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水管內에 長方形 障害物의 F.A.M解析에 關한 硏究

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Analysis of Flow in a Horizontal Channel with a Rectangular Block Using Finite Analytic Method

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요 약

많은 유체 유동현상중, 空洞을 지나는 흐름은 유체유동을 수치적으로 해석하고자 하는 연구자들의 관심의 대상으로, 이는 수치계산 기법의 적합성 여부 및 응용성을 검증하는 목적으로서 널리 이용되어 왔다. 이러한 空洞 공간상에서의 유동특성을 본 논문에서는 Finite Analytic Method를 이용하여 2차원 사각형 공동을 지나는 laminar channel flow에 대해 계산하여 해석하였다.

Finite Analytic Method는 풀고자 하는 문제 영역의 전 영역을 많은 작은 격자요소로 나누고 이들 작은 격자요소에 대한 부분해석 해를 구한 다음 이 부분해를 활용하여 한 격자점과 그 격자점을 둘러싸고 있는 격자계에 관한 대수방정식을 만들고 이 대수방정식을 풀므로써 전 영역에서의 해를 구해가는 수치계산 법이다. 본 연구에서는 공동내부의 x, y방향의 유속을 Reynolds number 2×10^3의 영역에 이르기까지 계산을 수행하였으며 또한, 전 구간에 걸친 압력분포 및 질량보존량을 구하였다. 계산과정에서 F.A.M은 해의 수렴속도가 빠르면서도 안정성에서도 우수함이 증명되었다. 수치계산상, Reynolds number의 크기와 밀접한 관계가 있는 시간증분 τ는 격자계수 300개 일때 Rn가 400에서는 0.5로 하였으며, Reynolds number에 따른 유장변화는 층류 범위내에서는 渦(와)의 중심이 Rn가 증가함에 따라 점차 앞으로 이동함을 알 수 있었다.

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Abstract

In the present study, Navier-Stokes equation is numerically solved by use of Finite Analytic Method to investigate the flow phenomena behind a rectangular obstacle in a horizontal channel.

The basic idea of F.A.M. is the incorporation of local analytic solutions in the numerical solution of linear or non-linear partial differential equations. In the F.A.M., the total problem is subdivided into a number of small elements. The local analytic solution is obtained for the small element in which the governing equation, if non-linear, to be linearized.

The local analytic solutions are then expressed in algebraic form and are overlapped to cover the entire region of the problem. The assembly of these local analytic solutions, which still preserve the overall non-linearity of the governing equations, result in a system of linear algebraic equations is then solved to provide the numerical solutions of the total problem.

The computed flow field shows the same characteristics as physical concept of flow phenomena.

Introduction

Among many flow phenomena, flow in channel, as well in cavity with and tinfinite medium has been the research theme of many researchers who are interested in numerical computation of fluid flow. In their studies with channel or cavity flow, the suitability and applicability of the newly proposed numerical method are examined.

In this paper, computational results of flow behind rectangular obstacle by F.A.M. and other numerical method are presented.

Streamlines and pressure contour obtaind from their computations agree well each other and with physical concept of view.

The above investigation were conducted for Reynolds number between 100 and 2×10^3 and effect of thermal buoyancy force on flow structure was not taken into account.

The purpose of present work is the demonstration on numerical computation by Finite Analytic Method for the flow in a two-dimensional horizontal channel partially enclosing a rectangular body.

The purpose of present work is the demonstration on numerical computation by Finite Analytic Method for the flow in a two-dimensional horizontal channel partially enclosing a rectangular body.

The two objectives of this study ard to verify the stability and convergence of CFD code F.A.M. and its applicability to a computation of fluid flow.

Principle of Finite Analytic Method

To illustrate the basic principle of F.A.M., A partial differential convective transport equation for unsteady two-dimensional flow.

$$L(\phi)=F$$

is considered as an example, where the operator L can be linear or non-linear, and F is an inhomogeneous source term. Let x, y and t be the independent variables in plane and time respectively. The PDE is to be solved in the region R shown Fig.1.

Let the boundary and initial conditions be specified so that problem is well-posed.

Nomenclature

A length of the rectangular block
a neighbour coefficient
B width of the rectangular block
b defined by equation(0)
D divergence of velocity vectors, eq.(1)
F inhomogeneous source term
f higher order correction term
G defined by equation(0)
H channel width
L differentiation operator
n iterations in time step
P pressure
Re Reynolds number($U_{av}B/\nu$)
t time
U constant velocity in the local element
U_0 inlet uniform velocity
u axial component of velocity
v normal component of velocity
x axial dimension of coordinates
y normal dimension of coordinates
ν kinematic viscosity of the fluid
τ nondimensional time, time increment

Superscripts

^ pseudo value
* guessed value
′ corrected value

Eq.0)

$$\phi_p=\frac{1}{G^{\phi}+Re/\tau b_p^{\phi}}$$

$$\left(b_{NW}\phi_{NW}+b_{NE}\phi_{NE}+\cdots\cdots+\frac{Re}{\tau}b_p^{\phi}\phi_p^{n-1}-b_p^{\phi}f_p^{\phi}\right)$$

$$b_{NW}=\bar{s}C_{NW}+\bar{s}(t-1)C_{SW}$$

$$\bar{s}=\frac{h_w h_w(e^{2Ah_E}+e^{-2Ah_E}-2)}{h_E\ h_w(e^{2Ah_E}-1)+h_E(e^{-2Ah_E}-1)+h_E(e^{-2Ah_E}-1)}$$

$$G=1-(2-S-\bar{S})C_{WC}-(2-t-\bar{t})C_{SC}-(2-S-\bar{S})(2-t-\bar{t})C_{SW}$$

C F.A. coefficient

Subscripts

nb neighbour points
p nodal point
E East, West, North, South(W, N, S)
e east, west, north, south(w, n, s)
x derivative(y, xx, yy, t)

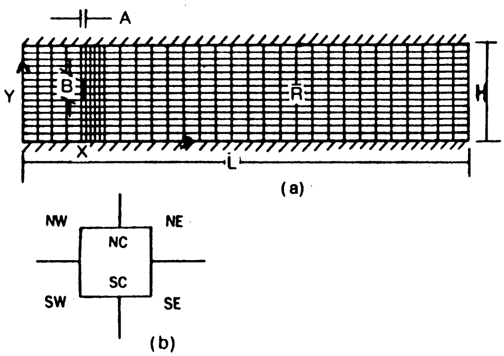

Fig. 1. (a) Obstacle in the computation domain R :
(b) Determination of the velocity and pre ssure locations on a staggered grid.

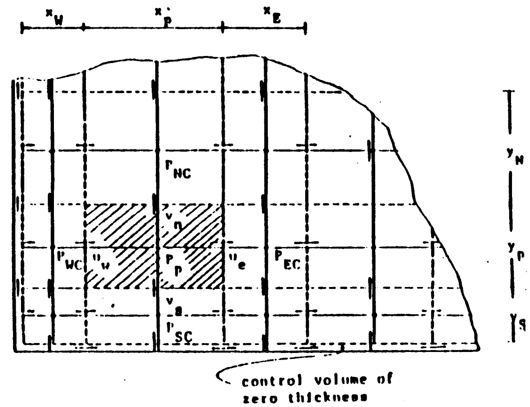

Fig. 2. Staggered grid coordinate system " · " ; pressure, "→" ; u "↑" ; v

In order to solve the problem with the F.A. method, the whole region of the problem is broken up into a number of small element where analytic solutions can be obtained.

A typical local element with the nodal point P(i, j, n) may be surround by neighbouring 8 points and those of previous time steps.

Once the region R has been subdivided into small rectangular elements, the analytic solution in each local element may be obtained if the boundary and initial conditions for that element are properly specified. In the case when the PDE is non-linear, the non-linear equation may be locally linearized in the small element. In this fashion, the overall non-linear effect can still be approximately presented by the assembling of local analytic solutions which constitute the numerical solution of the governing PDE over the whole region R.

Let $L_1(\phi)=F_1$ be governing equation of $L(\phi)=F$ in a small element, so that an analytic solution can be obtained for the local element as a function of the boundary and initial conditions. i.e., $\phi = f(f_E(y, t), f_W(y, t), f_N(x, t), f_S(x, t), f_I(x, y), h_E, h_W, h_N, h_S, \tau, x, y, t, F)$

where f_I is the initial condition and f_E, f_W, f_N and f_S are the east, west, north and south boundary conditions, respectively.

h_E, h_W, h_N, h_S and τ are respectively the grid sizes in x, y direction and the step size in time domain. For numerical purposes, the boundary and initial conditions may be approximately expressed in terms of the nodal values along the boundary.

Statement of the Problem

The system of interest is a horizontal channel. An obstacle in the form of rectangular block is placed inside it(Fig. 1a).

The dimensionless equations for continuity, momentum may be expressed in the following form

(1) Equation of continuity

$$D = u_x + v_y = 0 \tag{1}$$

(2) Momentum(Navier-Stokes) equations

$$u_t - uu_x + vu_y = -p_x + 1/Re(u_{xx} + u_{yy}) \tag{2}$$

$$v_t - uv_x + vv_y = -p_y + 1/Re(v_{xx} + v_{yy}) \tag{3}$$

No slip boundary conditions for velocities on solid walls are used. At the channel inlet, a normal component of velocity is assumed to be zero, and a equally distributed profile for the axial velocity is deployed.

There is no particlular difficulty in solving momentum equations by F.A. method. However, as well known, the real difficulty in obtaining the velocity field in primitive variable formulation lies in the unknown pressure field.

