

# TRACKING CONTROLLER DESIGN FOR SYSTEMS WITH DELAY IN THE CONTROL

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## Abstract

제어입력에 무시하지 못할 정도의 시간지연이 있는 시스템에 있어서의 강인한 제어기를 설계하고 실현하는 문제를 다루었다. 본 논문에서 제안한 제어기를 설계하기 위하여 전향 방향에 적분기 하나를 추가하고 빠른 응답 특성을 주기 위하여 상태변수  $x_1$  대신 오차신호  $y-y_r$ 를 궤환 하도록 하였으며 여기서 일어날 수 있는 오우버슈터를 줄이기 위하여 Integral Reset Scheme을 이용하였다.

제어기 실현을 위하여서는 궤환되어지는 모든 상태변수의 측정이 요구되나 실제 제어 문제에서는 경제적, 기술적인 문제로 그렇지 못하므로 측정 불가능한 상태변수는 상태관측기를 구성하여 이용하도록 하였다.

## I. Introduction

In many process control problems, the systems to be controlled can contain a significant time delay. Compared to systems without delay, the presence of delay in systems greatly complicates the analytical aspects of system design and makes control more difficult to achieve. In the practical design of a control system, the delay term introduces the phase shift and tends to destabilize the closed-loop system. To counteract this problem, the gain of the controller must be reduced below that of the controller of the system without delay. Hence the system will respond more slowly to set-point changes or disturbances.

The Smith Predictor(SP) firstly introduced by Smith<sup>1</sup> uses a mathematical model of the plant to compensate the current control of the system with time delay in control. The main advantage of the Smith predictor method is that the design problem for the system with delay can be converted to the one without delay. Thus, the controller can be designed without considering delay. However, the Smith predictor control uses only the limited information, i. e. the output feedback and cannot stabilize unstable plant with delay.

On the other hand, several alternatives for systems with delay have been proposed. Manitius et al.<sup>2</sup> considered the finite spectrum assignment problems of delayed systems. Donoghue<sup>3,4</sup>, Mee<sup>5</sup>, and Watanabe et al.<sup>6</sup> investigated the delayed systems using the optimal control design approach. Donoghue compared the characteristics of both the Smith predictor and the proposed control system for the accessible state. The resulting controller is sensitive to parameter variations because of the feedforward of the reference input. Mee suggested the controller designs of multivariable systems in which different delays exist. Watanabe et al. considered the process-model control system which is essentially based on the prediction of the output of the process.

In this paper, we restrict our attention to linear SISO systems with delay in the control subject to initial conditions. The concept of the augmented system is also used because of its advantages for the treatment of disturbances. An optimal control with robustness is obtained from a non-delayed problem under minimization of the quadratic cost function. The error feedback method which feeds back the error  $y-y_r$  instead of the state variable  $x_1$  with the same feedback gain is applied to lessen sluggish responses for the set-point changes. In addition, the integral reset scheme is used to cope with overshoot problem which may be introduced by the error feedback method. In practical, since all the state may be not available for feedback, an observer is constructed to give the state estimate.

## II. Problem Formulation

A single input/output system to be controlled is described by the linear state model with time delay in the control

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t-L)$$

$$(S) \quad (1)$$

$$y(t) = \mathbf{C}\mathbf{x}(t)$$

where  $\mathbf{x}(t)$  is the  $n \times 1$  state vector,  $u(t)$  is the control input,  $y(t)$  is the output, and  $L$  is the delay in the system.  $\mathbf{A}$  and  $\mathbf{B}$  are real constant matrices of appropriate orders and  $\mathbf{C} = [1 \ 0 \ \dots \ 0]$ . It is assumed that the pair  $(\mathbf{A}, \mathbf{B})$  is controllable and the pair  $(\mathbf{A}, \mathbf{C})$  is observable.

