On the effect of public investment on economic growth : Model building

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공공투자가 경제성장에 미치는 영향분석

유 일 선

I. Introduction

As Lucas, Jr(1988) indicated, I would like to regard the problem of economic growth as simply that of accounting for the observed pattern accross countries and time, in levels and rates of per capita income. In fact, there is a great difference among countries. The diversity accross countries in measured per capita income levels and growth rate is literally too great to be believed[1]. So I think it very important to solve that problem in dealing with growth theory: what causes a national economy to grow and make a great variety in levels and growth rate?

Since A. Smith, a lot of economists have so far posed that problem and have been trying to solve it in their own way. R. Solow(1956) who made an first attempt to solve the problem under the framework of neoclassical economics[2], showed that in the long run, the growth rate of per capita income converged to that of technology exogiously determined, in case capital accumulation should be proportional to output. Although this model laid an epoch-making foundation on growth theory-it is often referred to as the conventional growth theory, not a few economists recognize in common that the conventional theory failed to explain sufficiently the phenomenon observed in

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1) first of all, in levels of per capita income, compared to the 1980 average for what the World Bank calls the 'Industrial Market Economies'(Ireland up through Swiss) of $10,000, India's is $240, Haiti's is $270 and so on for the rest of the very poorest countries. Rates of growth of real per capita GNP is also diverse even over sustained periods. For 1960~1980, while the industrial economies averaged 3.6%, India 1.4% per year, Egypt 3.4%, South Korea 7.0%, Japan 7.1%, U.S 2.3% and so on.

2) The main assumption about production in neoclassical economics is that production function is homogeneous of degree one(constant returns to scale)

reality. So they begin to examine again what the neoclassical economics postulate and make an effort to suggest an alternative by qualifying through the extension or a new conceptual framework.

What I aim at in this paper is an extension of the neoclassical growth model based on Solow's model by adding public sector. According to Lee(1990), the public investment made by Korean government played a major role in economic growth, while the Korean economy has undergone a rapid growth process in the last forty years especially since the early sixtie's. So the extension made here is to include public sector as an argument in utility and production function of the agents. It is assumed that govermment spending filled with all sum of income tax should be always spent on public investment and public consumption should become the constant proportion of the service flow which stems from public investment. The former enters into production function and the latter does into utility fuction. This model is related to the recent works by P. Romer(1986) and Lucas, Jr(1988) in that all two models consider the case of Increasing Returns to Scale(IRS) as a mechanism to generate endogenous growth. In this paper, there may exist IRS due to the public goods which is characteristic of nonrivalry.

The organization of the paper is as follows. In section two, I summarize the trend of the recent growth theory actively presented by P. Romer(1986, 1990) and Lucas, Jr(1988).

In section three, I set up a simple growth model in which government spending is a major element determining the dynamic growth path of economy and conclude the paper in section four.

II. Recent trend of growth theory

Even granted its limitation, the simple neoclassical Solow's Model has made basic contributions to our thinking about economic growth. Qualitatively, it put a stress on a distinction between 'growth effect'-changes in parameters that alters growth rates along balanced path-and 'level effect'-changes that raise or lower balanced growth paths without affecting their slope-that is fundamental in thinking about policy changes.

The model also predicts that countries with the same preferences and technology will necessarily converge to identical levels of income and asymptotic rates of growth. Since this prediction does not accord at all with what is observed if we want to fit the theory to observed cross-country variation and time series data, we need to stipulate appropriate variation in model.

Unlike the Solow's model, Arrow(1965) suggested that an endogenous theory of changes in knowledge which underlie intertemporal and international shifts in production function, refer to the changes in knowledge as the concept of 'learnig by doing' which is the product of

experience and which can only take place through the attempt to solve a problem and therefore take place during activity. Taking advantage of this concept, he found that 'learning effect' could become one of important sources of economic growth and provide a new possibility of IRS production function, that is much contrasted with constant returns to scale(CRS) production function which the neoclassical economics stands on. So his pioneering works had an important effect on the recent growth theory in which Romer, Lucas Jr, Helpman and Grossman play a leading role.