The pressure field influences the velocity field through the pressure gradient terms in momentum equations. If the velocity component u and v are thought to be governed by the two momentum equations (2) and (3), then the pressure field should be, though indirectly, specified by the equation of continuity. In order to extract the pressure from the equation of continuity, a Poisson equation for pressure are derived. By taking the divergence of Navier−Stokes equations, a Poisson equation can be expressed.

$$p_{xx} + p_{yy} = 2(u_x v_y + u^{x}_{2} + v^{y}_{2}) + \frac{1}{Re}(D_{xx} + D_{yy}) - (uD_x + vD_y) \tag{4}$$

here, D=0 from the equation of continuity hence, equation (4) reduced to

$$p_{xx} - p_{yy} = 2(u_x v_y - v_x u_y) \tag{5}$$

Thus, we can solve the Poisson equaton (5) and two momentum equations (2) and (3) for the variables u, v and p.

Several methods of handling the pressure-velocity coupling problems used the velocity correction formulas and pressure correction equations to extract the pressure from the equation of continuity was proposed. Among them, the Pressure-Update-Patankar(PUP) scheme combined with Patanker-Spalding's P′ equation is known to give the best result.

In PUP scheme, a pseudo-velocity field obtained by omitting the pressure gradient term in Navier-Stokes equation is introduced so that the pressure field can be obtained directly from a guessed velocity field. In the present study, a staggered grid for velocity component is adopted to avoid the possible unrealistic pressure and velocity field result from the finite difference representation and also the equation of continuity.

Fig.2 shows the locations of staggered grid formation for u, v and p in x-y plane. The dashed lines represent the control volume faces and 'e', 'n' respectively denoting east, north nodes for velocity component ; u_e, v_n, In such a staggered grid system, 10-point F.A. formula for unsteady two dimensional momentum equation in x-direction becomes.

$$u_e = \frac{1}{G_u + Re/\tau b_e^u} \left[\sum_1^8 b_{nb}^u u_{nb} + \frac{Re}{\tau} b_e^u u_e^{n-1} - b_e^u (Re p_x + f_e^u) \right] \tag{6}$$

where the pressure gradient term p_x is approximated by

$$p_x = \frac{p_{EC} - p_P}{0.5(\Delta x_E + \Delta x_P)} \tag{6-a}$$

and the higher order correction term

$$f_e^u = Re[(u' u)_x + (v' u)_y]_e \tag{6-b}$$

is a representative constant value evaluated as

$$Re\left[\frac{(u' u)_e - (u' u)_w}{0.5(\Delta x_e + \Delta x_w)} + \frac{(v' u)_n - (v' u)_s}{0.5(\Delta y_n + \Delta y_s)}\right] \tag{6-c}$$

where, u′ and v′ are deviations of velocities in small element with respect to U and V i.e., $u' = u - U$.

In order to resolve the pressure-velocity coupling problem described before, pseudo-velocity field for u_e based on eq. (5) is introduced,

$$u_e = \frac{1}{G_u + Re/\tau b_e^u} \left[\sum_1^8 b_{nb}^u u_{nb} + \frac{Re}{\tau} b_e^u u_e^{n-1} - b_e^u (Re p_x + f_e^u) \right] \tag{7}$$

Eq. (7) defines the pseudo-velocity and is essentially eq. (6) without pressure. Therefore, the discretized momentum eq.(6) can be written as

$$u^e = u^e - d^e(P_{EC} - P_P)$$

$$d_e = \frac{Re\ b_e^u}{0.5(\Delta x_E + \Delta x_P)\ (G^u + Re/\tau\ b_e^u)} \tag{8-a}$$

here, b_{nb}^u, b_e^u and G^u are the FA coefficients.

Similarly, v_n can be written as

$$v_n = \hat{v}_n - d_n(p_{NC} - p_P) \tag{8-b}$$

The moment eq.(8-a) and (8-b) can be solved for u and v as long as the pressure field is some-what estimated. However, unless the correct pressure field is employed, the resulting velocity fiedl u_e, v_n etc. will not satisfy the equation of continuity. Let the imperfect velocity field based on a guessed pressure field p^* be u^* and v^*

$$u_e^* = u_e^* - d_e(p_{EC}^* - p_P^*) \tag{9-a}$$

$$v_n^* = v_n^* - d_n(p_{NC}^* - p_P^*) \tag{9-b}$$

By substracting eq. (8) from eq.(9), the velocity-correction formulas relating the velocity-corrections, $u_e - u_e^*$ and $v_n - v_n^*$ to the pressure-correction $p' = p - p^*$ can be obtained as follows

$$u_e - u_e^* = (\hat{u}_e - \hat{u}_e^*) - d_e(p'_{EC} + p'_P) \tag{10-a}$$

$$v_n - v_n^* = (\hat{v}_n - \hat{v}_n^*) - d_n(p'_{NC} + p'_P) \tag{10-b}$$

If we require the velocity field to satisfy the discretized equation of continuity of the form of

$$D = u_x + v_y = 0$$

$$D = \frac{u_e - u_w}{\Delta x_P} + \frac{v_n - v_s}{\Delta y_P} = 0 \tag{11}$$

an equation for p' in terms of u^*, v^* can be derived.

Since the velocity and pressure correction formulas become trival in the final converged solution where both the velocity and pressure corrections, i.e., $u - u^*$, $v - v^*$ and $p' = p - p^*$ are exactly zero, the pressure correction equation for p' can be considered as an intermediate algorithm that leads to correct pressure field p and have no direct effect on final solution. Thus, it is possible to simplify or to omit the part of the velocity-correction in eq.(10-a) and (10-b).

So that a simpler pressure correction formula for p' can be obtained. The final converged solution should not depend on the approximation made on velocity and pressure correction formulas during inter mediate calculations, although the rate of convergence will depend on the approximate formulation of p'.The simplest approximation as that used in SIMPLE or SIMPLER algorithm, is to omit the indirect influence $\hat{u}-\hat{u}^*$, $\hat{v}-\hat{v}^*$, in eq.(10-a) and (10-b), such that the velocity-corrections can be expressed explicitly in terms of the pressure correction p′ as,

$$u_e - u_e^* = -d_e(p'_{EC} - p'_P) \tag{12-a}$$

$$v_n - v_n^* = -d_n(p'_{NC} - p'_P) \tag{12-b}$$

If we require the approximate velocity correction formulas(12-a) and (12-b) to satisfy the discretized equation of continuity(11), then a Poisson equation for pressure-correction p′ can be derived.

$$a_P p'_P = a_e p'_{EC} + a_w p'_{WC} + a_n p'_{nC} + a_s p'_{SC} - D^* \tag{13}$$

were,

$$a_e = \frac{d_e}{\Delta x_P},\ a_w = \frac{d_w}{\Delta x_P},\ a_n = \frac{d_n}{\Delta y_P},\ a_s = \frac{d_s}{\Delta y_P} \tag{13-a}$$

$$a_P = a_e + a_w + a_n + a_s \tag{13-b}$$

$$D^* = \frac{u_e - u_w}{\Delta x_P} + \frac{v_n - v_s}{\Delta y_P} \tag{13-c}$$

After obtaining the pressure-correction p′ form eq.(13), we may update the pressure by letting

$$p = p^* + \alpha p' \tag{14}$$

with a relaxation parameter α. In the SIMPLER algorithm, due to the approximation made on velocity-correction formulas, many iteraions are needed to obtain a converged solution even if the correct pressure field p* is used as an initial guess.

Since the pressure-correction equation does a fair job in correcting velocities, but a rather poor job in updating the pressure itself, as an alternative of SIMPLE, the SIMPLER algorithm in adopted to update the pressure field. By requiring eq. (8-a) and (8-b) to satisfy the discritized equation of continuity(11), we obtain a Poisson equation for pressure as

$$a_P p_P = a_e p_{EC} + a_w p_{WC} + a_n p_{NC} + a_s p_{SC} - \hat{D} \tag{15}$$

where

$$\hat{D} = \frac{u_e - u_w}{\Delta x_P} + \frac{v_n - v_s}{\Delta y_P} \tag{15-a)'$$

and a_P, a_e, a_w, a_s and a_n are defined in eq.(13)

In eq.(15), there is no approximation. Thus, if a corrected velocity field is employed as an initial guess, eq.(15) would at once give the correct pressure field. The pressure-corrrection(13) is used to correct only the velocity field. For unsteady fluid flow problems, the flow field is required to satisfy the equation of continuity at each time step.

For the problems where the initial and boundary conditions are properly specified, the transient numerical soulutions can be obtained in the following manner.

(1) Discretize the domain of calculation into suitable number of small local elements.

(2) Specify the initial condition for velocity field.

(3) The velocity field at $(n-1)^{th}$ time step is employed as an initial guess for the velocity field at n^{th} time step.

(4) Calculate the FA coefficients b^u_{nb}, b^v_{nb}, and also the FA coefficients for pressure and pressure-correction equations, i.e., a_e, a_w etc.

(5) Calculate the higher order correction terms f^u and f^v, if needed.

(6) Calculate the pseudovelocities $\hat{u}$, $\hat{v}$ from eq.(7) etc, in terms of velocity field of $(n-1)^{th}$ time step.

(7) Calculate $\hat{D}$ frodm eq.(15-a) and solve the pressure eq.(15) by tridiagonal algorithm to obtain the pressure field ; p.

(8) Treating this pressure field, p, as guessed pressure field p^*, solve the momentum eq. (9-a) and (9-b) to obtain the starred velocity field u^* and v^*. The system of algebraic equations is solved by tridiagonal algorithm.

