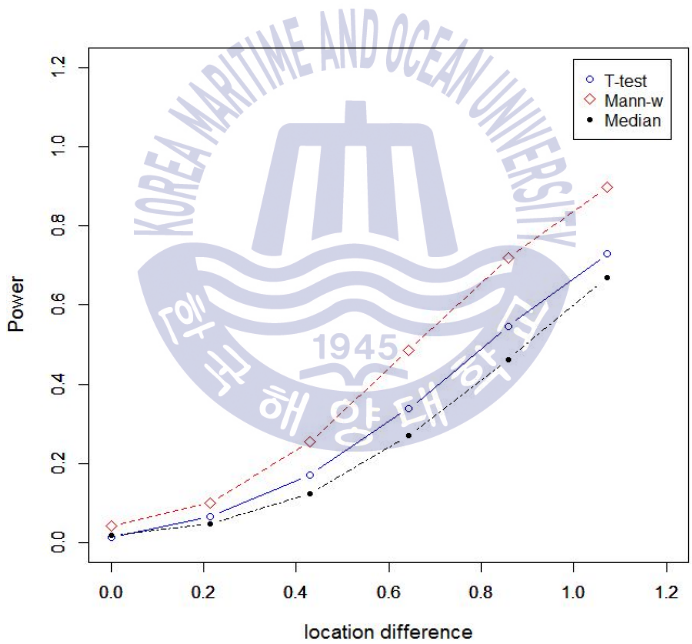

Fig. 15 Log-normal distribution (n=20)

Table 16. Estimated Powers of the t-test, Mann-Whitney test and the median test for $\ln(0,1)$ with equal samples of size 25 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.2145	0.0720	0.1098	0.0482	0.6557	2.2780	1.4938
0.4291	0.2096	0.3004	0.1451	0.6977	2.0702	1.4445
0.6436	0.4191	0.5765	0.3124	0.7270	1.8454	1.3415
0.8581	0.6526	0.8233	0.5516	0.7927	1.4926	1.1831
1.0727	0.8141	0.9500	0.7616	0.8569	1.2539	1.0689

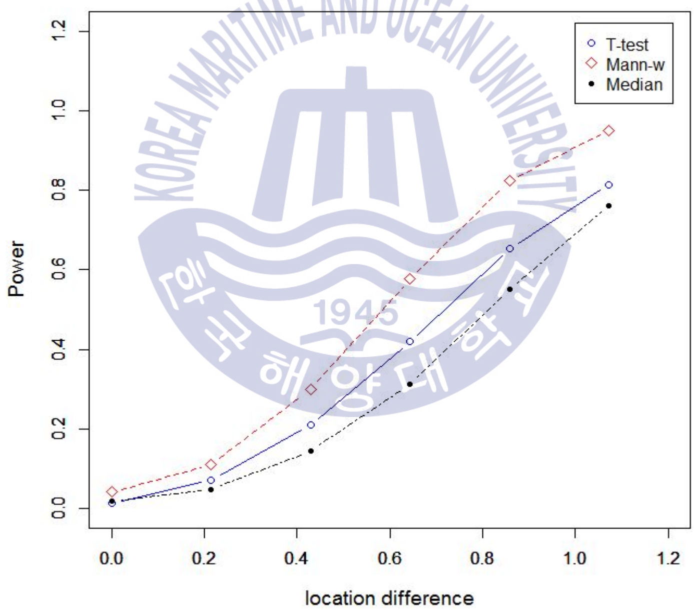

Fig. 16 Log-normal distribution (n=25)

Table 17. Estimated Powers of the t-test, Mann-Whitney test and the median test for $U(0,1)$ with equal samples of size 10 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.0960	0.0635	0.0556	0.0257	1.1421	2.1634	2.4708
0.1920	0.1117	0.0954	0.0409	1.1709	2.3325	2.7311
0.2881	0.2192	0.1858	0.0749	1.1798	2.4806	2.9266
0.3841	0.3584	0.2883	0.1285	1.2431	2.2436	2.7891
0.4801	0.5537	0.4390	0.2281	1.2613	1.9246	2.4274

