附 録 IV

Stowage Factors and Permeabilities(from C&R No. 8)

Commodity	Type of packing	Stow-age factor, cu. ft. per ton	Perme-ability of cargo in full hold
Autos(knock down)	Case(4 ton)	110	80
Autos	Case(2 ton)	220	84
Tractors	Case	200	75
Auto parts	Case	90	70
Autos	Open	…	95
Apples	Barrels	104	61
Apples	Boxes	72	40
Acid	Drums	45	40
Acid	Barrels	50	35
Bard wire	Rolls	55	85
Beans	Bags	60	50
Biscuits	Cases	142	79
Blankets	Bales	153	78
Butter	Boxesn	56	20
Canned goods	Cans in cases	50	30
Cable	Reels	31	50
Cardboard	Bundles	210	88
Cartridges	Boxes	30	30
Castings	Boxes	22	50
Castings	Barrels	31	70
Cement	Bags	35	63
Cement	Barrels	36	72
Chain	Barrels	12	60
Cheese	Boxes	45	30
Coffee	Bags	58	42
Conduits	Boxes	31	78
Copper	Slabs	7	18
Copper	Bars	10	26
Cork	Bales	187	24
Corn	Barrels	65	54
Corn	Bags	55	42
Dates	Boxes	45	30
Dry fruit	Boxes	45	30
Dry goods	Boxes	100	60
Earth	Bags	56	30
Eggs	Cases	100	45
Electric motors	Boxes	50	40
Fish	Boxes	65	70
Fish	Barrels	53	42
Flour	Bags	48	29
Flour	Barrels	73	44
Furniture	Boxes	156	80
Gasoline	Drums	61	40
General cargo		70	60
Grape fruit	Boxes	70	46
Grape juice	Cases(bottles)	70	46
Hardware	Boxes	50	50
Hay	Bales	120	60
Hides	Bales	102	30
Iron	Pigs	10	17
Lanterns	Cases	375	80
Lard	Boxes	45	20
Laths	Bundles	107	37
Leather	Bales	80	35
Lime	Bags	52	45
Linoleum	Rolls	70	30
Linseed	Bags	60	50

Commodity	Type of packing	Stow-age factor, cu. ft. per ton	Perme-ability of cargo in full hold
Machinery	Cases	50	85
Machinery	Boxes	46	55
Magazines	Bundles	75	70
Meat	Cold storage	90-100	66
Motors, gasoline	Cases	110	80
Newspapers	Bales	120	63
Nitrate	Bags	24	55
Nuts	Bags	70	55
Oats	Bags	77	48
Oil	Barrels	50	35
Oil	Cases	50	34
Oil	Drums	45	40
Onions	Barrels	104	60
Onions	Bags	78	48
Overcoats	Bales	160	40
Oranges	Boxes	78	46
Paper	Rolls	80	70
Paper	Bales	80	70
Paper	Boxes	60	52
Paint	Druns	24	40
Paint	Barels	28	30
Paint	Cans	36	30
Peas	Bags	55	55
Poultry	Boxes	95	60
Potatoes	Bags	60	49
Potatoes	Barrells	75	61
Plumbing Fixtures	Crates	100	60
Rags	Bales	149	76
Rails	Nond	15	50
Raisins	Boxes	54	50
Rice	Bags	58	55
Roof paper	Rolls	80	30
Rope	Coil	72	55
Rubber	Bundles	140	25
Rugs	Bales	146	70
Silk	Bolts	80	40
Soap	Boxes	45	20
Soap powder	Boxes	90	70
Sugar	Bags	47	48
Sugar	Barrels	58	60
Starch	Boxes	59	55
Steel rods	Nond	12	28
Tallow	Barrels	66	35
Tasajo(dried beef)	Bales	90	40
Tea	Boxes	91	80
Thread	Cases	60	45
Tile	Boxes	50	20
Tin	Sheets	7	15
Tires	Bundles	168	85
Tobacco	Boxes	134	60
Transformers	Cases	30	30
Typewriters	Cases	110	80
Waste(cotton)	Bales	175	81
Wax, vegetable	Bags	50	25
Wax	Barrels	70	35
Wheat	Bulk	47	45
Wool	Bales	160	30
Zinc	Slabs	7	15

單相關係數에 關한 漸近分布의 共分散行列

李 鍾 厚

Asymptotic distribution of sample correlation coefficients.

Jonghoo Lee

Abstract

Suppose $X'=(\{x_{1i}, x_{2i}, \cdots, x_{ki}\}', \quad i=1,2,\cdots,n)$ $k\leqq n$ is a sample from the k-dimensional distribution $N(\{\mu_i\},\Lambda)$ $\Lambda=(\lambda_{ij})$ and let $V/(n-1)=(X'X-n\bar{x}'\bar{x})/(n-1)=S$ be an unbiased covariance matrix of the sample. Let $D_{\sqrt{\lambda}}=\mathrm{diag}\{\sqrt{\lambda_{11}}, \cdots, \sqrt{\lambda_{kk}}\}, U=D^{-1}_{\sqrt{\lambda}}SD^{-1}_{\sqrt{\lambda}}, \ P=D^{-1}_{\sqrt{\lambda}}\Lambda D^{-1}_{\sqrt{\lambda}}$,