(9) Calculate te mass source term D^* in eq.(13-c), and hence solve the pressure-correction eq. (13) to obtain p'. The system of algebraic equations is solved by the tridiagonal algorithm also.

(10) Correct the velocity field using velocity-correction formulas (12-a) and (12-b), but do not correct the pressure. The velocity field u and v thus obtained should satisfy the equation of continuity (11).

(11) Return to step (4) and repeat the steps (4) to (10) until convergence criterion is achivevd. i.e., $\max |D^*_{ij}| < \varepsilon$.

(12) Stop if the steady-state criterion is achieved (i. e., $\max |u^n_{ij} - u^{n-1}_{ij}| < \varepsilon$ etc.) or the time t exceeds the maximum time period assigned.

(13) If not, return to step (3) for $(n+1)^{th}$ time step calculation.

In studing two-dimensional fluid flow problems, F.A.formula based on exponential and linear boundary approximations is employed to solve the flow behind an obstacle in channel where comparison with other numerical calculation is available.

The channel flow is a flow where the fluid is set into motion by the viscous shear and viscosity, and non-linear convection affect the flow region.

The simplicity of geometry of channel and block is chosen for the purpose of testing new numerical schemes.

This simple configuration has the advantage that the basic phisics regarding vortex street will be maintained, but at the same time the numerical result will not be too complicated to interpret and understand.

For the comparison with the results obtained by other CFD code, computation have been carried out with one of the application package program NISA for the 2-D channel of length L/H=5.

A calculation domain of 1.0L×0.21L is used to simulate the infinite extent of the whole region. The rectangular block or 0.01L is located at a distance 0.125L from inlet. The aspect ratio(A/B) of the block is 1/6, and the blockage ration of channel (B/H) is 0.2857.

A non-uniform grid of 0.03125(4 nodes), 0.010(5nodes) and 0.0337(24 nodes) is used in the x-direction, while uniform grid of 0.015 is used in the y-direction. In the present calculation of F.A. solution, the channel flows for Reynolds number between 100 and 2000 are solved in a staggered grid coordinate system using non-uniform meshes ranging from 18×9 to 34×15.

The Reynolds number is defned by U_oB/ν, where U_o is the uniform oncoming velocity. In order to apply the no-slip boundary conditions exactly on the boundaries, control volume of zero thickness are chosen along the wall (see Fig.2). The 10-point F.A. formula for unsteady two-dimensional convective transport equations based on non-uniform grid local element is employed to descretize the momentum equations.

A pseudo-velocity field described in eq. (7) is then introduced so that the pressure field can be obtained via the euation of continuity. The guessed flow field is corrected by velocity-correction formulars (10-a) and (10-b) to obtain a continuity-satisfied flow field. In all calcuations, zero initial velocity field is specified at the beginning.

From the experience obtained in solving channel flow problems, boundary condition derived from impermeable and no-ship condition is used along the boundary of the rectangular block and two walls.

For all calculations, uniformly distributed oncoming velocity u=1.0, v=0.0 is specified at the entrance. A time increment of 1.0 and 0.1 is used throughout the time steps for Re=1,000 and 150 respectively. The convergence criterion is made on mass conservation, and specified as m.c$<10^{-5}$ at each time step for all field points between two internal iterations. From the present F.A. calculation, unsteady separation flows behind the rectangular block are predicted. It should be remarked that the boundary conditions posed for the problem do not stipulate the symmetry conditions at the plane of geometric symmetry, therefore the prediction does produce the asymmetric flow pattern such as vortex shedding phenomenon.

It is found that the flow pattern at the initial stage of flow is symmetric, however, the flow pattern becomes asymmetric and oscillatory. For example, at time step 2 (Re=1,000) the separation bulb appear and begins to show asymmetric pattern and eventually the F.A. calculation predicts vortex shedding.

The fluid in the wake of obstacle moves up and turns back at some distance aft, creating any number of separated zones and dead-water zone, depending on the relative magnitude of the Reynolds number. In the separation zones, some flow particles advance backward. The mass conservation then dictates an increased fluid velocity outside the zones. The recirculating wake extends a full block width in the downstream, but it is steady (nearly symmetrical), and vortex shedding has not yet started. At the rear of the block the flow separates due to sudden enlargement of flow area, where some fluid particles reattache with small velocity gradient. However, most of them fail to reattach giving rise to eddies behind obstacle. When the effects of seperation zones come into play, the wake loses its original symmetry. Oscillations in the wake grow in magnitude. While the eddies arised at the corner of obstacle interact with vortex street and vortex shedding is carried downstream.

In the subsequent time of calculation, this vorticity convected downstream while simultaneously diffused away. In case of Reynold number=1,000, the streamlines and pressure contour show nearly the same pattern over the timestep 40 where the steady static pattern is observed. For this case, the velocity profiles at different axial locations in the channel are nearly symmetrical about channel axis. Thus, the stability of the finite analytic transient numerical solution is demonstrated. With different Reynolds number and time increment, the resultant velocity and pressure contours are qualitatively similar each other. However, at a large value of Reynold number, vortex performming becomes less and less prominent and may be considered uniform flow far be hind obstacle. For both Reynolds number, the vorticity is suppressed far downstream of the obstacle by the side walls and flow at channel exit again tends to become steady parabolic.

Conclusions

The 10-point F.A. formular for unsteady two-dimensional convective transportation is employed to study the vortex shedding processes behind a rectangular block. Transient solutions are given for Re=150 and 1,000.

Streamlines, pressure contours and flow vector profiles are plotted in the figures. The results agree well with those obtained in other CFD code or experiments. In the test problems considered, including cavity flow problem (References, 10), the finite analytic numerical solutions are shown to be accurate and stable.

Significant results are summerized in the following.

(1) In unsteady two-dimensional case, the F.A. solution gives all positive F.A. coefficients which means reasonal and requires less computation time. Thus, the extension to unsteady three -dimensional case becomes practical.

(2) The 10-point F.A. formula for unsteady 2D. convective transport equation derived in a non -uniform grid spacing local element gives physically realistic all positive coefficients and exhibits a desyred upwind shift.

(3) Higher order corrections for the convective terms in Navier-Stokes equations are considered. It significantly improves the linearizaton scheme and the accuracy of the F.A. solutions.

(4) For unsteady fluid flow problems, large time increment can be used to obtain the F.A. solution for transient problems. Thus, the stability and computational efficiency of the present F. A. method is demonstrated.

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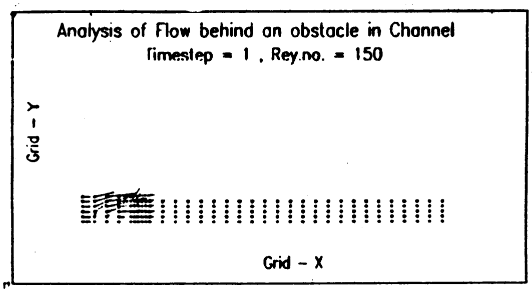
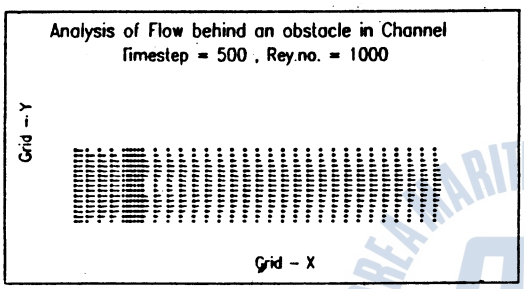
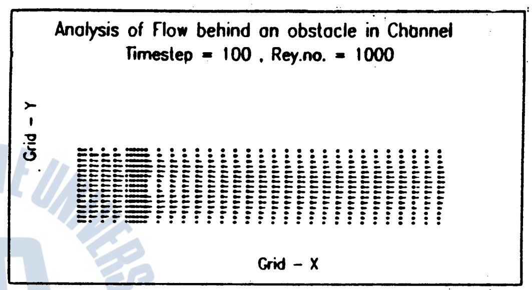
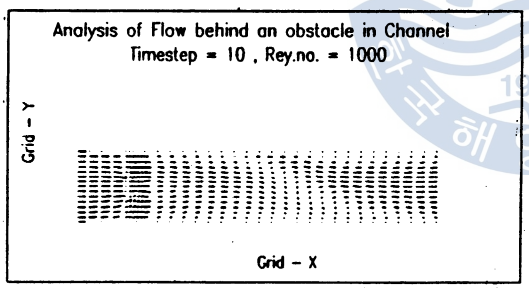
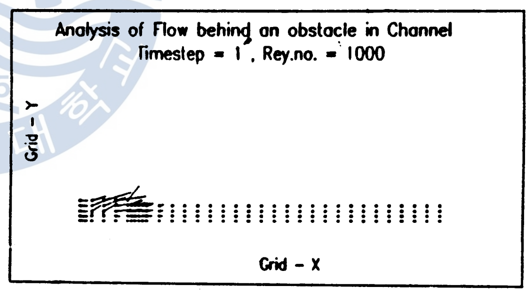

Analysis of Flow behind an obstacle in Channel
Timestep = 1 , Rey.no. = 150
Grid - Y
Grid - X

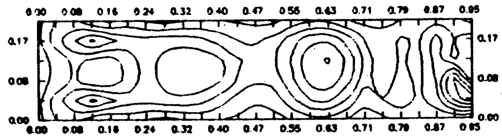

Analysis of Flow behind an obstacle in Channel
Timestep = 500 , Rey.no. = 1000
Grid - Y
Grid - X