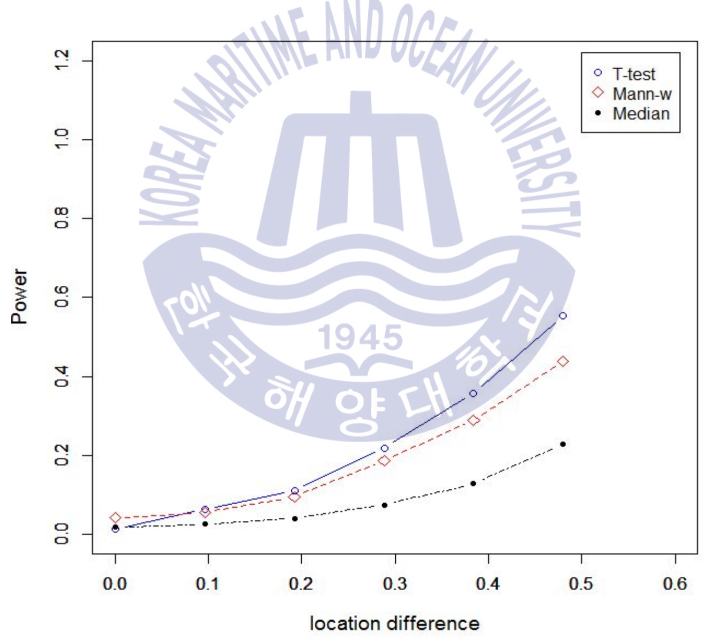

Fig. 17 Uniform distribution (n=10)

Table 18. Estimated Powers of the t-test, Mann-Whitney test and the median test for $U(0,1)$ with equal samples of size 15 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.0960	0.0722	0.0648	0.0322	1.1142	2.0124	2.2360
0.1920	0.1499	0.1294	0.0529	1.1584	2.4461	2.8336
0.2881	0.3171	0.2568	0.1096	1.2348	2.3431	2.8932
0.3841	0.5455	0.4411	0.2054	1.2367	2.1475	2.6558
0.4801	0.7771	0.6285	0.3489	1.2364	1.8014	2.2273

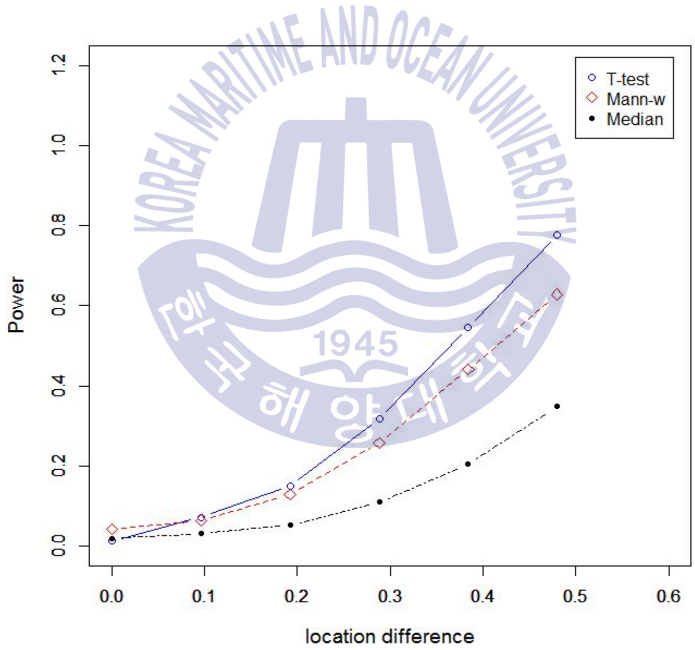

Fig. 18 Uniform distribution (n=15)

Table 19. Estimated Powers of the t-test, Mann-Whitney test and the median test for $U(0,1)$ with equal samples of size 20 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.0960	0.0848	0.0825	0.0364	1.0279	1.2891	2.3297
0.1920	0.1966	0.1779	0.0635	1.1220	2.8016	3.0961
0.2881	0.4088	0.3430	0.1279	1.1918	2.6818	3.1962
0.3841	0.6830	0.5666	0.2539	1.2054	1.0512	2.6900
0.4801	0.8968	0.7740	0.4508	1.1587	1.7169	1.9894