and let

$$u=(u_{11}, u_{22}, \cdots, u_{kk}, u_{12}, u_{13}, \cdots, u_{1k}, u_{23}, \cdots, u_{k-1,k})$$
$$\rho^*=(1, \cdots, 1, \rho_{12}, \rho_{13}, \cdots, \rho_{1k}, \rho_{23}, \cdots, \rho_{k-1,k})$$

where u_{ij} is respectively independent components in U and ρ_{ij} in P. Further more let

$$r=(r_{12}, \cdots, r_{1k}, r_{23}\cdots, r_{k-1,k})$$

where

$$r_{ij}=f_{ij}(u)=\frac{u_{ij}}{\sqrt{u_{ii}\ u_{jj}}}$$

then r is asymptotically distributed according to $\frac{1}{2}k(k-1)$-dimensional distribution $N(\rho, \ ||\mathrm{cov}(r_{ij}, r_{lm})||)$,

where

$$\rho=(\rho_{12}, \rho_{13}, \cdots, \rho_{1k}, \rho_{23}, \cdots, \rho_{2k}, \cdots, \rho_{k-1}, \rho_k)$$

$$\mathrm{cov}(r_{ij}, r_{lm}) \doteqdot \frac{1}{\nu}\{\frac{1}{2}\rho_{ij}\rho_{lm}(\rho_{il}^2+\rho_{jm}^2+\rho_{jl}^2+\rho_{jm}^2)+\rho_{il}\rho_{jm}+\rho_{im}\rho_{jl} - (\rho_{ij}\rho_{il}\rho_{im}+\rho_{ji}\rho_{jl}\rho_{jm}+\rho_{li}\rho_{lj}\rho_{lm}+\rho_{mi}\rho_{mj}\rho_{ml})\}.$$

이 論文은 標本 單相關係數에 關한 漸近分布가 正規分布에 收斂한다는 것과 共分散行列을 具體的으로 求하는데 있다.

Lemma 1. (中心極限定理) 有限平均 $(\mu_i, \ i=1,\cdots,k)$, 陽値形式의 共分散行列 $||\sigma_{ij}||$, $i,j=1,\cdots,k$ 를 가지는 k 變量分布에서 取한 크기 n의 標本確率變量을 $(x_{1\xi},\cdots,x_{k\xi};\xi=1,\cdots,n)$라 하면, $\left(z_i=\sum_{\xi=1}^{n}x_{i\xi}, \ i=1,\cdots,k\right)$, $\left(\bar{x}_i=\frac{1}{n}z_i, \quad i=1,\cdots,k\right)$는 各各 k變量分布 $N(\{n\mu_i\}, \ ||n\sigma_{ij}||)$ 및 $N(\{\mu_i\}, \ ||\sigma_{ij}/n||)$에 漸近的으로 收斂한다.

(證明) 一變量分布의 中心極限定理에 依하여

$$\lim_{n\to\infty}P\left(\frac{(z_i-n\mu_i)}{\sqrt{n}}\leq y_i,\ i=1,\cdots,k\right)$$
$$=\lim_{n\to\infty}P\left((\bar{x}_i-\mu_i)\sqrt{n}\leq y_i,\ i=1,\cdots,k\right)$$
$$=\frac{\sqrt{|\sigma^{ij}|}}{(2\pi)^{\frac{k}{2}}}\int_{-\infty}^{y_k}\cdots\int_{-\infty}^{y_1}\exp\left(-\frac{1}{2}\sum_{i,j=1}^{k}\sigma^{ij}u_iu_j\right)du_1\cdots du_k \tag{1.1}$$

이므로

$$S_1z,\cdots,z_k)\sim N(\{n\mu_i\},\ ||n\sigma_{ij}||) \tag{1.2}$$
$$(\bar{x}_1,\cdots,\bar{x}_k)\sim N(\{\mu_i\},\ ||\sigma_{ij}/n||) \tag{1.3}$$

이다.

Lemma 2. $\boldsymbol{u}(n)=(u_1(n),\cdots,u_m(n))$을 確率 Vector라 하고 $\sqrt{n}\{\boldsymbol{u}(n)-\boldsymbol{b}\}$가 漸近的으로 $N(\boldsymbol{O},\Psi)$ $\Psi=(E\{\sqrt{n}(u_i-b_i)\sqrt{n}(u_j-b_j)\})$에 收斂한다고 한다. 지금 $\boldsymbol{f}(\boldsymbol{u}(n))=(f_1(\boldsymbol{u}(n),\cdots,f_k(\boldsymbol{u}(n))\ k\leq m$의 各 成分이 $\boldsymbol{u}(n)=\boldsymbol{b}$의 近傍에서 一次, 二次의 微係數를 가지는 $\boldsymbol{u}(n)$의 函數라 하면 $\sqrt{n}\{\boldsymbol{f}(\boldsymbol{u}(n))-\boldsymbol{f}(\boldsymbol{b})\}$의 極限分布는 $N(\boldsymbol{O},\boldsymbol{A}\Psi\boldsymbol{A}')$이다. 여기 $\boldsymbol{A}=(a_{ij})$는 $k\times m$行列이고 그 要素는 $a_{ij}=[\partial f_i(\boldsymbol{u})/\partial u_j]_{=b}$이다.