Recent growth theory focuses its attention on the economic environment with increasing returns to scale when investment takes place. It is well known that the marginal product of input factor need not decline over time to the level of factor prices in case of IRS, quite differently from competitive equilibrium. Then, the incentive to accumulate capital may persist indefinitely. These astute observations have revitalized the theory of economic growth. Research attention has focused primarily on the processes of accumulation of 'knowledge capital' which turns homogeneous input factor into various quality in part because the public good aspects of knowledge as information naturally create IRS in many contexts.

So, there has been an attempt to understand the determinant of long-run growth based on investment in new technology, that in human capital and public investment. First of all, as to the investment in new technology, several common features distinguish recent effort to endogenize innovation within general equilibrium models of long-run growth.

In general, this approach has two common supposition: first, a rigorous accounting of the resources used up in creating new knowledge and second, explicit consideration of the profit motive that derives private investment in R & D(Research and Development). In these matter, the new theory draws on modelling approaches developed by industrial organization economists[3].

Their modeling of R & D incorporates some of the spillovers inherent to the process of knowledge generation. As is well known, technology bears many of the characteristics of public good because knowledge as a commodity is nonrivalrous, In particular, an industrial innovation will have difficulty in preventing others from taking advantage of more general forms of scientific and engineering knowledge that are generated in the course of developing some specific product of process. As Romer(1990) has emphasized, these spillovers may cause aggregate investment in

3) In the recent work on product innovation.{Helpman & Grossman(1989), Romer(1990)} under the assumption that an entrepreneur must develop the design for a new differentiated product before it can be produced, they treat R & D as an ordinary economic activity specifying a technology that relates inputs (primary factors of production) to outputs (blueprints for news products). In other works{Helpman & Grossman (1989. c,d) and Grossman(1988)}, they model the process of quality upgrading as a set of concurrent industry-specific patent races, each aimed at developing the next generation of product.

knowledge to exhibit IRS and so allow innovation to be a sustainable process in the long run.

Next, the economists emphasize the role of human capital[4] including 'learning by doing' on economic growth. Lucas Jr, (1990) introduces human capital through schooling into the model which involves spelling out both the way human capital levels affect current production and the way the current time allocation affects the accumulation of human capital whose average level also contribute to the productivity of all factors of production[5]. He suggests that the latter create IRS and have a major effect on growth, compared to Solow's model.

In the formation of human capital, another concept, learning by doing appears to be at least as important as schooling. Based on this recognition, he also introduces an example of a system in which all human capital accumulation is due to learning by doing. Thinking about economies with many consumption goods, the model is opening up interesting new possibility for interaction between international trade and economic growth[6]. Romer(1990) classifies human capital into two different kinds of educational activities and four different product activities. Among them, especially he emphasized the facts that 'applied science' that represents the outcome of applied P & D is characteristic of being partly rivalrous and partly excludable and 'basic science' is both nonrival and nonexcludable. Due to this property output increases more than proportionally with increases in all of the inputs. So the marginal product of each input times the quality of that input summed over all inputs, yields a quantity that is greater than output which may induce persistent growth. Another factor that affects growth is public sector.

G. King & S. Rebelo(1990) showes that national taxation can substantially affect long run growth rate, within two sector endogenous growth model which has its origin in the microeconomic literature on human capital formation. In particular for small open economy with substantially capital mobility, national taxation can readily lead to 'development trap'(in which countires stagnate or regress) or to 'growth miracles' (in which countries shift from little growth to rapid expansion).

Lee(1990) paid an attention to the rapid economy growth which Korea has experienced since 1960's. After summarizing the basic facts concerning government spending activities in Korea, he then investigated the relationship between economic growth on the one hand and the relative size and compositions of government spending on the other hand. This investigation is performed by setting up a simple endogenous growth model and testing its implications. Conclusively he found that government spending had an real effect on Korean economic growth.

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4) if you want to know of 'human capital' in detail, refer to G. Becker *Human Capital*(2ed) *NBER*, 1975
5) Lucas, Jr (1988) calls the former the "internal effect of human capital" and the latter, the "external effect".
6) more detail, refer to Nancy, L. Stocky (1988)

To sum up the recent growth theory, it is closely related with analysis of the sources which creates IRS : what factor creates IRS which induce the spillover effect and externality over the whole economy and what factors really affect economc growth insteady of input factor ratio.