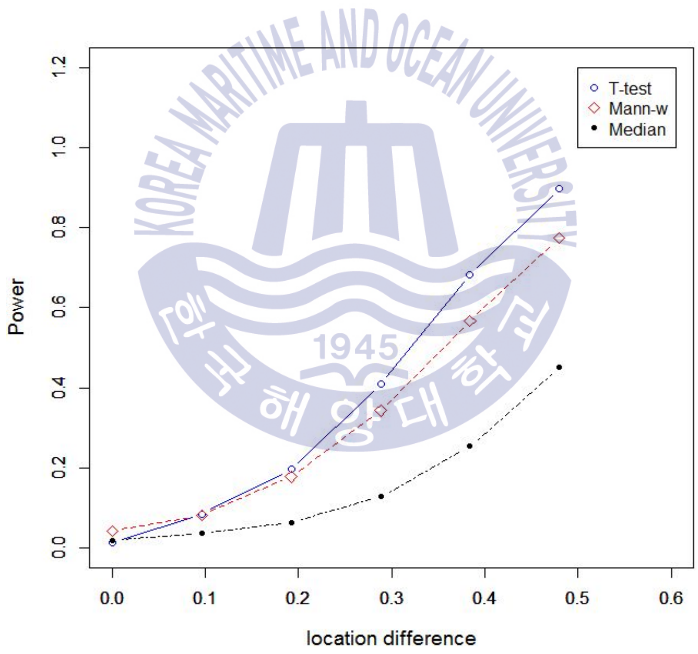

Fig. 19 Uniform distribution (n=20)

Table 20. Estimated Powers of the t-test, Mann-Whitney test and the median test for $U(0,1)$ with equal samples of size 25 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.0960	0.0908	0.0876	0.0306	1.0365	2.8627	2.9673
0.1920	0.2387	0.2175	0.0693	1.0975	3.1385	3.4444
0.2881	0.5069	0.4270	0.1498	1.1871	2.8505	3.3838
0.3841	0.7879	0.6684	0.2919	1.1788	2.2898	2.6992
0.4801	0.9500	0.8539	0.5087	1.1125	1.6886	1.8675

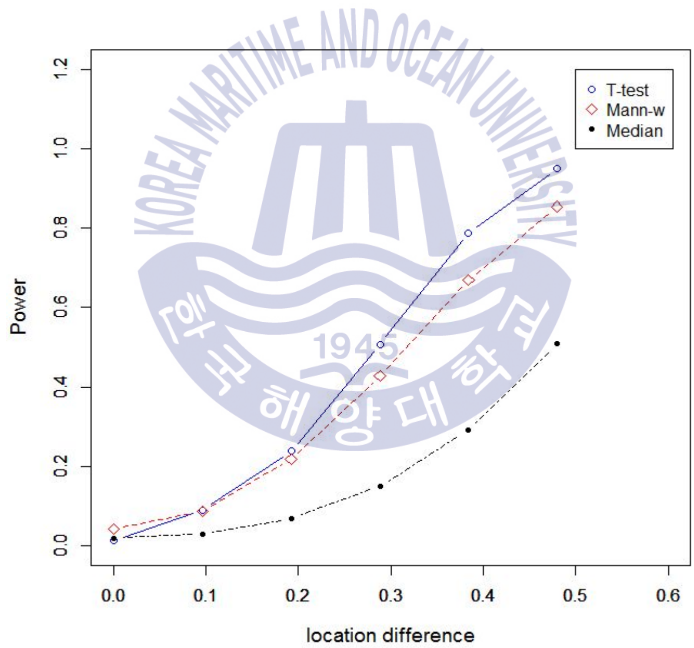

Fig. 20 Uniform distribution (n=25)

Table 21. Estimated Powers of the t-test, Mann-Whitney test and the median test for $\text{Exp}(1)$ with equal samples of size 10 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.4085	0.0787	0.0912	0.0470	0.8629	1.9404	1.6745
0.8169	0.1731	0.1803	0.0924	0.9601	1.9513	1.8734
1.2254	0.2659	0.2820	0.1464	0.9429	1.9262	1.8163
1.6338	0.3743	0.3899	0.2153	0.9600	1.8110	1.7385
2.0423	0.4605	0.4801	0.2761	0.9592	1.7389	1.6679