(證明)

$$\sqrt{n}\{\boldsymbol{f}(\boldsymbol{u}(n))-\boldsymbol{f}(\boldsymbol{b})\}$$
$$=(\sqrt{n}\{f_1(\boldsymbol{u})-f_1(\boldsymbol{b})\}.\cdots\sqrt{n}\{f_i(\boldsymbol{u})-f_i(\boldsymbol{b})\},\cdots,\sqrt{n}\{f_k(\boldsymbol{u})-f_k(\boldsymbol{b})\}) \tag{2.1}$$

의 i 成分에 대해서 생각하자.

假定에서 $\boldsymbol{u}(n)$은 n이 充分히 클 때 $\boldsymbol{b}$에 確率收斂하므로

$$E(u_j-b_j)=O(\frac{1}{n^q}),\ q>0,\ j=1,\cdots,m$$

이 成立한다. Tchebycheff의 不等式에 依하여 $\varepsilon>0$을 取하면

$$P(|u_j-b_j|\geqq\varepsilon)<\frac{M}{\varepsilon n^q} \qquad (\because \text{ Ref 3, P182})$$

여기서 M은 n과 ε에 無關係한 定數이다. 지금 Z를 $|u_j-b_j|<\varepsilon\ (j=1,\cdots,m)$을 滿足하는 m次元의 集合이라 하면

$$P(Z)>1-\frac{mM}{\varepsilon n^q} \tag{2.2}$$

이 成立한다. 그리고 $\varepsilon=n^{-q+q'}$, $q=2q'+\frac{1}{2}$, $q'>0$으로 두면

$$P(Z)>1-mMn^{-q'} \tag{2.3}$$

가 된다.

n이 充分히 클 때 $f_i(\boldsymbol{u})$의 $\boldsymbol{b}$ 둘레(즉 Z안에서)의 Taylor 展開를 하면

$$\sqrt{n}\{f_i(\boldsymbol{u})-f_i(\boldsymbol{b})\}=\sum_{j=1}^{m}\left.\frac{\partial f_i(\boldsymbol{u})}{\partial u_j}\right|_{\boldsymbol{u}=\boldsymbol{b}}\sqrt{n}(u_j-b_j)+\sqrt{n}\ R \tag{2.4}$$

$$R=\sum_{\mu,\nu}^{m}\frac{\partial^2 f_i(\boldsymbol{u}_1)}{\partial u_\mu\partial u_\nu}_{\boldsymbol{u}_1\varepsilon Z}(u_\mu-b_\mu)(u_\nu-b_\nu)$$

$|\sqrt{n}R|<\sqrt{n}Km^2\varepsilon^2=m^2Kn^{-q}$, 이다. 이것은 $\sqrt{n}R$이 $P(Z)\to1$에 收斂하는 것보다 더 빨리 0에 收斂함을 말하고 있다.

따라서 $\sqrt{n}\{f_i(\boldsymbol{u})-f_i(\boldsymbol{b})\}$는 $n\to\infty$인 極限에서

$$\sum_{j=1}^{m}\frac{\partial f_i(\boldsymbol{u})}{\partial u_j}\Big|_{\boldsymbol{u}=\boldsymbol{b}}\sqrt{n}(u_j-b_j)=\sum_{j=1}^{m}a_{ij}\sqrt{n}(u_j-b_j) \tag{2.5}$$

가 된다.

위의 結果는 모든 $\sqrt{n}\{f_i(\boldsymbol{u})-f_i(\boldsymbol{b})\}$, $i=1,\cdots,k$에 對해서 成立하므로 (2.1)式에서

$$\sqrt{n}\{\boldsymbol{f}(\boldsymbol{u})-\boldsymbol{f}(\boldsymbol{b})\}\sim\left(\sum_{j=1}^{m}a_{1j}\sqrt{n}(u_j-b_j),\cdots,\ \sum_{j=1}^{m}a_{kj}(u_j-b_j)\right)=\sqrt{n}(\boldsymbol{u}(n)-\boldsymbol{b})\boldsymbol{A}'$$

을 얻는다. 그러므로

$$\sqrt{n}\{\boldsymbol{f}(\boldsymbol{u}(n))-\boldsymbol{f}(\boldsymbol{b})\}\sim N(\boldsymbol{O},\boldsymbol{A}\boldsymbol{\Psi}\boldsymbol{A}')$$

가 된다.

k變量 正規分布 $N(\boldsymbol{\mu},\boldsymbol{\Lambda})$를 가지는 母集團에서의 크기 n의 任意標本을 $\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n$라 한다. 지금

$$\boldsymbol{X}=\begin{bmatrix}\boldsymbol{x}_1\\ \vdots\\ \boldsymbol{x}_n\end{bmatrix}=\begin{bmatrix}x_{11},x_{21}\cdots x_{k1}\\ \cdots\cdots\cdots\\ x_{1n}\ x_{2n}\cdots x_{kn}\end{bmatrix} \tag{3.1}$$

$$\boldsymbol{M}=\begin{bmatrix}\boldsymbol{\mu}\\ \vdots\\ \boldsymbol{\mu}\end{bmatrix}=\begin{bmatrix}\mu_1\ \ \mu_2\cdots\mu_k\\ \cdots\cdots\cdots\\ \mu_1\ \ \mu_2\cdots\mu_k\end{bmatrix} \tag{3.2}$$

로 表示하면 $\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n$의 同時密度函數는

$$(2\pi)^{-\frac{nk}{2}}|\boldsymbol{\Lambda}|^{-\frac{n}{2}}\exp\{-\frac{1}{2}\mathrm{tr}\,\boldsymbol{\Lambda}^{-1}(\boldsymbol{X}-\boldsymbol{M})'(\boldsymbol{X}-\boldsymbol{M})\} \tag{3.3}$$

으로 表示된다. $\boldsymbol{X}$를 正規觀測行列이라 한다. 즉 各 行이 서로 獨立이고 k變量正規分布 $N(\boldsymbol{\mu},\boldsymbol{\Lambda})$인 $n\times k$ 行列이다. $\boldsymbol{X}$의 同時密度函數가 式(3.3)임을 $\boldsymbol{X}\sim N_n(\boldsymbol{M};\boldsymbol{\Lambda})$로 表示한다.