III. Public investment and economic growth

To explain how public sector contributes to growth, this section sets up a simple model of growth based on Lucar, Jr(1988) and Lee(1990). I consider an economy populated with a large number of homogenous consumers who have the following utility function. At date t, there are N (t) persons or equivalently manhours devoted to production.

$$u=\int_0^{\infty} e^{-et}\frac{[C(t)^{\alpha}GC(t)^{1-\alpha}]^{1-\sigma}-1}{1-\sigma}N(t)\,dt \quad \cdots\cdots(1)$$

where C(t) denotes per capita consumption of private goods, GC(t) = private consumption of public goods and parameter e, α, σ respectively represent the time discount rate, the share of consumption of private goods vs public goods and the degree of risk aversion of composite commodity. Thus the welfare of representative consumer depends on his life time consumption of private and public goods.

The representative firm has the form of Cobb-Douglas technology

$$Y(t)=A(t)K(t)^{\beta}N(t)^{1-\beta}GK(t)^{\varepsilon} \quad \cdots\cdots(2)$$

where A(t) denotes technical change, K(t) = the total stock of capital, GK(t) = public capital stock and parameter β and ε represent respectively the productivity of private and public capital. It is assumed that government in this economy imposes income tax on the agents at the rate of τ and makes full investment on the increase of public capital stock. Each consumer in the economy then, after paying taxes to the government, enjoys the proportional service flow which stems from GK(t).

Now total production is divided into three parts : total consumption, capital accumulation($\dot{K}$(t)) and public investment ($G\dot{K}(t)$)[7].

Thus

$$N(t)\ C(t)+K(t)=(1-\tau)\ A(t)\ K^{\beta}(t)\ N(t)^{1-\beta}\ GK(t)^{\varepsilon} \quad \cdots\cdots(3)$$

7) · means the rate of change of given variables

and GK(t) should be constantly equal to all sum of income tax

$$\dot{GK}(t)=\tau\ A(t)\ K^{\beta}(t)\ N(t)^{1-\beta}GK(t)^{\varepsilon} \quad \cdots\cdots(4)$$

By assumption GC(t) is proportional to GK(t)

$$GC(t)=i\ GK(t) \quad \cdots\cdots(5)$$

The paths A(t) and N(t) are given exogeneously and let each growth rate be μ and λ.

Moreover, since GK(t) and GC(t) which inplies public goods as nonrivalous is provided by government, each individual agent would regard the time part of GC, GK as given, when he intend to get the optimal path. Therefore, the resource allocation problem faced by this simple economy is as follows

$$max\ \int_0^{\infty} e^{-et}\frac{[C^{\alpha}GC^{1-\alpha}]^{1-\sigma}-1}{1-\sigma}N\ dt$$

subject to $NC+K=(1-\tau)\ Y$[8]

it can be solved by using the current value Hamiltonian H defined by

$$H(K,\theta,\ C,\ t)=\frac{N}{1-\sigma}\left([C^{\alpha}GC^{1-\alpha}]^{1-\sigma}-1\right)+\theta[(1-\tau)\ Y-N\cdot C]$$

which is just the sum of current-period utility and the rate of increase of capital valued at the potential price of capital θ.

The first order condition for maximizing H with respect to C is

$$C^{\alpha(1-\alpha)-1}\cdot GC^{(1-\sigma)(1-\alpha)}=\theta \quad \cdots\cdots(6)$$

Costate equation is that the price θ must satisfy

$$\dot{\theta}=e\theta-\frac{\partial H}{\partial K}$$

$$=[e-(1-\tau)\beta AN^{1-\beta}\ K^{\beta-1}\ GK^{\varepsilon}]\theta \quad \cdots\cdots(7)$$

These two equations are a pair of first-order differential equations in K(t) and its price θ(t),

8) Although this solution of the problem may be optimal path to individual, it cannot be so to social planner. For every variable involved in this problem should be choice variable to social planner. So this problem cannot but be suboptimal in the whole society's view

solving this system there will be a one-parameter family of paths[K(t), $\theta(t)$] satisfying the given initial condition on K(0).

The unique member of this family that satisfies the transversality condition.

$$\lim_{t\to\infty} e^{-et}\theta(t)\ K(t)=0 \quad \cdots\cdots(8)$$

is the optimal path.

In an attempt to solve this system, let's consider balanced growth path Lacar, Jr(1988) defined: the particular solution {K(t) θ(t), C(t)} such that the rates of growth in each of these variable are constant.