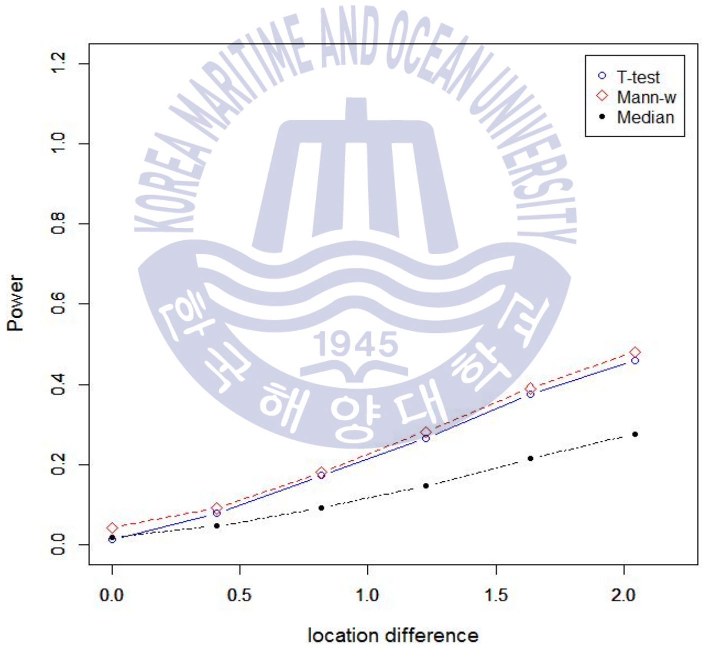

Fig. 20 Exponential distribution (n=10)

Table 22. Estimated Powers of the t-test, Mann-Whitney test and the median test for Exp(1) with equal samples of size 15 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.4085	0.1190	0.1151	0.0620	1.0339	1.8565	1.9194
0.8169	0.2764	0.2560	0.1372	1.0797	1.8659	2.0146
1.2254	0.4674	0.4236	0.2381	1.1034	1.7791	1.9630
1.6338	0.6128	0.5570	0.3429	1.1002	1.6244	1.7871
2.0423	0.7297	0.6787	0.4400	1.0751	1.5425	1.6584

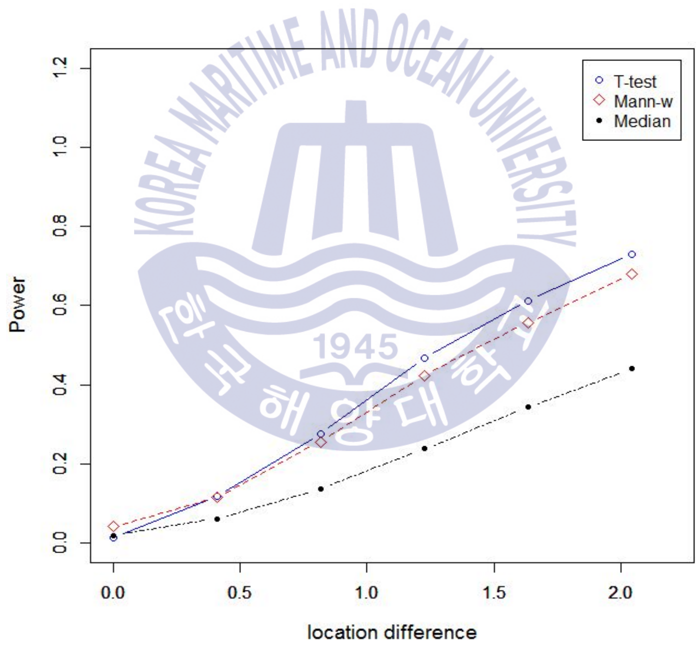

Fig. 22 Exponential distribution (n=15)

Table 23. Estimated Powers of the t-test, Mann-Whitney test and the median test for $\mathrm{Exp}(1)$ with equal samples of size 20 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.4085	0.1562	0.1470	0.0699	1.0626	2.1030	2.2346
0.8169	0.3950	0.3511	0.1740	1.1250	2.0178	2.2701
1.2254	0.6244	0.5525	0.3148	1.1301	1.7551	1.9835
1.6338	0.7852	0.7142	0.4533	1.0994	1.5756	1.7322
2.0423	0.8800	0.8219	0.5694	1.0707	1.4434	1.5455