任意의 直交行列 $\boldsymbol{L}(n\times n)$에 依한 變換 $\boldsymbol{Z}=\boldsymbol{L}\boldsymbol{X}$를 생각하면 $\boldsymbol{X}'\boldsymbol{X}=\boldsymbol{X}'\boldsymbol{L}'\boldsymbol{L}\boldsymbol{X}=\boldsymbol{Z}'\boldsymbol{Z}$이고. $J(\boldsymbol{X};\boldsymbol{Z})=1$이므로 $\boldsymbol{Z}\sim N_n(\boldsymbol{L}\boldsymbol{M};\boldsymbol{\Lambda})$임을 쉽게 알 수 있다. 지금 $\boldsymbol{L}$의 第n行이 $(1/\sqrt{n},\cdots,1/\sqrt{n})$인 것을 쓰면

$$\boldsymbol{Z}=\underset{k}{\begin{bmatrix}\boldsymbol{Y}\\ \cdots\cdots\\ \sqrt{n}\,\bar{\boldsymbol{x}}\end{bmatrix}}\begin{matrix}n-1\\ \\ 1,\end{matrix}\qquad\qquad \boldsymbol{L}\boldsymbol{M}=\underset{k}{\begin{bmatrix}\boldsymbol{O}\\ \cdots\cdots\\ \sqrt{n}\,\boldsymbol{\mu}\end{bmatrix}}\begin{matrix}n-1\\ \\ 1\end{matrix} \tag{3.4}$$

이 된다. 여기서 $\bar{\boldsymbol{x}}=\sum_{\alpha=1}^{n}\boldsymbol{x}_\alpha/n$ 은 標本平均 Vector이다. 따라서

$$\boldsymbol{X}'\boldsymbol{X}=\boldsymbol{Z}'\boldsymbol{Z}=\boldsymbol{Y}'\boldsymbol{Y}+n\bar{\boldsymbol{x}}'\bar{\boldsymbol{x}}$$

$$\therefore\quad \boldsymbol{Y}'\boldsymbol{Y}=\boldsymbol{X}'\boldsymbol{X}-n\bar{\boldsymbol{x}}'\bar{\boldsymbol{x}}=\sum_{\alpha=1}^{n}(\boldsymbol{x}_\alpha-\bar{\boldsymbol{x}})'(\boldsymbol{x}_\alpha-\bar{\boldsymbol{x}})=\boldsymbol{V} \tag{3.5}$$

이다. $\boldsymbol{V}/n=\boldsymbol{S}^*$는 標本共分散行列, $\boldsymbol{V}/(n-1)=\boldsymbol{S}$는 不偏共分散行列로 알려져 있다.

Lemma 3. $\boldsymbol{S}=\boldsymbol{V}/\nu$를 正規標本에 있어서의 不偏共分散行列이라 하면 $\sqrt{\nu}(\boldsymbol{S}-\boldsymbol{\Lambda})=(1/\sqrt{\nu})(\boldsymbol{V}-\nu\boldsymbol{\Lambda})$의 漸近分布는 $N(\boldsymbol{O},\boldsymbol{\Psi})$이고 $\boldsymbol{\Psi}$의 要素는

$$\mathrm{cov}(S_{ij},S_{lm})\doteqdot(\lambda_{il}\lambda_{jm}+\lambda_{im}\lambda_{jl})/\nu \tag{3.6}$$

이다. (단 "$\doteqdot$"은 漸近分布에 있어서의 값임을 表示한다)

(證明) 自由度 $\nu=n-1$의 不偏共分散行列을 $\boldsymbol{S}(k\times k)$라 하고 $\nu\boldsymbol{S}=\boldsymbol{V}$라 하면 $\boldsymbol{Y}(k\times\nu)=(\boldsymbol{y}'_1,\boldsymbol{y}'_2,$

$\cdots, \boldsymbol{y}'_\nu) \sim N_\nu(\boldsymbol{O}; \boldsymbol{\Lambda}=(\lambda_{ij}))$인 $\boldsymbol{Y}$에 依하여 $\boldsymbol{V}=\boldsymbol{Y}'\boldsymbol{Y}=\sum_{\alpha=1}^{\nu} \boldsymbol{y}'_\alpha \boldsymbol{y}_\alpha$로 表示된다. (∵ Ref 1. P24. T3. 1)