Analysis of Flow behind an obstacle in Channel
Timestep = 100 , Rey.no. = 1000
Grid - Y
Grid - X

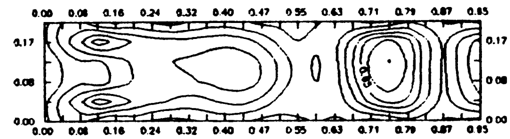

Analysis of Flow behind an obstacle in Channel
Timestep = 10 , Rey.no. = 1000
Grid - Y
Grid - X


Analysis of Flow behind an obstacle in Channel
Timestep = 1 , Rey.no. = 1000
Grid - Y
Grid - X

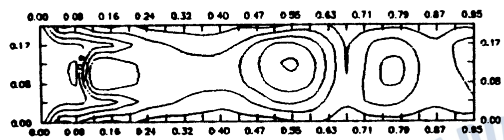
Resultant velocity , Timestep = 20 , Rey.no. = 150



Resultant velocity , Timestep = 30 , Rey.no. = 150

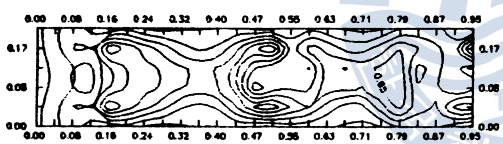


Resultant velocity , Timestep = 40 , Rey. no.= 150

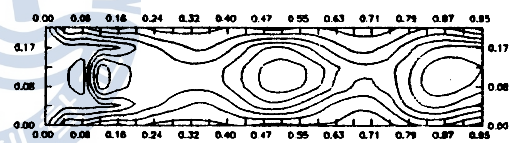


Resultant velocity , Timestep = 50 , Rey.no. = 150

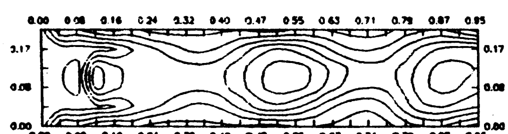
Resultant velocity , Timestep = 100 , Rey.no. = 150



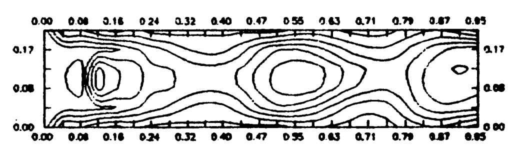
Resultant velocity , Timestep = 200 , Rey.no. = 150



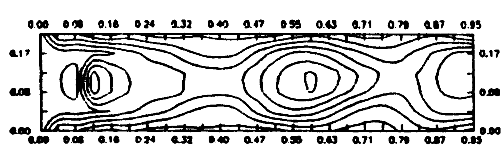
Resultant velocity , Timestep = 300 , Rey.no. = 150



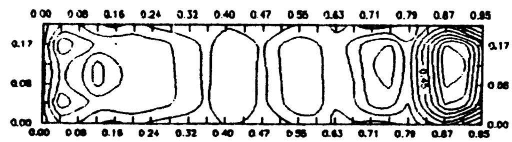
Resultant velocity , Timestep = 400 , Rey.no. = 150



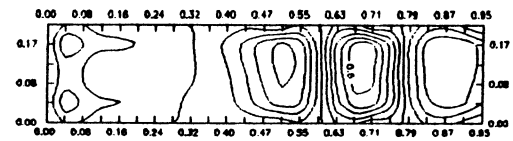
Resultant velocity , Timestep = 500 , Rey.no. = 150



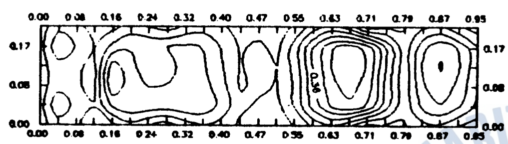
Pressure contour , Timestep = 20 , Rey.no. = 150



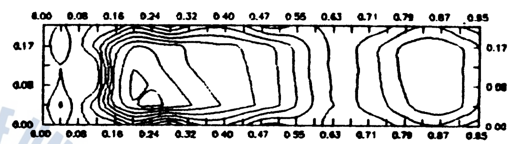
Pressure contour , Timestep = 30 , Rey.no. = 150



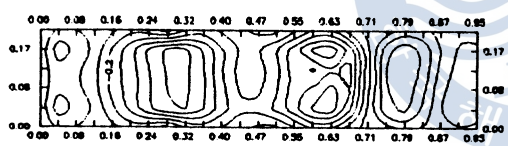
Pressure contour , Timestep = 40 , Rey.no. = 150



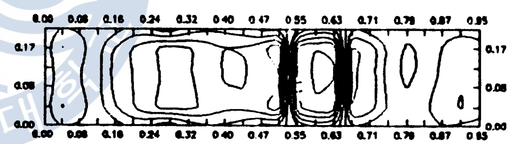
Pressure contour , Timestep = 50 , Rey.no. = 150



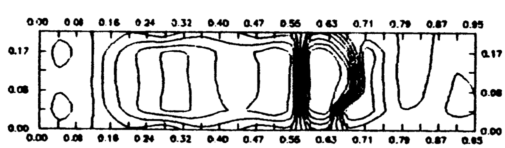
Pressure contour , Timestep = 100 , Rey.no. = 150



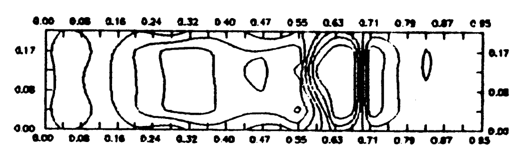
Pressure contour , Timestep = 200 , Rey.no. = 150



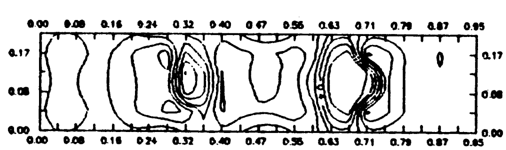
Pressure contour , Timestep = 300 , Rey.no. = 150



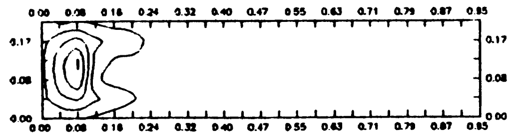
Pressure contour , Timestep = 400 , Rey.no. = 150



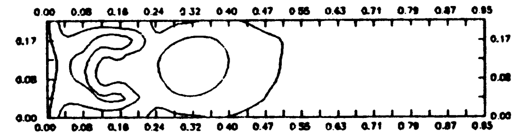
Pressure contour , Timestep = 500 , Rey.no. = 150



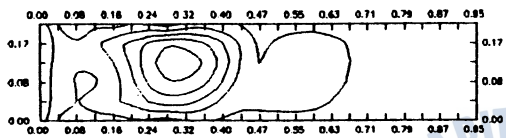
Resultant velocity , Timestep = 1 , Rey.no. = 150



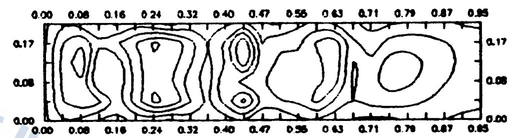
Resultant velocity , Timestep = 2 , Rey.no. = 150



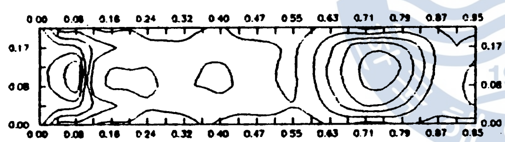
Resultant velocity , Timestep = 3 , Rey.no. = 150



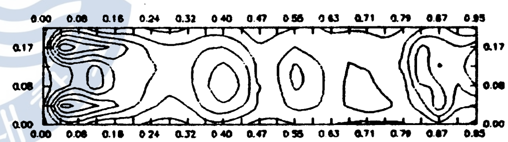
Resultant velocity , Timestep = 4, Rey.no = 150



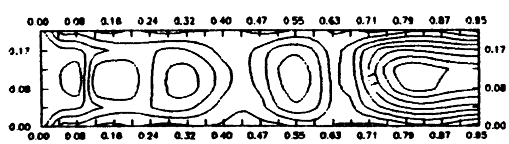
Resultant velocity , Timestep = 5 , Rey.no. = 150



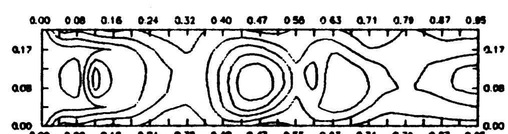
Resultant velocity , Timestep = 6 , Rey.no. = 150



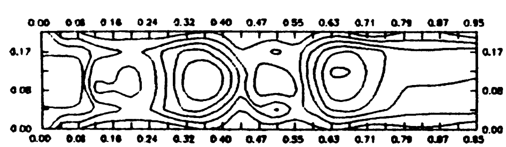
Resultant velocity , Timrstep = 7 , Rey.no. = 150



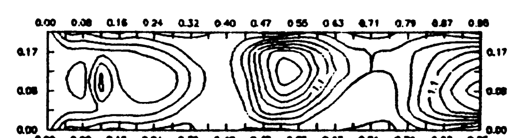
Resultant velocity , Timestep = 8 , Rey.no. = 150



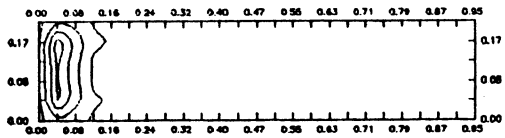
Resultant velocity , Timestep = 9 , Rey.no. = 9