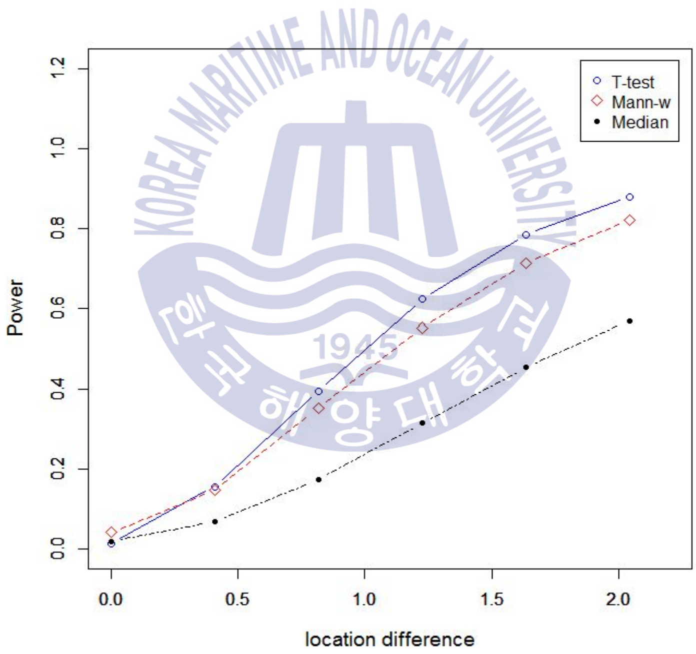

Fig. 23 Exponential distribution (n=20)

Table 24. Estimated Powers of the t-test, Mann-Whitney test and the median test for $\mathrm{Exp}(1)$ with equal samples of size 25 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.4085	0.1953	0.1763	0.0771	1.1078	2.2866	2.5331
0.8169	0.4831	0.4159	0.2005	1.1616	2.0743	2.4095
1.2254	0.7371	0.6494	0.3708	1.1350	1.7153	1.9879
1.6338	0.8840	0.8113	0.5249	1.0896	1.5456	1.6841
2.0423	0.9500	0.8994	0.6582	1.0563	1.3665	1.4433

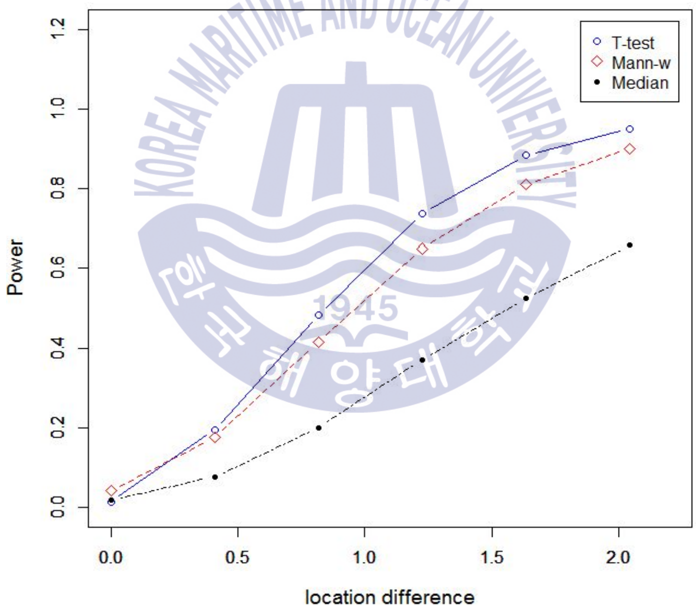

Fig. 24 Exponential distribution (n=25)

Table 25. Estimated Powers of the t-test, Mann-Whitney test and the median test for $Cauchy(0)$ with equal samples of size 10 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.4503	0.0268	0.0740	0.0518	0.3622	1.4286	0.5174
0.9006	0.0538	0.1610	0.1286	0.3342	1.2519	0.4184
1.3508	0.0960	0.2770	0.2516	0.3466	1.1010	0.3816
1.8011	0.1561	0.4219	0.4053	0.3700	1.0410	0.3851
2.2514	0.2065	0.5337	0.5390	0.3869	0.9902	0.3831