지금 $\boldsymbol{Z}_\alpha=(y_{1\alpha}^2, y_{1\alpha}y_{2\alpha}, \cdots, y_{2\alpha}^2, \cdots, y_{k\alpha}^2)$이라 두면 $E(\boldsymbol{Z}_\alpha)=(\lambda_{11}, \lambda_{12}, \cdots, \lambda_{22}, \cdots, \lambda_{kk})$이고 또 共分散行列 $E(\boldsymbol{Z}_\alpha-E(\boldsymbol{Z}_\alpha))'(Z_\alpha-E(Z_\alpha))$의 要素는 $\boldsymbol{x}(1\times k)\sim N(\boldsymbol{\mu}, \boldsymbol{\Lambda})$의 特性函數 $\phi(\boldsymbol{t})=\exp\{i\boldsymbol{t}\boldsymbol{\mu}-\frac{1}{2}\boldsymbol{t}\boldsymbol{\Lambda}\boldsymbol{t}'\}$에서 積率을 求하는 公式

$$E(x_1-\mu_1)^{j1}\cdots(x_k-\mu_k)^{jk}=\frac{1}{(i)^{j1+\cdots+jk}}\left.\frac{\partial^{j1+\cdots+jk}\phi(\boldsymbol{t})}{\partial t_1^{j1}\cdots\partial t_k^{jk}}\right|_{\boldsymbol{t}=\boldsymbol{0}}$$

에 依하여

$$E(x_i-\mu_i)(x_j-\mu_j)(x_l-\mu_l)(x_m-\mu_m)=E(y_{i\alpha}y_{j\alpha}y_{l\alpha}y_{m\alpha})=\lambda_{ij}\lambda_{lm}+\lambda_{il}\lambda_{jm}+\lambda_{im}\lambda_{jl}$$

을 代入하면

$$E(y_{i\alpha}y_{j\alpha}-\lambda_{ij})(y_{l\alpha}y_{m\alpha}-\lambda_{lm})=E(y_{i\alpha}y_{j\alpha}y_{l\alpha}y_{m\alpha})-\lambda_{ij}\lambda_{lm}=\lambda_{il}\lambda_{jm}+\lambda_{im}\lambda_{jl} \quad 3.7)$$

이다.

그리고 $E(\sqrt{\nu}(\boldsymbol{S}-\boldsymbol{\Lambda}))=\boldsymbol{0}$ 이므로 *Lemma* 1에 依하여 要素

$$\operatorname{cov}(S_{ij}, S_{lm}) \doteqdot (\lambda_{il}\lambda_{jm}+\lambda_{im}\lambda_{jl})/\nu$$

를 가지는 共分散行列 Ψ에 對하여

$$\sqrt{\nu}(\boldsymbol{S}-\boldsymbol{\Lambda})\sim N(\boldsymbol{O}, \Psi) \quad (3.8)$$

이다.

$\boldsymbol{P}=(\rho_{ij})$ ($\rho_{ij}=\lambda_{ij}/\sqrt{\lambda_{ii}\lambda_{jj}}$), $\boldsymbol{D}_{\sqrt{\lambda}}=diag\{\sqrt{\lambda_{11}}, \cdots, \sqrt{\lambda_{kk}}\}$라 하면 $\boldsymbol{\Lambda}=\boldsymbol{D}_{\sqrt{\lambda}}\boldsymbol{P}\boldsymbol{D}_{\sqrt{\lambda}}$이다. $\boldsymbol{U}=\boldsymbol{D}_{\sqrt{\lambda}}^{-1}\boldsymbol{S}\boldsymbol{D}_{\sqrt{\lambda}}^{-1}$라 두면 $\sqrt{\nu}(\boldsymbol{U}-\boldsymbol{P})=\boldsymbol{D}_{\sqrt{\lambda}}^{-1}\{\sqrt{\nu}(\boldsymbol{S}-\boldsymbol{\Lambda})\}\boldsymbol{D}_{\sqrt{\lambda}}^{-1}$의 漸近分布는 *Lemma* 3에 依하여 平均 $\boldsymbol{O}$, 共分散行列의 要素로서

$$\nu\operatorname{cov}(u_{ij}, u_{lm}) \doteqdot (\rho_{il}\rho_{jm}+\rho_{im}\rho_{jl}) \quad (3.9)$$

을 가지는 正規分布이다. $\boldsymbol{U}$와 $\boldsymbol{P}$의 獨立인 要素를 一行으로 놓은 것을

$$\boldsymbol{u}=(u_{11}, \cdots, u_{kk}, u_{12}, \cdots, u_{1k}, u_{23}, \cdots, u_{2k}, \cdots, u_{k-1,k})$$

$$\boldsymbol{\rho}^*=(1, \cdots, 1, \rho_{12}, \cdots\rho_{1k}, \rho_{23}, \cdots, \rho_{2k}, \cdots\rho_{k-1,k})$$

라 하면 $\sqrt{\nu}(\boldsymbol{u}-\boldsymbol{\rho}^*)$는 漸近的으로 平均 $\boldsymbol{O}$, 共分散行列의 要素를 式 (3.9)로 하는 正規分布에 收斂한다.

이 結果를 適用하여 單相關係數의 漸近分布를 求하자.

$$\left.\begin{array}{l} r_{ij}=f_{ij}(\boldsymbol{u})=u_{ij}/\sqrt{u_{ii}u_{jj}} \\ \boldsymbol{r}=(r_{12}, r_{13}, \cdots, r_{1k}, r_{23}, \cdots, r_{2k}, \cdots, r_{(k-1)k}) \\ \boldsymbol{\rho}=(\rho_{12}, \rho_{13}, \cdots, \rho_{1k}, \rho_{23}, \cdots, \rho_{2k}, \cdots, \rho_{(k-1)k}) \end{array}\right\} \quad (4.1)$$

로 두면 다음 定理를 얻는다.