## III. Review of the Existing Control Systems

The Smith predictor used widely in industrial process control is shown in figure 1. In controlling a process using the SP control technique, we must implement the controller  $G_c(s)$  as well as the plant model  $G(s)(1-e^{-Ls})$ , where  $G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$ . The main advantage of the Smith predictor method is that the design problem for the delayed

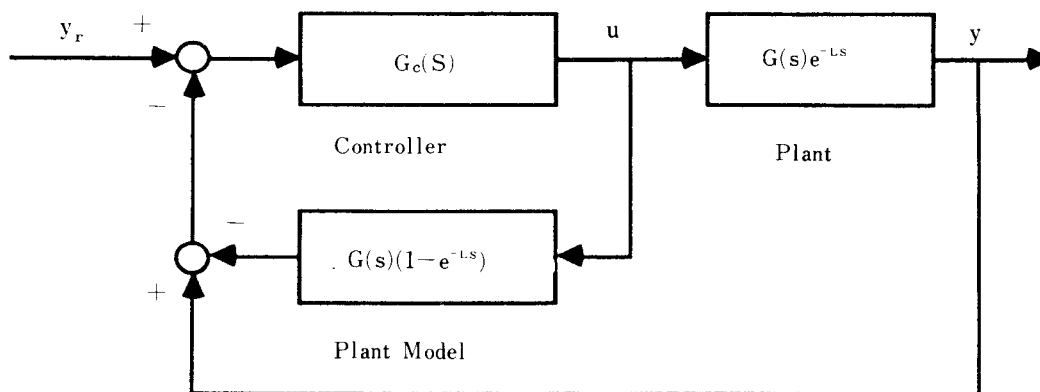


Fig. 1. Smith predictor control system

system can be converted to the undelayed one. However, it is pointed out that the Smith predictor control cannot stabilize unstable plant with delay in the control.

Donoghue<sup>3</sup> proposed an alternative design technique based on the optimal control theory for systems with delay in the control. The optimal control is given as

$$u(t) = -Kx(t+L) + H_c^{-1}(0)y_r \quad (2)$$

In equation (2),  $K$  is the feedback gain matrix,  $H_c^{-1}(0) = C[-(A-BK)]^{-1}B$ , and the state predictor is represented by

$$\begin{aligned} x(t+L) &= F(L)x(t) + \int_{t-L}^t F(t-\tau)Bu(\tau)d\tau \\ &= F(L)x(t) + \int_0^t F(t-\tau)Bu(\tau)d\tau - F(L) \int_0^{t-L} F(t-L-\tau)Bu(\tau) \\ &\quad d\tau \end{aligned} \quad (3)$$

where  $F(L) = \exp(AL)$  and  $G(s) = (sI-A)^{-1}B$ . The proposed control is implemented as shown in figure 2.