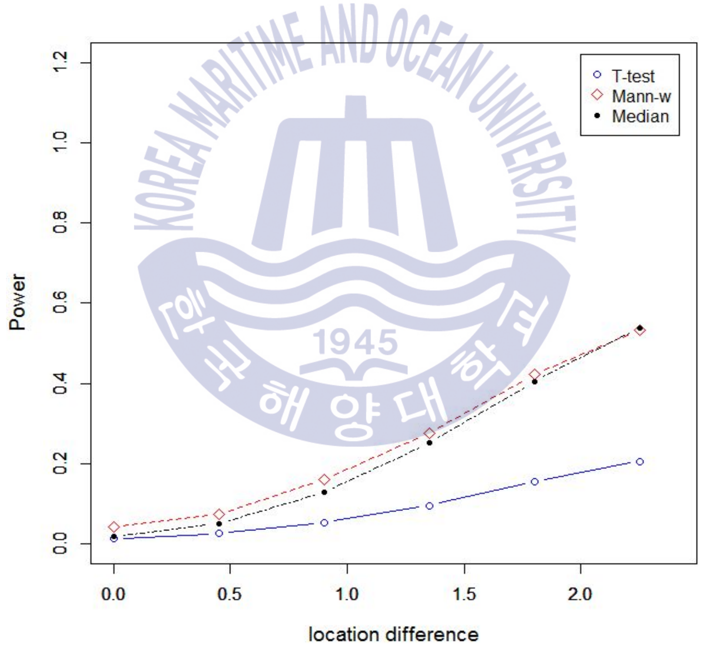

Fig. 25 Cauchy distribution (n=10)

Table 26. Estimated Powers of the t-test, Mann-Whitney test and the median test for $Cauchy(0)$ with equal samples of size 15 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.4503	0.0269	0.0924	0.0688	0.2911	1.3430	0.3910
0.9006	0.0596	0.2311	0.2123	0.2579	1.0886	0.2807
1.3508	0.1018	0.4197	0.4268	0.2426	0.9834	0.2385
1.8011	0.1673	0.6025	0.6334	0.2777	0.9512	0.2641
2.2514	0.2221	0.7249	0.7710	0.3064	0.9402	0.2881

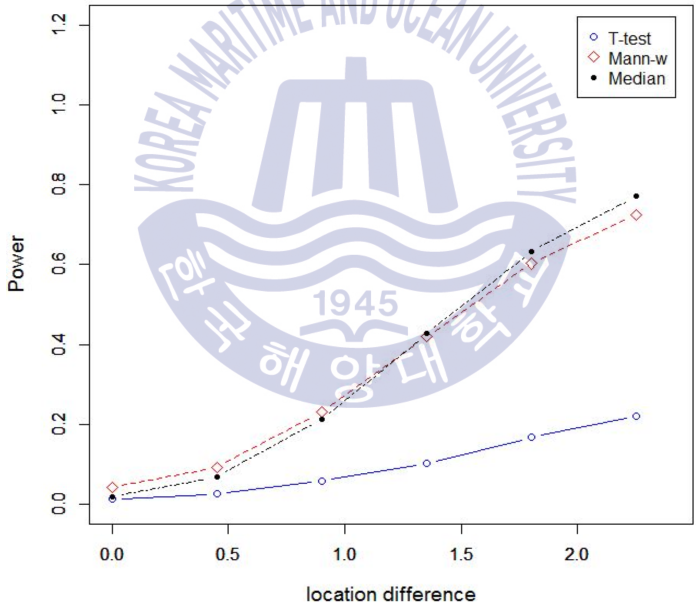

Fig. 26 Cauchy distribution (n=15)

Table 27. Estimated Powers of the t-test, Mann-Whitney test and the median test for $Cauchy(0)$ with equal samples of size 20 and $\alpha = 0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.4503	0.0283	0.1112	0.0835	0.2545	1.3317	0.3389
0.9006	0.0586	0.3068	0.2803	0.1910	1.0945	0.2091
1.3508	0.1074	0.5477	0.5525	0.1961	0.9913	0.1944
1.8011	0.1640	0.7404	0.7703	0.2215	0.9612	0.2129
2.2514	0.2299	0.8516	0.8880	0.2700	0.9590	0.2589

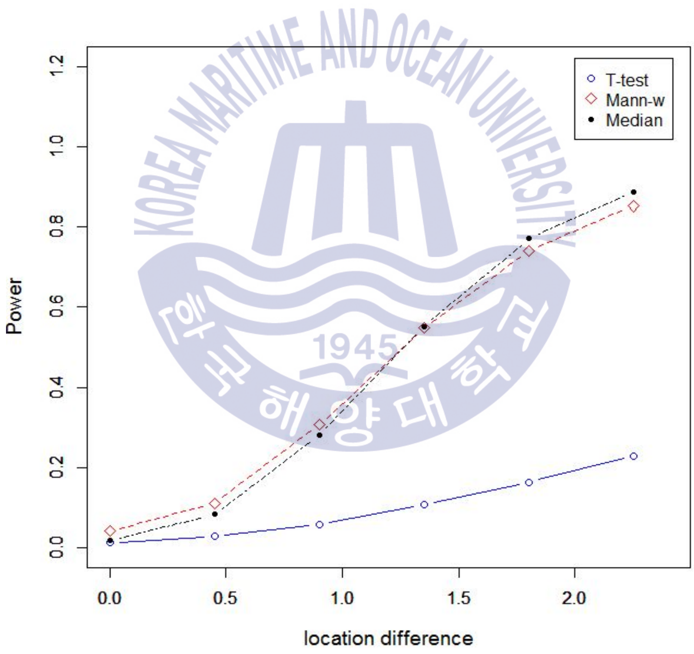

Fig. 27 Cauchy distribution (n=20)

Table 28. Estimated Powers of the t-test, Mann-Whitney test and the median test for $Cauchy(0)$ with equal samples of size 25 and $\alpha=0.05$.