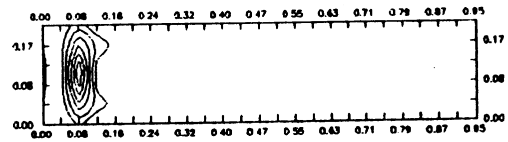
Resultant velocity , Timestep = 10 , Rey.no. = 150



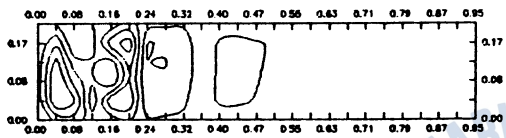
Pressure contour , Timestep = 1 , Rey.no. = 150



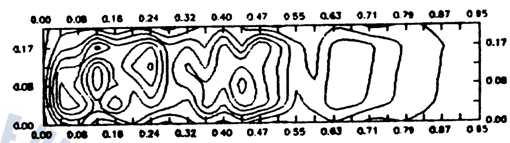
Pressure contour , Timestep = 2 , Rey.no. = 150



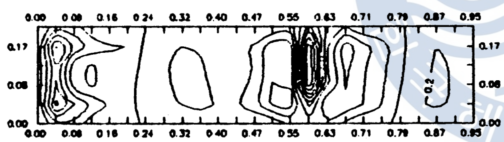
Pressure contour , Timestep = 3 , Rey.no. = 150



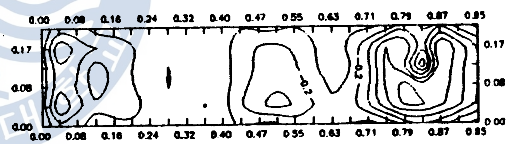
Pressure contour , Timestep = 4 , Rey.no. = 150



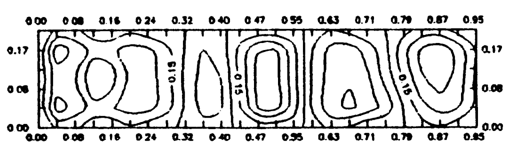
Pressure contour , Timestep = 5 , Rey.no. = 150



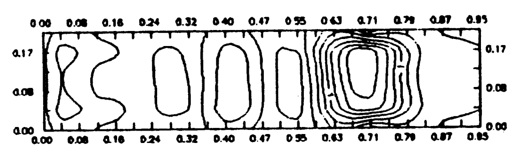
Pressure contour , Timestep = 6 , Rey.no. = 6



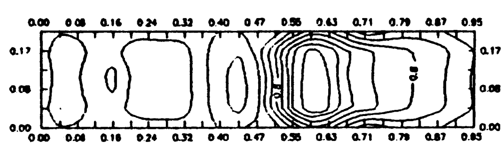
Pressure contour , Timestep = 7 , Rey.no. = 150



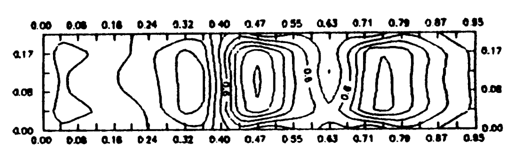
Pressure contour , Timestep = 8 , Rey.no. = 150



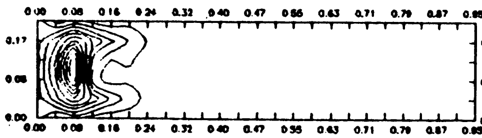
Pressure contour , Timestep = 9 , Rey.no. = 9



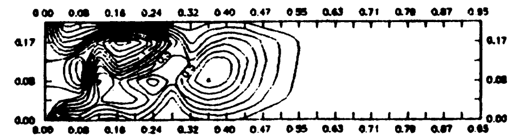
Pressure contour , Timestep = 10 , Rey.no. = 150



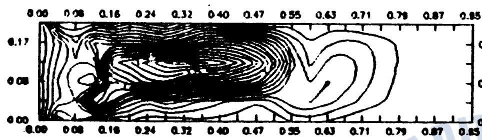
Resultant velocity , Timestep = 1 , Rey.no. = 1000



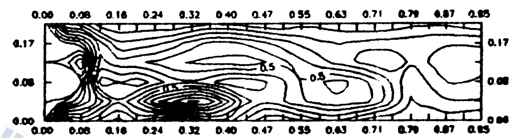
Resultant velocity , Timestep = 2 , Rey.no. = 1000



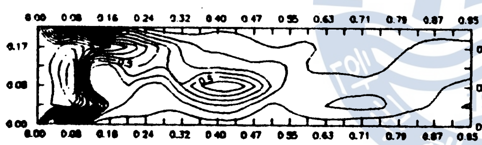
Resultant velocity , Timestep = 3 , Rey.no. = 1000



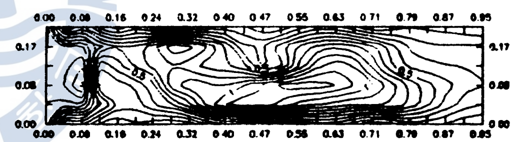
Resultant velocity , Timestep = 4 , Rey.no. = 1000



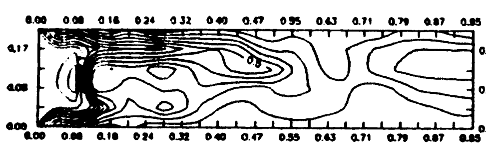
Resultant velocity , Timestep = 5 , Rey.no. = 1000



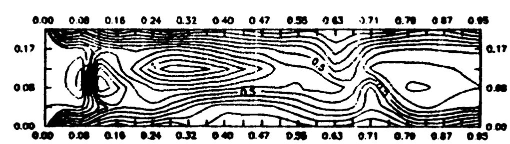
Resultant velocity , Timestep = 6 , Pey.no. = 1000



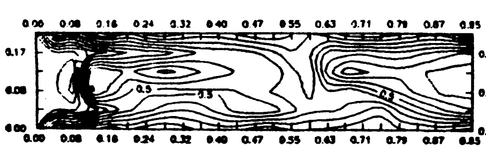
Resultant velocity , Timestep = 7 , Rey.no. = 1000



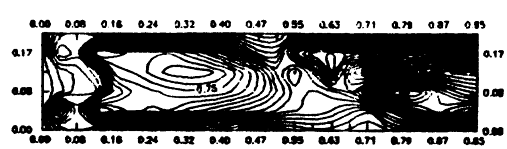
Resultant velocity , Timestep = 8 , Rey.no. = 1000



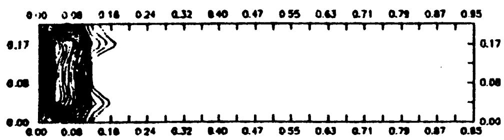
Resultant velocity , Timestep = 9 , Rey.no. = 1000



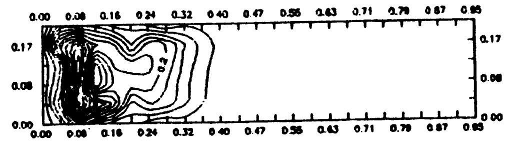
Resultant velocity , Timestep = 10 , Rey.no. = 1000



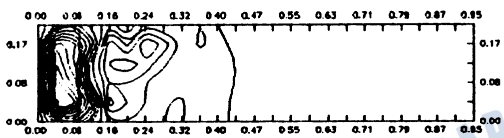
Pressure contour , Timestep = 1 , Rey.no. = 1000



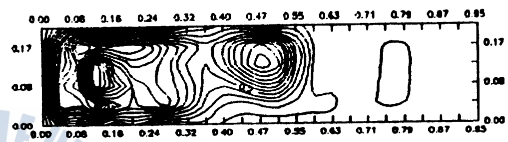
Pressure contour , Timestep = 2 , Rey.no. = 1000



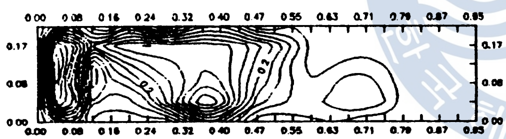
Pressure contour , Timestep = 3 , Rey.no. = 1000



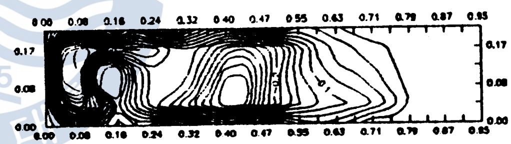
Pressure contour , Timestep = 4 , Rey.no. = 1000



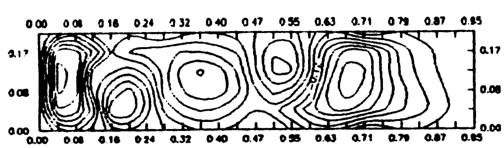
Pressure contour , Timestep = 5 , Rey.no. = 1000



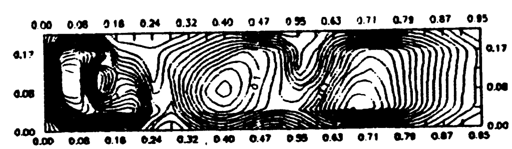
Pressure contour , Timestep = 6 , Rey.no. = 1000



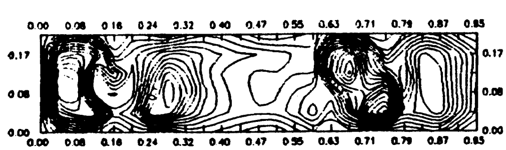
Pressure contour , Timestep = 7 , Rey.no. = 1000