(定理) $\boldsymbol{r}$은 漸近的으로 平均 $\boldsymbol{\rho}$,

$$\operatorname{cov}(r_{ij}, r_{lm}) \doteqdot \frac{1}{\nu}\{\frac{1}{2}\rho_{ij}\rho_{lm}(\rho_{il}^2+\rho_{im}^2+\rho_{jl}^2+\rho_{jm}^2)+\rho_{il}\rho_{jm}+\rho_{im}\rho_{jl}$$
$$-\{\rho_{ij}\rho_{il}\rho_{im}+\rho_{ji}\rho_{jl}\rho_{jm}+\rho_{li}\rho_{lj}\rho_{lm}+\rho_{mi}\rho_{mj}\rho_{ml})\} \quad (4.2)$$

$i=l$ 또는 m일 때는

$$\operatorname{cov}(r_{ij}, r_{im}) \doteqdot \frac{1}{\nu}\{\frac{1}{2}(2\rho_{jm}-\rho_{ij}\ \rho_{im})(1-\rho_{ij}^2-\rho_{im}^2-\rho_{jm}^2)+\rho_{jm}^3\} \quad (4.3)$$

또 $i=l$, $j=m$일 때는

$$\mathrm{cov}(r_{ij},\ r_{ij})=\mathrm{var}(r_{ij})\quad \frac{1}{\nu}(1-\rho_{ij}^2)^2 \tag{4.4}$$

을 共分散行列의 要素로 하는 正規分布에 收斂한다

(證明) *Lemma* 2에 依하여 $\sqrt{\nu}(\boldsymbol{r}-\boldsymbol{\rho})$는 漸近的으로 平均 $\boldsymbol{O}$인 $k(k-1)/2$變量의 正規分布에 收斂함은 明白하다. 問題는 共分散行列 $\boldsymbol{A\Psi A'}$를 計算하는 것이다.

$\boldsymbol{A}$의 要素는 $[\partial r_{ij}/\partial u_{lm}]_0$, Ψ의 要素는 $\nu\mathrm{cov}(u_{ij},\ u_{lm})\doteqdot\rho_{il}\rho_{jm}+\rho_{im}\rho_{jl}$이다. 먼저 四變數의 境遇$(r_{12},\ r_{13},\ r_{14},\ r_{23},\ r_{24},\ r_{34})$에 對한 $\boldsymbol{A}$를 求해보자. 여기서는 $E(u_{11})=E(u_{22})=E(u_{33})=E(u_{44})=1$로 두었으므로 $\boldsymbol{A}$는

	(11)	(22)	(33)	(44)	(12)	(13)	(14)	(23)	(24)	(34)
(12)	$-\frac{1}{2}\rho_{12}$	$-\frac{1}{2}\rho_{12}$	0	0	1	0	0	0	0	0
(13)	$-\frac{1}{2}\rho_{13}$	0	$-\frac{1}{2}\rho_{13}$	0	0	1	0	0	0	0
(14)	$-\frac{1}{2}\rho_{14}$	0	0	$-\frac{1}{2}\rho_{14}$	0	0	1	0	0	0
(23)	0	$-\frac{1}{2}\rho_{23}$	$-\frac{1}{2}\rho_{23}$	0	0	0	0	1	0	0
(24)	0	$-\frac{1}{2}\rho_{24}$	0	$-\frac{1}{2}\rho_{24}$	0	0	0	0	1	0
(34)	0	0	$-\frac{1}{2}\rho_{34}$	$-\frac{1}{2}\rho_{34}$	0	0	0	0	0	1

行列 $\boldsymbol{A}$의 (ij)行 (lm)列의 要素를 $((ij)(lm))$로 表示하면 一般的으로 (ij)行의 要素는 $((ij)(ii))=((ij)(jj))=-\frac{1}{2}\rho_{ij}$, $((ij)(ij))=1$이고 그외의 要素는 0이다.

다음에 $(r_{12},\ r_{13},\ r_{14},\ r_{23},\ r_{24},\ r_{34})$에 對한 $\boldsymbol{\Psi}$를 셈하면 行列은 (10×10)型이고 $E(u_{ii})=1$, $i=1,2,3,4$, $\nu\mathrm{cov}(u_{ij},\ u_{lm})\doteqdot(\rho_{il}\rho_{jm}+\rho_{im}\rho_{jl})$이므로 다음과 같이 된다.