Location Difference Between Pop 1 and Pop 2	Estimated Powers					
	t-test	Mann-W	Median	Power Ratio [(2)/(3)]	Power Ratio [(3)/(4)]	Power Ratio [(2)/(4)]
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.0000	0.0136	0.0420	0.0191	0.3238	2.1990	0.7120
0.4503	0.0317	0.1339	0.0930	0.2367	1.4398	0.3409
0.9006	0.0603	0.3723	0.3336	0.1620	1.1160	0.1808
1.3508	0.1151	0.6323	0.6409	0.1820	0.9866	0.1796
1.8011	0.1693	0.8274	0.8594	0.2046	0.9628	0.1970
2.2514	0.2366	0.9221	0.9500	0.2566	0.9706	0.2491

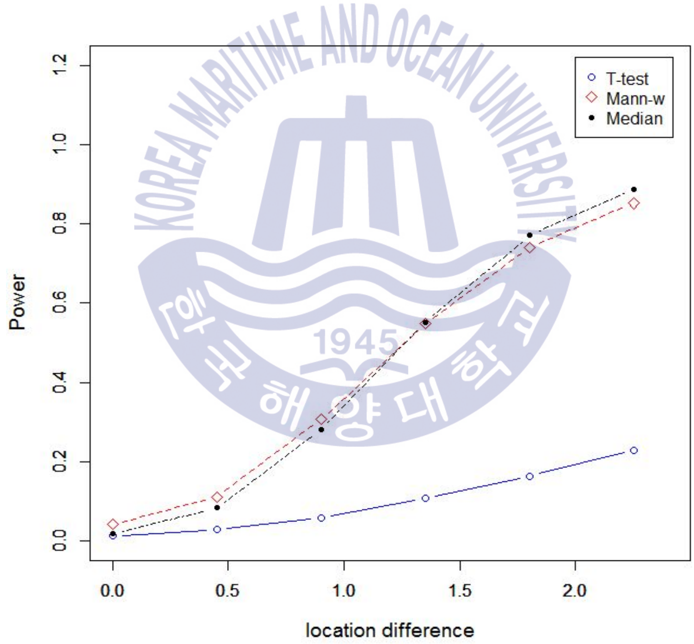

Fig. 28 Cauchy distribution (n=25)

B. Sample of program

```
n.generate<-function(p){x<-rnorm(p[1],p[2],p[3])}
u.generate<-function(p){x<-runif(p[1],p[2],p[3])}
e.generate<-function(p){x<-rexp(p[1],p[2])}
l.generate<-function(p){x<-rlnorm(p[1],p[2],p[3])}
c.generate<-function(p){x<-rcauchy(p[1],p[2])}
mix.generate<-function(p){
        n1<-round(p[1]*p[2],0)
        n2<-p[1]-n1
        x<-rnorm(n1,p[3],p[4])
        x<-c(x,rnorm(n2,p[5],p[6]))  }

t<-function(x,y){
    z<-c(x,y)
    g <- rep(1:2, c(length(x),length(y)))
    t.test(z~g)$p.value  }

w<-function(x,y){
    z<-c(x,y)
    g <- rep(1:2, c(length(x),length(y)))
    wilcox.test(z~g)$p.value  }

m<-function(x,y){
    z<-c(x,y)
    g <- rep(1:2, c(length(x),length(y)))
    me<-median(z)
    fisher.test(z<me,g)$p.value  }

n<-10
set.seed(1)
u<-runif(1,0,6)
for(i in 1:5){
        p1<-c(n,0,1)
```

```
        p2<-c(n,(u/5)*i,1)                      ## normal distribution

        p1<-c(n,0.75,0,1,3,6)
        p2<-c(n,0.75,(u/5)*i,1,3+(u/5)*i,6)     ## mixed normal distribution I

        p1<-c(n,0.80,0,1,2,5)
        p2<-c(n,0.80,(u/5)*i,1,2+(u/5)*i,5)    ## mixed normal distribution II

        p1<-c(n,0,1)
        p2<-c(n,(u/5)*i,1)                     ## Log-normal distribution

        p1<-c(n,1)
        p2<-c(n,1+(u/5)*i)                     ## exponential distribution

        p1<-c(n,0)
        p2<-c(n,(u/5)*i)                       ## Cauchy distribution

c<-c(0,0,0)
for(i in 1:10000){
                x<-n.generate(p1)
                y<-n.generate(p2)               ## normal distribution

                x<-mix.generate(p1)
                y<-mix.generate(p2)             ## mixed normal distribution I, II

                x<-l.generate(p1)
                y<-l.generate(p2)               ## Log-normal distribution

                p1<-c(n,0,1)
                p2<-c(n,(u/5)*i,1)              ## exponential distribution

                p1<-c(n,1)
                p2<-c(n,1+(u/5)*i)              ## Cauchy distribution

                if(t(x,y)<0.05){c[1]<-c[1]+1}
```

```
                    if(w(x,y)<0.05){c[2]<-c[2]+1}
                    if(m(x,y)<0.05){c[3]<-c[3]+1}   }
print(c[1]/10000)
print(c[2]/10000)
print(c[3]/10000)   }
```