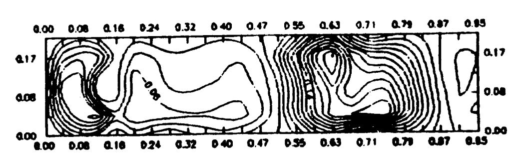
Pressure contour , Timestep = 8 , Rey.no. = 1000



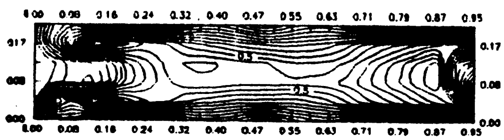
Pressure contour , Timestep = 9 , Rey.no. = 1000



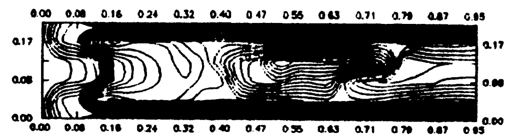
Pressure contour , Timestep = 10 , Rey.no. = 1000



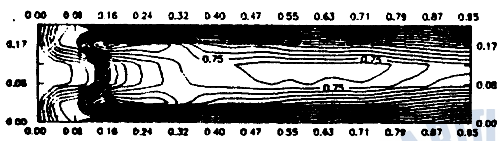
Resultant velocity , Timestep = 20 , Rey.no. = 1000



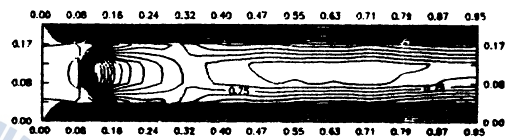
Resultant velocity , Timestep = 30 , Rey.no. = 1000



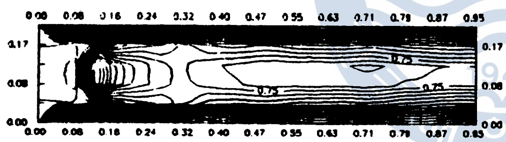
Resultant velocity , Timestep = 40 , Rey.no. = 1000



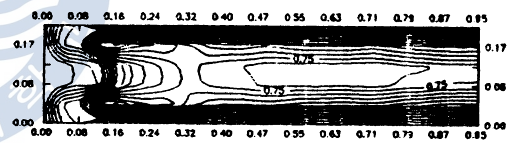
Resultant velocity , Timestep = 50 , Rey.no. = 1000



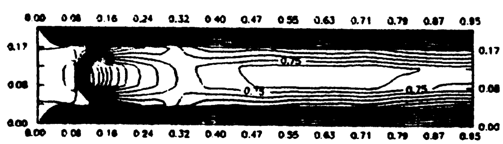
Resultant velocity , Timestep = 100 , Rey.no. = 1000



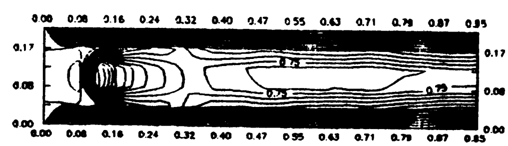
Resultant velocity , Timestep = 200 , Rey.no. = 1000



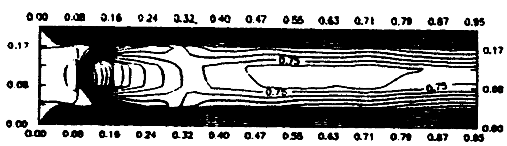
Resultant velocity , Timestep = 300 , Rey.no. = 1000



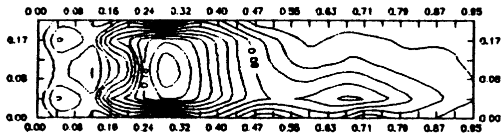
Resultant velocity , Timestep = 400 , Rey.no. = 1000



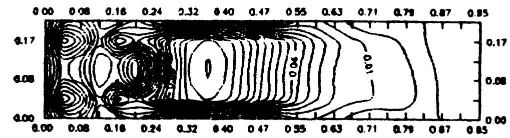
Resultant velocity , Timestep = 500 , Rey.no. = 1000



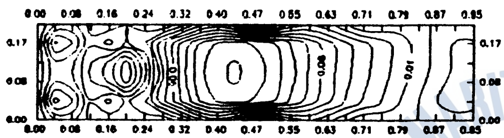
Pressure contour , Timestep = 20 , Rey.no. = 1000



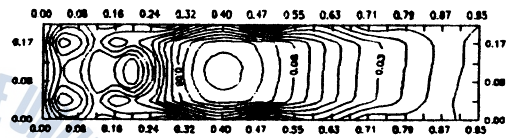
Pressure contour , Timestep = 30 , Rey.no. = 1000



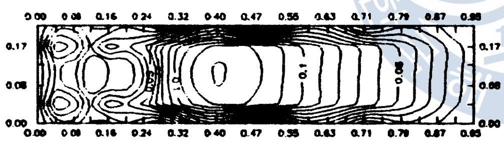
Pressure contour , Timestep = 40 , Rey.no. = 1000



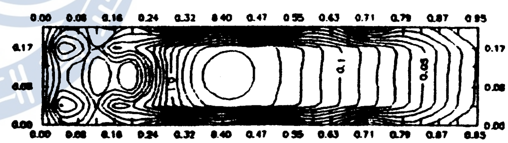
Pressure contour , Timestep = 50 , Rey.no. = 1000



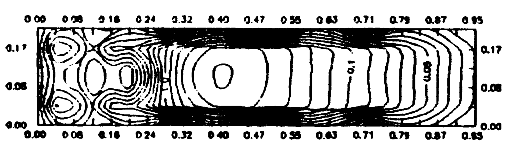
Pressure contour , Timestep = 100, Rey.no. = 1000



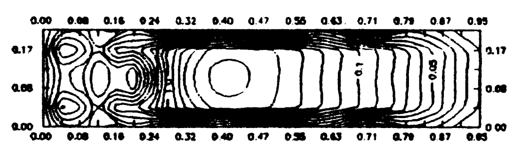
Pressure contour , Timestep = 200 , Rey.no. = 1000



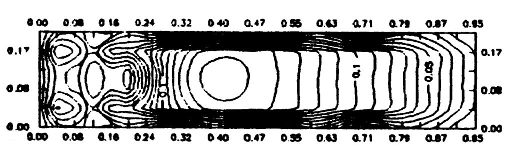
Pressure contour , Timestep = 300 , Rey.no. = 1000



Pressure contour , Timestep = 400 , Rey.no. = 1000



Pressure contour , Timestep = 500 , Rey.no. = 1000



<Appendix>

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C      ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C      ANALYSIS OF 2-DIMENSIONAL FLOW FIELD IN THE SQUARE CAVITY
C      USING A FINITE ANALYTIC METHOD IN PRIMITIVE VARIABLE FORMULATION
C      ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C
C
C      ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C
       IMPLICIT REAL*8(A-H,O-Z)
       COMMON/ABC1/U(34,10),V(34,10)
       COMMON/ABC2/U1(34,10),V1(34,10)
       COMMON/ABC3/U2(34,10),V2(34,10)
       COMMON/ABC4/FX(34,10),FY(34,10)
       COMMON/ABC5/CU(34,10),CV(34,10)
       COMMON/ABC6/DS(34,10),PE(34,10),PP(34,10)
       COMMON/ABC7/AA(34),BB(34),CC(34),DD(34),TT(34)
      &,HX(34),HY(34)
       COMMON/ABC8/UX(34,10),VY(34,10)
       COMMON/AAA/CF(3,3)
       COMMON/UC1/UMP(34,10),UNP(34,10),UPP(34,10)
       COMMON/UC2/UMN(34,10),UNN(34,10),UPN(34,10)
       COMMON/UC3/UMM(34,10),UNM(34,10),UPM(34,10)
       COMMON/VC1/VMP(34,10),VNP(34,10),VPP(34,10)
       COMMON/VC2/VMN(34,10),VNN(34,10),VPN(34,10)
       COMMON/VC3/VMM(34,10),VNM(34,10),VPM(34,10)
       DIMENSION X(34),Y(10),Q(34,10)
C
       OPEN(5,FILE='FCC.IN')
       OPEN(6,FILE='FCC.OUT')
C
       IXMAX=33
       IYMAX=9
       IXP1=IXMAX+1
       IYP1=IYMAX+1
       IXM1=IXMAX-1
       IYM1=IYMAX-1
       IXMM=(IXMAX+1)/2
       ITERP=10
       ITERU=9
       ITERV=9
       IEND=50
       NM=5
       EPE=0.0001
       HX(1)=0.
       HX(IXP1)=0.
```

```
      HY(1)=0.
      DO 41 IX=2,IXMAX
   41 HX(IX)=1./IXM1
      DO 411 IY=2,IYP1
  411 HY(IY)=0.5/IYM1
      TAU=1.0
      RN=2000.
      ROT=RN/TAU
      WRITE(6,50)RN,TAU
      WRITE(6,350)(HX(IX),IX=1,IXP1)
      WRITE(6,350)(HY(IY),IY=1,IYP1)
   50 FORMAT(//5X,6E12.4)
      DO 90 IX=1,IXP1
      DO 90 IY=1,IYP1
      U(IX,IY)=0.
      V(IX,IY)=0.
      DS(IX,IY)=0.
      PP(IX,IY)=0.
      CU(IX,IY)=0.
      CV(IX,IY)=0.
      UX(IX,IY)=0.
      VY(IX,IY)=0.
   90 PR(IX,IY)=0.
C
      DO 123 IX=1,IXMAX
  123 READ(5,351) (U(IX,IY),IY=1,IYP1)
      DO 124 IY=1,IYMAX
  124 READ(5,351)(V(IX,IY),IX=1,IXP1)
      DO 126 IY=1,IYP1
  126 READ(5,351)(PR(IX,IY),IX=1,IXP1)
      DO 27 IX=1,IXP1
      DO 27 IY=1,IYP1
   27 U(IX,1)=1.
      DO 25 IY=1,IYP1
      DO 25 IX=1,IXP1
      U1(IX,IY)=U(IX,IY)
      U2(IX,IY)=U(IX,IY)
      V1(IX,IY)=V(IX,IY)
   25 V2(IX,IY)=V(IX,IY)
      MM=0
C
      DO 1000 IT=1,IEND
      MM=MM+1
      DO 80 IX=2,IXMAX
      DO 80 IY=2,IYMAX
      HEE=0.5*(HX(IX+1)+HX(IX))
      HWW=0.5*(HX(IX-1)+HX(IX))
      HNN=0.5*(HY(IY+1)+HY(IY))
```