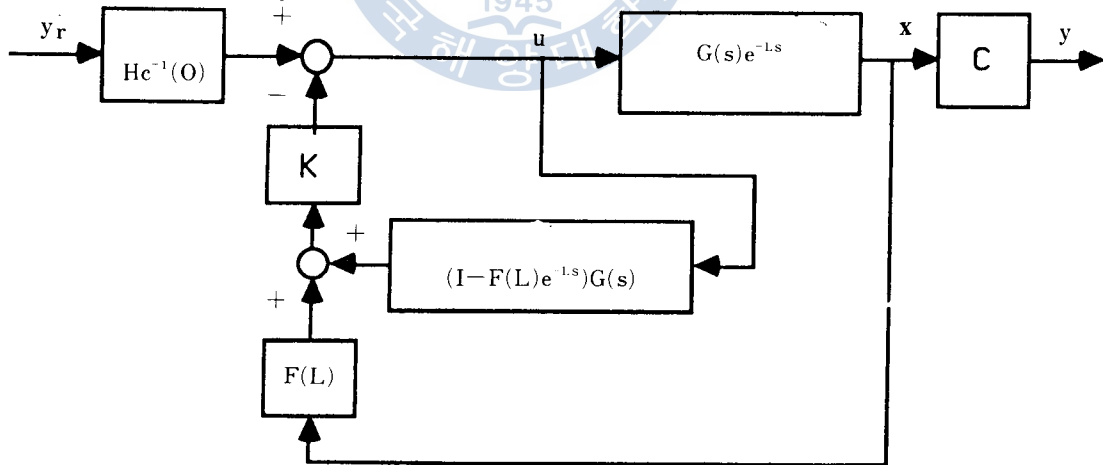


Fig. 2. Control system proposed by Donoghue

## IV. Tracking Controller Design

It is desired to design a tracking controller such that the output  $y$  becomes equal to a step reference input  $y_r$  in the steady-state. It has been shown that if the system is augmented by one integrator in the forward loop, then the output  $y$  can be forced to track the step reference input  $y_r$ .<sup>7,11</sup> If we define a new state variable as

$$\dot{z}(t) = y(t) - y_r(t), \quad (4)$$

then the augmented system is represented by

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}u(t-L) - \begin{pmatrix} 0 \\ 1 \end{pmatrix} y_r(t) \\ (\tilde{\mathbf{S}}) \quad y(t) &= \tilde{\mathbf{C}}\tilde{\mathbf{x}}(t) \end{aligned} \quad (5)$$

where

$$\tilde{\mathbf{A}} = \begin{pmatrix} \mathbf{A} & 0 \\ \mathbf{C} & 0 \end{pmatrix}, \quad \tilde{\mathbf{B}} = \begin{pmatrix} \mathbf{B} \\ 0 \end{pmatrix}, \quad \tilde{\mathbf{C}}^T = \begin{pmatrix} \mathbf{C}^T \\ 0 \end{pmatrix}, \quad \text{and} \quad \tilde{\mathbf{x}} = \begin{pmatrix} \mathbf{x} \\ z \end{pmatrix}$$

It is shown that the augmented system  $(\tilde{\mathbf{S}})$  is controllable<sup>10</sup> if and only if the system  $(\mathbf{S})$  is controllable and

$$\text{Rank} \begin{pmatrix} \mathbf{B} & \mathbf{A} \\ 0 & \mathbf{C} \end{pmatrix} = n+1.$$

We synthesize a controller as

$$u(t-L) = -\mathbf{K}\tilde{\mathbf{x}}(t). \quad (6)$$

Substituting equation (6) into equation (5) and then differentiating once, the closed-loop tracking system becomes

$$\begin{aligned} \ddot{\tilde{\mathbf{x}}}(t) &= (\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{K})\dot{\tilde{\mathbf{x}}}(t) - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \dot{y}_r(t) \\ &= (\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{K})\dot{\tilde{\mathbf{x}}}(t). \end{aligned} \quad (7)$$

If the control law of equation (6) is synthesized in such a way that all the eigenvalues of the closed-loop tracking system are assigned in the left half plane, from equation (7)

$$\dot{\tilde{\mathbf{x}}}(t) = \begin{pmatrix} \dot{\tilde{\mathbf{x}}} \\ \dot{\tilde{z}} \end{pmatrix} \rightarrow \mathbf{0} \text{ as } t \rightarrow \infty, \text{ i. e.}$$

$$\dot{\tilde{z}} = y - y_r \rightarrow 0 \text{ as } t \rightarrow \infty$$

This means that equation (6) is a satisfactory control which can track the step command inputs. It has been shown that there always exists such a control which can stabilize the system  $(\tilde{S})$  and an optimal control can be obtained<sup>9</sup> with slight modification of the system  $(\tilde{S})$ , i. e. with  $y_r = 0$ . Define the cost function  $J$  as

$$J = \int_L^\infty [\tilde{\mathbf{x}}^T(t) Q \tilde{\mathbf{x}}(t) + r u^2(t-L)] dt \quad (8)$$

where  $Q = Q^T$  is a positive semidefinite weighting matrix and  $r$  is a positive coefficient. The lower limit on  $J$  is taken to be  $L$  because, first, the control input  $u(t)$  for  $0 \leq t < \infty$  can only affect the states for  $L \leq t < \infty$  and secondly, we are choosing the control input for  $0 \leq t < \infty$ <sup>3,4</sup>. Then the optimal control law which minimizes  $J$  becomes equation (6). In equation (6), the gain matrix  $K$  is  $\tilde{B}^T P / r$  and the matrix  $P$  is the solution of the matrix Riccati equation.  $K$  is determined by the choice of the weighting matrix  $Q$  and the coefficient  $r$ .