Let γ denote the rate of growth of per capita consumption $\dot{C}/C$ and $\delta=\dot{G}K/GK$ on a balanced growth path.

Then from (5) and (6) we have $\dot{\theta}/\theta=\{\alpha(1-\sigma)-1\}\alpha+(1-\sigma)(1-\alpha)\delta$ Next by using (7)

$$e-\{\alpha(1-\sigma)-1\}\gamma-(1-\sigma)(1-\alpha)\delta=(1-\tau)AK^{\beta-1}N^{1-\beta}GK^{\varepsilon} \quad \cdots\cdots(9)$$

(9) means that along the balanced path, the marginal product of capital must be constant. With Cobb-Douglas technology, the marginal product of capital is proportional to the average product, so that dividing (3) through by K and applying (6) we obtain

$$\frac{NC}{K}+\frac{\dot{K}}{K}=(1-\tau)\ Y(t)=\frac{e-\{\alpha(1-\sigma)-1\}r-\{\gamma-(1-\sigma)(1-\alpha)\delta\}}{\beta} \quad \cdots\cdots(10)$$

By definition of a balanced path, $\dot{K}/K$ is constant.

Therefore (10) implies that NC/K is constant. If differentiating that

$$\dot{K}/K=\frac{\dot{N}}{N}+\frac{\dot{C}}{C}=\lambda+\gamma \quad \cdots\cdots(11)$$

Thus per capita consumption capital grow at the common γ. To solve this common rate, differentiate both (4) and (10) to gain

$$r=\frac{\varepsilon\lambda+\mu}{1-(\varepsilon+\beta)} \quad \delta=\frac{(1-\beta)+\mu}{1-(\varepsilon+\beta)} \quad \cdots\cdots(12)$$

differentiating (2) and substituting (12) into the result we can obtain the growth rate of per capita income along a balanced path

$$\dot{y}/y=\frac{\mu+\varepsilon\lambda}{1-(\varepsilon+\beta)}$$ [9]

Next consider balanced net savings rate s defined by

$$s=\frac{\dot{K}(t)+G\dot{K}(t)}{Y(t)}$$

By using (10), we can derive the following easily

$$s=\frac{\beta(1-\tau)(\gamma+\lambda)}{e-\{\alpha(1-\sigma)-1\}\gamma-(1-\alpha)(1-a)\delta}+\tau \quad \cdots\cdots(13)$$

of course, the first term is saving rate achieved by private sector and the second term done by public sector. Hence along a balanced growth path, the growth rate of per capita income is proportional to the given rate of technical change μ and that of population λ.

When $\beta+\varepsilon$ do not exceed one, the high share of public sector increases the per capita growth rate and vice versa. It suggests that the excessive expenditure on public sector, distorts the private saving and also decreases its marginal product, which may affect the decrease of per capita growth rate. The rate of time preference e, the degree of risk aversion σ and the relative inportance between private goods and public goods in consumption α have no bearing on this balanced growth rate. But low time preference e and low risk aversion σ induce a high saving rates s and high savings is, in turn, associated with relatively high output levels in a balanced path. So to speak, a thrifty society will, in the long run, be wealthier than an impatient one but it will not grow faster. However it indicates that tax rate τ and the share of public sector (ε) which exists in the form of the function of γ and δ have an ambiguous effect on saving rate, for private saving is probably trade-off related with public saving.

IV. Conclusion

First of all, in this paper I sum up the recent trend of growth theory. In general, the recent growth theory focuses on the sources different from the conventional growth model in that it creates IRS through spillover effect and externality of knowledge or public goods. The subject that the economists make a general reference to as the sources which induce IRS is about the investment in technology, that in human capital and public investment, So I make a simple attempt

9) for this growth rate to become the unique solution it must safisfy the transversality condition (8), that is
$(\gamma+\lambda)<e-\{\alpha(1-\sigma)-1\}\gamma-(1-\alpha)(1-\sigma)\delta$

to analyze what public sector has a real effect on growth rate by setting up two-sector dynamic growth model. Then I find public sector is considerably related with growth rate and per capita level. Incidentally, if the sum of output elasticity of capital and that of public investment is below one($\beta+\varepsilon<1$), the increase of GK may promotes growth rate.

The more inelastic capital structure a country has, the faster growth it may experience.

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