	(11)	(22)	(33)	(44)	(12)	(13)	(14)	(23)	(24)	(34)
(11)	2	$2\rho_{12}^2$	$2\rho_{13}^2$	$2\rho_{14}^2$	$2\rho_{12}$	$2\rho_{13}$	$2\rho_{14}$	$2\rho_{12}\rho_{13}$	$2\rho_{12}\rho_{14}$	$2\rho_{13}\rho_{14}$
(22)	$2\rho_{12}^2$	2	$2\rho_{23}^2$	$2\rho_{24}^2$	$2\rho_{12}$	$2\rho_{12}\rho_{23}$	$2\rho_{12}\rho_{24}$	$2\rho_{23}$	$2\rho_{24}$	$2\rho_{23}\rho_{24}$
(33)	$2\rho_{13}^2$	$2\rho_{23}^2$	2	$2\rho_{34}^2$	$2\rho_{13}\rho_{23}$	$2\rho_{13}$	$2\rho_{13}\rho_{34}$	$2\rho_{23}$	$2\rho_{23}\rho_{34}$	$2\rho_{34}$
(44)	$2\rho_{14}^2$	$2\rho_{24}^2$	$2\rho_{34}^2$	2	$2\rho_{14}\rho_{24}$	$2\rho_{14}\rho_{34}$	$2\rho_{14}$	$2\rho_{24}\rho_{34}$	$2\rho_{24}$	$2\rho_{34}$
(12)	$2\rho_{12}$	$2\rho_{12}$	$2\rho_{13}\rho_{23}$	$2\rho_{14}\rho_{24}$	$1+\rho_{12}^2$	$\rho_{23}+\rho_{12}\rho_{13}$	$\rho_{24}+\rho_{12}\rho_{14}$	$\rho_{13}+\rho_{12}\rho_{23}$	$\rho_{14}+\rho_{12}\rho_{24}$	$\rho_{13}\rho_{24}+\rho_{23}\rho_{14}$
(13)	$2\rho_{13}$	$2\rho_{12}\rho_{23}$	$2\rho_{13}$	$2\rho_{14}\rho_{34}$	$\rho_{23}+\rho_{12}\rho_{13}$	$1+\rho_{13}^2$	$\rho_{34}+\rho_{13}\rho_{14}$	$\rho_{12}+\rho_{13}\rho_{23}$	$\rho_{12}\rho_{34}+\rho_{14}\rho_{23}$	$\rho_{14}+\rho_{13}\rho_{34}$
(14)	$2\rho_{14}$	$2\rho_{12}\rho_{24}$	$2\rho_{14}\rho_{34}$	$2\rho_{14}$	$\rho_{24}+\rho_{12}\rho_{14}$	$\rho_{34}+\rho_{13}\rho_{14}$	$1+\rho_{14}^2$	$\rho_{12}\rho_{34}+\rho_{13}\rho_{24}$	$\rho_{12}+\rho_{14}\rho_{24}$	$\rho_{13}+\rho_{14}\rho_{34}$
(23)	$2\rho_{12}\rho_{13}$	$2\rho_{23}$	$2\rho_{23}$	$2\rho_{24}\rho_{34}$	$\rho_{12}\rho_{23}+\rho_{13}$	$\rho_{12}+\rho_{13}\rho_{23}$	$\rho_{12}\rho_{34}+\rho_{13}\rho_{24}$	$1+\rho_{23}^2$	$\rho_{34}+\rho_{23}\rho_{24}$	$\rho_{24}+\rho_{23}\rho_{34}$
(24)	$2\rho_{12}\rho_{14}$	$2\rho_{24}$	$2\rho_{24}\rho_{34}$	$2\rho_{24}$	$\rho_{12}\rho_{24}+\rho_{14}$	$\rho_{12}+\rho_{14}\rho_{24}$	$\rho_{12}+\rho_{14}\rho_{24}$	$\rho_{34}+\rho_{23}\rho_{24}$	$1+\rho_{24}^2$	$\rho_{23}+\rho_{24}\rho_{34}$
(34)	$2\rho_{13}\rho_{14}$	$2\rho_{23}\rho_{24}$	$2\rho_{34}$	$2\rho_{34}$	$\rho_{13}\rho_{24}+\rho_{23}\rho_{14}$	$\rho_{14}+\rho_{13}\rho_{34}$	$\rho_{13}+\rho_{14}\rho_{34}$	$\rho_{24}+\rho_{23}\rho_{34}$	$\rho_{23}+\rho_{24}\rho_{34}$	$1+\rho_{34}^2$

여기서 $\boldsymbol{A\Psi A'}$의 [(13)(24)]要素를 求해보자. 먼저 $\boldsymbol{A\Psi}$의 (13)行의 (22)要素, (44)要素, (24)要素를 求하면

$((13)(22)):\ -\rho_{12}^2\rho_{13}-\rho_{13}\rho_{23}^2+2\rho_{12}\rho_{23}$

$((13)(44)):\ -\rho_{13}\rho_{24}^2-\rho_{14}\rho_{34}^2+2\rho_{14}\rho_{34}$

$((13)(24)):\ -\rho_{12}\rho_{13}\rho_{14}-\rho_{13}\rho_{23}\rho_{34}+\rho_{12}\rho_{34}+\rho_{14}\rho_{23}$

이므로 $\boldsymbol{A\Psi A'}$의 [(13)(24)] 要素는

$$-\frac{1}{2}\rho_{24}(-\rho_{12}^2\ \rho_{13}-\rho_{13}\rho_{23}^2+2\rho_{12}\rho_{23})-\frac{1}{2}\rho_{24}(-\rho_{13}\rho_{24}^2-\rho_{14}\rho_{34}^2+2\rho_{14}\rho_{34})$$
$$-\rho_{12}^2\rho_{14}-\rho_{13}\rho_{23}\rho_{34}+\rho_{12}\rho_{34}+\rho_{14}\rho_{23}$$
$$=-\frac{1}{2}\rho_{13}\rho_{24}\ (\rho_{12}^2+\rho_{14}^2+\rho_{23}^2+\rho_{34}^2)+\rho_{12}\rho_{34}+\rho_{14}\rho_{23}$$
$$-(\rho_{12}\rho_{13}\rho_{14}+\rho_{21}\rho_{23}\rho_{24}+\rho_{31}\rho_{32}\rho_{34}+\rho_{41}\rho_{42}\rho_{43})$$

을 얻는다. 이것은 (4.2)의 特別한 境遇이다. 以上의 準備에서 一般의 境遇를 생각하자.