```
      HSS=0.5*(HY(IY-1)+HY(IY))
      AUX=0.5*RN*UX(IX,IY)
      BVY=0.5*RN*VY(IX,IY)
      IF(ABS(AUX).LT.EPE)AUX=SIGN(EPE,AUX)
      IF(ABS(BVY).LT.EPE)BVY=SIGN(EPE,BVY)
      EPAUX=EXP(0.5*AUX*HX(IX))
      EPBVY=EXP(0.5*BVY*HY(IY))
      UX(IX,IY)=(U(IX-1,IY)*EPAUX+U(IX,IY)/EPAUX)
     &/(EPAUX+1./EPAUX)
      VY(IX,IY)=(V(IX,IY-1)*EPBVY+V(IX,IY)/EPBVY)
     &/(EPBVY+1./EPBVY)
   80 CONTINUE
C
      DO 120 IX=2,IXMAX
      HE=HX(IX+1)
      HW=HX(IX)
      DO 120 IY=2,IYMAX
      HN=0.5*(HY(IY+1)+HY(IY))
      HS=0.5*(HY(IY-1)+HY(IY))
      VN=(HE*V(IX,IY)+HW*V(IX+1,IY))/(HE+HW)
      VS=(HE*V(IX,IY-1)+HW*V(IX+1,IY-1))/(HE+HW)
      UN=(HY(IY+1)*U(IX,IY)+HY(IY)*U(IX,IY+1))/2./HE
      US=(HY(IY-1)*U(IX,IY)+HY(IY)*U(IX,IY-1))/2./HW
      AE=0.5*RN*U(IX,IY)
      BE=0.5*RN*V(IX,IY)
      CALL COEFF(AE,BE,HE,HW,HN,HS)
      UMM(IX,IY)=CF(1,1)
      UMN(IX,IY)=CF(1,2)
      UMP(IX,IY)=CF(1,3)
      UNM(IX,IY)=CF(2,1)
      UNN(IX,IY)=CF(2,2)
      UNP(IX,IY)=CF(2,3)
      UPM(IX,IY)=CF(3,1)
      UPN(IX,IY)=CF(3,2)
      UPP(IX,IY)=CF(3,3)
      CU(IX,IY)=UNN(IX,IY)*RN/(1.+ROT*UNN(IX,IY))
     &/0.5/(HE+HW)
C
      FX(IX,IY)=RN*(((UX(IX+1,IY )-U(IX,IY))*UX
     1(IX+1,IY )-(UX(IX,IY)-U(IX,IY))*UX(IX,IY))
     2/0.5/(HE+HW)+((VN-VX)*UN-(VS-VX)*US)/HY(IY))
C
      UU1=0.
      CF(2,2)=0.
      DO 99 JX=1,3
      DO 99 JY=1,3
   99 UU1=UU1+CF(JX,JY)*U(IX+JX-2,IY+JY-2)
  120 U1(IX,IY)=(UU1+UNN(IX,IY)*(ROT*U1(IX,IY)
     &-FX(IX,IY)))/(1.+ROT*UNN(IX,IY))
```

```
C
      DO 121 IY=2,IYMAX
      HN=HY(IY+1)
      HS=HY(IY)
      DO 121 IX=2,IXMAX
      HE=0.5*(HX(IX+1)+HX(IX))
      HW=0.5*(HX(IX-1)+HX(IX))
      UE=(HN*U(IX,IY)+HS*U(IX,IY+1))/(HN+HS)
      UW=(HN*U(IX-1,IY)+HS*U(IX-1,IY+1))/(HN+HS)
      VE=(HX(IX+1)*V(IX,IY)+HX(IX)*V(IX+1,IY))/2./HN
      VW=(HX(IX-1)*V(IX,IY)+HX(IX)*V(IX-1,IY))/2./HS
      AR=0.5*RN*U(IX,IY)
      BR=0.5*RN*V(IX,IY)
      CALL COEFF(AR,BR,HE,HW,HN,HS)
      VMM(IX,IY)=CF(1,1)
      VMN(IX,IY)=CF(1,2)
      VMP(IX,IY)=CF(1,3)
      VNM(IX,IY)=CF(2,1)
      VNN(IX,IY)=CF(2,2)
      VNP(IX,IY)=CF(2,3)
      VPM(IX,IY)=CF(3,1)
      VPN(IX,IY)=CF(3,2)
      VPP(IX,IY)=CF(3,3)
      CV(IX,IY)=VNN(IX,IY)*RN/(1.+ROT*VNN(IX,IY))
     &/0.5/(HN+HS)
C
      FY(IX,IY)=RN*(((UE-UY)*VE-(UW-UY)*VW)/HX(IX)
     1+((VY(IX,IY+1)-V(IX,IY))*VY(IX,IY+1)-
     2(VY(IX,IY)-V(IX,IY))*VY(IX,IY))/0.5/(HN+HS))
C
      VV1=0.
      CF(2,2)=0.
      DO 98 JX=1,3
      DO 98 JY=1,3
   98 VV1=VV1+CF(JX,JY)*V(IX+JX-2,IY+JY-2)
  121 V1(IX,IY)=(VV1+VNN(IX,IY)*(ROT*V1(IX,IY)
     &-FY(IX,IY)))/(1.+ROT*VNN(IX,IY))
C
      DO 148 IX=2,IXMAX
      DO 148 IY=2,IYMAX
  148 DS(IX,IY)=(U1(IX,IY)-U1(IX-1,IY))/HX(IX)
     &+(V1(IX,IY)-V1(IX,IY-1))/HY(IY)
C
      DO 51 ITER=1,ITERP
      DO 55 IY=2,IYMAX
      DO 60 IX=2,IXMAX
      AA(IX)=-CU(IX-1,IY)/HX(IX)
      BB(IX)=(CU(IX,IY)+CU(IX-1,IY))/HX(IX)+
     &(CV(IX,IY)+CV(IX,IY-1))/HY(IY)
```

```
      CC(IX)=-CU(IX,IY)/HX(IX)
   60 DD(IX)=(CV(IX,IY-1)*PR(IX,IY-1)+CV(IX,IY)
     &+PR(IX,IY+1))/HY(IY)-DS(IX,IY)
      CALL TRIDAG(2,IXMAX,AA,BB,CC,DD,TT)
      DO 61 IX=2,IXMAX
   61 PR(IX,IY)=TT(IX)
   55 CONTINUE
   51 CONTINUE
C
      DO 31 IX=2,IXMAX
      DO 31 IY=2,IYMAX
   31 FX(IX,IY)=FX(IX,IY)+RN*(PR(IX+1,IY)-PR(IX,IY
     &))/0.5/(HX(IX)+HX(IX+1))
      DO 311 IY=2,IYMAX
      DO 311 IX=2,IXMAX
  311 FY(IX,IY)=FY(IX,IY)+RN*(PR(IX,IY+1)-PR(IX,IY
     &))/0.5/(HY(IY)+HY(IY+1))
C
      DO 35 ITER=1,ITERU
      DO 33 IY=2,IYMAX
      DO 32 IX=2,IXMAX
      AA(IX)=-UMN(IX,IY)
      BB(IX)=1.+ROT*UNN(IX,IY)
      CC(IX)=-UPN(IX,IY)
   32 DD(IX)=UMP(IX,IY)*U2(IX-1,IY+1)+UNP(IX,IY)
     1*U2(IX,IY+1)+UPP(IX,IY)*U2(IX+1,IY+1)
     2+UMM(IX,IY)*U2(IX-1,IY-1)+UNM(IX,IY)*U2(IX,
     3IY-1)+UPM(IX,IY)*U2(IX+1,IY-1)
     4+UNN(IX,IY)*(ROT*U(IX,IY)-FX(IX,IY))
      DD(2)=DD(2)-AA(2)*U2(1,IY )
      DD(IXMAX)=DD(IXMAX)-CC(IXMAX)*U2(IXP1,IY)
      CALL TRIDAG(2,IXMAX,AA,BB,CC,DD,TT)
      DO 33 IX=2,IXMAX
   33 U2(IX,IY)=TT(IX)
      DO 36 IY=1,IYP1
   36 U2(IXP1,IY )=U2(IXMAX,IY)
   35 CONTINUE
      DO 45 ITER=1,ITERV
      DO 43 IX=2,IXMAX
      DO 42 IY=2,IYMAX
      AA(IY)=-VNM(IX,IY)
      BB(IY)=1.+ROT*VNN(IX,IY)
      CC(IY)=-VNP(IX,IY)
   42 DD(IY)=VMP(IX,IY)*V2(IX-1,IY+1)+VPP(IX,IY)*V2(IX+1,IY+1)
     1+VMN(IX,IY)*V2(IX-1,IY)+VPN(IX,IY)*V2(IX+1,IY)+VMM(IX,IY)
     2*V2(IX-1,IY-1)+VPM(IX,IY)*V2(IX+1,IY-1)
     3+VNN(IX,IY)*(ROT*V(IX,IY)-FY(IX,IY))
      DD(2)=DD(2)-AA(2)*V2(IX,1)
```