※ 그래프

```
x1<-c(0.0000,0.2101,0.4203,0.6304,0.8406,1.0507)        ## Location Difference
t1<-c(0.0506,0.0725,0.1436,0.2596,0.4212,0.5987)        ## t-test L.D.
w1<-c(0.0455,0.0666,0.1295,0.2324,0.3849,0.5549)        ## Mann-Whitney test L.D.
m1<-c(0.0219,0.0351,0.0651,0.1195,0.2102,0.3324)        ## median test L.D.

plot(x1,t1, xlim=c(0,1.3), ylim=c(0,1.2), xlab="location difference", ylab="Power",
        cex.lab=1.2, main="Normal distribution(n=10)", type="b", col="blue",
        pch=1, lty=1)
par(new=TRUE)
plot(x1,w1, xlim=c(0,1.3), ylim=c(0,1.2), xlab="", ylab="", main="", type="b",
        col="red", pch=5,lty=2)
par(new=TRUE)
plot(x1,m1, xlim=c(0,1.3), ylim=c(0,1.2), xlab="", ylab="", main="", type="b",
        col="black", pch=20, lty=4)
par(new=TRUE)
legend(x=1.09, y=1.22, c("T-test", "Mann-w", "Median"), pch=c(1,5,20),
        col=c("blue", "red","black"))
```

감사의 글

졸업논문을 마무리하고 나니 지난 대학원 생활들이 머릿속에 스쳐지나갑니다. 되돌아보면 2년이란 시간동안 놓친 것, 후회되는 일이 많은 나날들이였지만 그만큼 더 성장할 수 있었던 시간이었습니다. 제가 이 자리에 오기까지 많은 분들의 도움과 격려가 있었기에 가능하다고 생각합니다.

먼저 석사 과정 2년 동안 연구에 매진할 수 있도록 아낌없는 격려와 지도를 해주신 박찬근 교수님께 진심으로 존경과 감사의 마음을 올립니다. 학문적인 깨우침뿐만 아니라 인생 선배로써 조언을 아끼지 않으시고 배움에 대해 노력하는 자세와 자긍심을 일깨워 주셨습니다. 또한, 부족한 저의 논문의 심사를 맡아주시고, 충고와 조언을 해주신 김재환 교수님과 장길웅 교수님께도 감사드립니다.

그리고 학부와 대학원 생활을 하는 동안 좋은 말씀으로 격려해주신 배재국 교수님, 홍정희 교수님, 김익성 교수님, 손미정 교수님께도 감사드립니다. 또한, 세심한 배려로 항상 지켜봐주시고 많은 도움을 주신 정지영 조교선생님께도 감사의 말씀 전합니다. 2년 동안 같은 연구실 생활을 하면서 배움에 대한 열정과 의지를 함께 해준 연구실 식구들에게도 고맙다는 말 꼭 전하고 싶습니다.

끝으로 저를 끝까지 믿고 많은 선택의 기회를 주신 부모님께 정말 감사드리며 이 논문을 바칩니다. 사랑합니다.