A, Ψ의 各(i,j)行과 (l,m)列의 交叉點의 要素를 各各$((i,j)(l,m))$, $[(ij)(lm)]$로 表示하기로 한다. 이때

$$\left.\begin{array}{l}((ij)(ii))=((ij)(jj))=-\frac{1}{2}\rho_{ij},\quad ((ij)(ij))=1\\ ((ij)(lm))=0\ \ (i\neq j,\ \ i\neq l,\ \ j\neq m)\\ [(ij)lm)]=\rho_{il}\rho_{jm}+\rho_{im}\rho_{jl}\end{array}\right\}\qquad (4.5)$$

이다.

$A\Psi A'$의 (ij)行 (lm)列의 交叉點의 要素를 求하려면 먼저 $A\Psi$의 (ij)行의 (ll), (mm), (lm)째 要素를 求하여야 한다. 따라서 各 要素는

$$(ij)(ll):\quad ((ij)(ii))[(ii)(ll)]+((ij)(jj))\ [(jj)(ll)]+((ij)(ij))[(ij)(ll)]$$
$$(ij)(mm):((ij)(ii))[(ii)(mm)]+((ij)(jj))[(jj)(mm)]+((ij)(ij))[(ij)(mm)]$$
$$(ij)(lm):\quad ((ij)(ii))[(ii)(lm)]+((ij)(jj))[(jj)(lm)]+((ij)(ij))[(ij)(lm)]$$

이므로 $A\Psi A'$의 (ij)行 (lm)列의 交叉點의 要素는

$$\{((ij)(ii))[(ii)(ll)]+((ij)(jj))[(jj)(ll)]+((ij)(ij))[(ij)(ll)]\}((lm)(ll))$$
$$+\{((ij)(ii))[(ii)(mm)]+((ij)(jj))[(jj)(mm)]+((ij)(ij))[(ij)(mm)]\}((lm)(mm))$$
$$+\{((ij)(ii))[(ii)(lm)]+((ij)(jj))[(jj)(lm)]+((ij)(ij))[(ij)(lm)]\}((lm)(lm))$$

으로 表示된다. 이것을 實際로 計算하면 $A\Psi A'$의 $(ij)(lm)$要素 即 $\nu\text{cov}(r_{ij}, r_{lm})$은 漸近的으로 다음과 같이 되다.

$$\nu\text{cov}(r_{ij},\ \ r_{lm})\fallingdotseq[-\frac{1}{2}\rho_{ij}(2\rho_{il}^2)-\frac{1}{2}\rho_{ij}(2\rho_{jl}^2)+2\rho_{il}\rho_{jl}](-\frac{1}{2}\rho_{lm})$$
$$+[-\frac{1}{2}\rho_{ij}(2\rho_{im}^2)-\frac{1}{2}\rho_{ij}(2\rho_{jm}^2)+2\rho_{im}\rho_{jm}](-\frac{1}{2}\rho_{lm})$$
$$+[-\frac{1}{2}\rho_{ij}(2\rho_{il}\rho_{im})-\frac{1}{2}\rho_{ij}(2\rho_{jl}\rho_{jm})+\rho_{il}\rho_{jm}+\rho_{im}\rho_{jl}]$$
$$=\frac{1}{2}\rho_{ij}\rho_{lm}(\rho_{il}^2+\rho_{im}^2\ \ +\rho_{jl}^2+\rho_{jm}^2)+\rho_{il}\rho_{jm}+\rho_{im}\rho_{jl}$$
$$-(\rho_{ij}\rho_{il}\rho_{im}+\rho_{ji}\rho_{jl}\rho_{jm}+\rho_{li}\rho_{lj}\rho_{lm}+\rho_{mi}\rho_{mj}\rho_{ml})$$

이것은 (4.2)式이다.

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船舶擔保物權에 관한 比較法的 硏究

裵 炳 泰

A Comparative Study of the Maritime Lien and the Ship Mortgage in the Law of Admiralty

By

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〈 目 次 〉

Abstract

Two kinds of legal devices for obtaining loans in the transactions of shipping business are stipulated in the Korean Code of Commerce (SS. 861-874).

They are securities or incumbrances of a ship, the one Maritime Lien, the other Ship Mortgage.

These legal systems, especially of Maritime Lien, are identical with and derived from The International Convention for the Unification of certain Rules relating to Maritime Liens and Mortgages, 1926.

In this article, the writer intends to examine the major foreign legal systems relating to securities of a ship for the correct legal reasoning of the maritime lien, and to suggest a few legal problems for supplemental legislature relating to the ship mortgage in the Korean law.

In resspect to the legal systems of securities of a ship, Anglo-American Law, German Law, French Law and International Conventions are chosen to examine to limit the scope of the