```
      DD(IYMAX)=DD(IYMAX)-CC(IYMAX)*V2(IX,IYP1)
      CALL TRIDAG(2,IYMAX,AA,BB,CC,DD,TT)
      DO 43 IY=2,IYMAX
   43 V2(IX,IY)=TT(IY)
   45 CONTINUE
C
      DO 68 IX=2,IXMAX
      DO 68 IY=2,IYMAX
   68 DS(IX,IY)=(U2(IX,IY)-U2(IX-1,IY ))/HX(IX)
     &+(V2(IX,IY)-V2(IX,IY-1 ))/HY(IY)
C
      DO 151 ITER=1,ITERP
      DO 155 IY=2,IYMAX
      DO 160 IX=2,IXMAX
      AA(IX)=-CU(IX-1,IY)/HX(IX)
      BB(IX)=(CU(IX,IY)+CU(IX-1,IY))/HX(IX)+(CV
     &(IX,IY)+CV(IX,IY-1))/HY(IY)
      CC(IX)=-CU(IX,IY)/HX(IX)
  160 DD(IX)=(CV(IX,IY-1)*PP(IX,IY-1)+CV(IX,IY)*
     &PP(IX,IY+1))/HY(IY)-DS(IX,IY)
      CALL TRIDAG(2,IXMAX,AA,BB,CC,DD,TT)
      DO 71 IX=2,IXMAX
   71 PP(IX,IY)=TT(IX)
  155 CONTINUE
  151 CONTINUE
C
      DO 170 IX=1,IXMAX
      DO 170 IY=2,IYMAX
      U1(IX,IY)=U(IX,IY)
  170 U(IX,IY)=U2(IX,IY)-CU(IX,IY)*
     &(PP(IX+1,IY)-PP(IX,IY))
      DO 171 IY=1,IYP1
  171 U(IXP1,IY)=U(IXMAX,IY)
      DO 175 IY=1,IYMAX
      DO 175 IX=2,IXMAX
      V1(IX,IY)=V(IX,IY)
  175 V(IX,IY)=V2(IX,IY)-CV(IX,IY)*
     &(PP(IX,IY+1)-PP(IX,IY))
      MM=0
      WRITE(6,161)IT
  161 FORMAT(//5X,'NO. OF TIME STEPS =',I5)
      WRITE(6,211)
  211 FORMAT(///5X,'VELOCITY IN X-DIRECTION U=')
      DO 111 IX=1,IXMAX
  111 WRITE(6,352) (U(IX,IY),IY=1,IYP1)
      WRITE(6,212)
  212 FORMAT(///5X,'VELOCITY IN Y-DIRECTION V=')
      DO 112 IY=1,IYMAX
```

```
  112 WRITE(6,352)(V(IX,IY),IX=1,IXP1)
      WRITE(6,214)
  214 FORMAT(///5X,'PRESSURE FIELD PR=')
      DO 114 IY=1,IYP1
  114 WRITE(6,350)(PR(IX,IY),IX=1,IXP1)
      WRITE(6,215)
  215 FORMAT(///5X,'CONSERVATION OF MASS ')
      DO 115 IY=2,IYMAX
  115 WRITE(6,350)(DS(IX,IY),IX=2,IXMAX)
  350 FORMAT(6E13.4)
  351 FORMAT(10F5.2)
  352 FORMAT(6F13.3)
 1000 CONTINUE
      STOP
      END
C
C
C
C
      SUBROUTINE TRIDAG(II,L,A,B,C,D,V)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(34),B(34),C(34),D(34),V(34),BETA(34),
     &GAMMA(34)
      BETA(II)=B(II)
      GAMMA(II)=D(II)/BETA(II)
      IIP1=II+1
      DO 1 I=IIP1,L
      BETA(I)=B(I)-A(I)*C(I-1)/BETA(I-1)
    1 GAMMA(I)=(D(I)-A(I)*GAMMA(I-1))/BETA(I)
      V(L)=GAMMA(L)
      LAST=L-II
      DO 2 K=1,LAST
      I=L-K
    2 V(I)=GAMMA(I)-C(I)*V(I+1)/BETA(I)
      RETURN
      END
C
C
C
C
      SUBROUTINE COEFF(AE,BE,HE,HW,HN,HS)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/AAA/CF(3,3)
      PI=3.141592653589793D0
      EPE=0.0001
      MAX=5
      JX=1
      JY=1
```

```
      IF(HE.LT.HW) GO TO 2
      JX=-1
      AR=-AR
    2 IF(HN.LT.HS) GO TO 4
      JY=-1
      BR=-BR
    4 IF(ABS(AR).LT.EPE)AR=SIGN(EPE,AR)
      IF(ABS(BR).LT.EPE)BR=SIGN(EPE,BR)
      AB2=AR*AR+BR*BR
      HX=AMIN(HE,HW)
      HY=AMIN(HN,HS)
      AH=AR*HX
      BK=BR*HY
      EPA=EXP(-AH)
      EPB=EXP(-BK)
      COSHA=0.5*EPA+0.5/EPA
      COSHB=0.5*EPB+0.5/EPB
      COTHA=(1.+EPA*EPA)/(1.-EPA*EPA)
      COTHB=(1.+EPB*EPB)/(1.-EPB*EPB)
      IF(HX.GT.HY) GO TO 20
      BX2=0.0
      DO 10 I=1,MAX
      ZA=(I-0.5)*PI
      PWR=(-1.)**I*ZA
      AB=EXP((AB2+ZA*ZA/HX/HX)**0.5*HY)
   10 BX2=BX2-2.*PWR/(AB+1./AB)/(AH*AH+ZA*ZA)**2
      BY2=BX2*HX*HX/HY/HY+(1./BK/COTHB-HX*HX/HY/HY/AH/COTHA)
     & /4./COSHA/COSHB
      GO TO 15
   20 BY2=0.0
      DO 16 I=1,MAX
      ZA=(I-0.5)*PI
      PWR=(-1.)**I*ZA
      AB=EXP((AB2+ZA*ZA/HY/HY)**0.5*HX)
   16 BY2=BY2-2.*PWR/(AB+1./AB)/(BK*BK+ZA*ZA)**2
      BX2=BY2*HY*HY/HX/HX+(1./AH/COTHA-HY*HY/HX/HX/BK/COTHB)
     & /4./COSHA/COSHB
   15 E=0.25/COSHA/COSHB-AH*COTHA*BX2-BK*COTHB*BY2
      EA=2.*AH*COSHA*COTHA*BX2
      EB=2.*BK*COSHB*COTHB*BY2
      CNN=(AH/COTHA+BK/COTHB-4.*COSHA*COSHB*(AH*AH*BX2
     & +BK*BK*BY2))/2./AB2
      CNW=E/EPA*EPB
      CNE=E*EPA*EPB
      CSW=E/EPA/EPB
      CSE=E*EPA/EPB
      CNC=EA*EPB
      CSC=EA/EPB
      CWC=EB/EPA
```

```
      CBC=EB*EPA
      IF(ABS(HE-HW).LT.EPB.AND.ABS(HN-HS).LT.EPB.AND.
     &ABS(HT-HB).LT.EPB) GO TO 500
      HX1=AMAX1(HE,HW)
      HY1=AMAX1(HN,HS)
      AH1=AK*HX1
      BK1=BR*HY1
      SEW=HX1*(EXP(2.*AH)-1.)+HX*(EXP(-2.*AH1)-1.)
      TNS=HY1*(EXP(2.*BK)-1.)+HY*(EXP(-2.*BK1)-1.)
      S=(EPA*EPA+1./EPA/EPA-2.)*HX1/SEW
      S1=S-1.
      S2=S*HX/HX1
      S3=1.-S1-S2
      T=(EPB*EPB+1./EPB/EPB-2.)*HY1/TNS
      T1=T-1.
      T2=T*HY/HY1
      T3=1.-T1-T2
      FP=1.-S3*CWC-T3*CSC-S3*T3*CSW
      CF(2+JX,2+JY)=(CNE+S1*CNW+T1*CSE+S1*T1*CSW)/FP
      CF(2-JX,2+JY)=S2*(CNW+T1*CSW)/FP
      CF(2+JX,2-JY)=T2*(CSE+S1*CSW)/FP
      CF(2-JX,2-JY)=S2*T2*(CSW)/FP
      CF(2+JX,2)=(CEC+S1*CWC+T3*CSE+S1*T3*CSW)/FP
      CF(2-JX,2)=S2*(CWC+T3*CSW)/FP
      CF(2,2+JY)=(CNC+S3*CNW+T1*CSC+S3*T1*CSW)/FP
      CF(2,2-JY)=T2*(CSC+S3*CSW)/FP
      CF(2,2)=CNN/FP
      GO TO 501
  500 CF(2+JX,2+JY)=CNE
      CF(2+JX,2-JY)=CSE
      CF(2-JX,2+JY)=CNW
      CF(2-JX,2-JY)=CSW
      CF(2+JX,2)=CEC
      CF(2-JX,2)=CWC
      CF(2,2+JY)=CNC
      CF(2,2-JY)=CSC
      CF(2,2)=CNN
  501 RETURN
      END
```