Equation (6) can be rewritten as

$$\begin{aligned} u(t) &= -K_1 \mathbf{x}(t+L) - k_2 z(t+L) \\ &= -K_1 \mathbf{x}(t+L) - k_2 \int_0^t [y(\tau+L) - y_r] d\tau \end{aligned} \quad (9)$$

where  $K = [K_1 \ k_2]$ . An important aspect in equation (9) is that the control input at time  $t$  depends on the state predicted  $L$  time units ahead into the future. The state predictor at time  $t+L$  can be given as equation (3). An implementation of the state predictor and the optimal controller from equations (3) and (9) is shown in figure 3.

This controller is less sensitive for parameter variations than that of Donoghue since there is no feedforward for the reference input but unfortunately, tends to give a slower response for set-point changes. Therefore, the error feedback technique is employed to speed up its response. The basic idea is that  $y - y_r$  is fed back instead of the state variable  $x_1$  on the left hand side of equation (9) with the same feedback gain. Then, equation (9) can be rewritten as

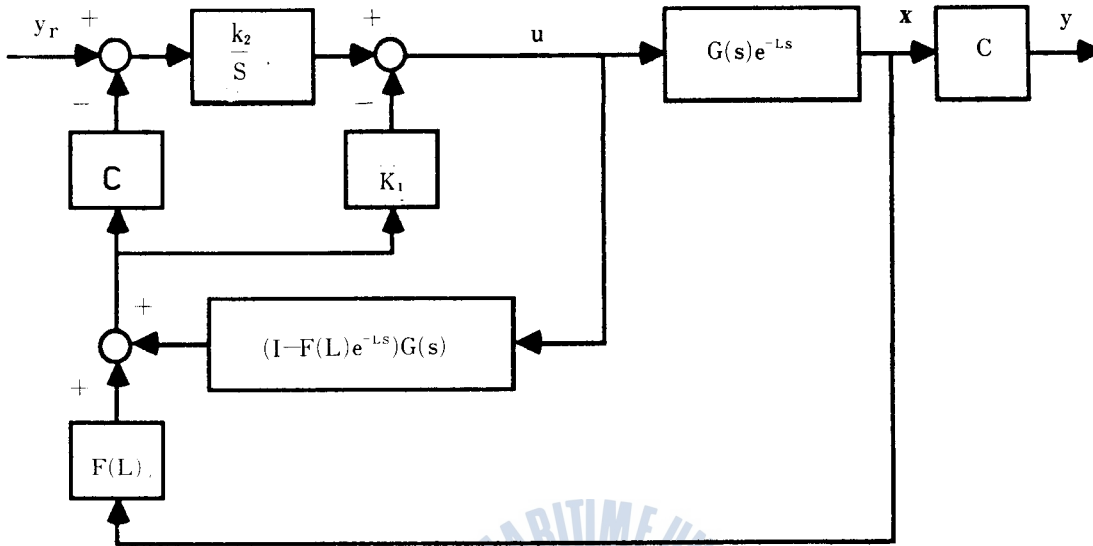


Fig. 3. Optimal tracking control system with delay in the control

$$\begin{aligned}
 u(t) &= -K_1 D[y(t+L) - y_r] - K_1 [\mathbf{x}(t+L) - D\mathbf{x}_1(t+L)] - k_2 \int_0^t [y(\tau + L) - y_r] d\tau \\
 &= -K_1 \mathbf{x}(t+L) - k_2 \int_0^t [y(\tau + L) - y_r] d\tau + K_1 D y_r
 \end{aligned} \quad (10)$$

where  $D = [1 \ 0 \ \dots \ 0]^T$ . The modified closed-loop system based on equation (10) is shown in figure 4.

As far as the stability is kept, the resulting controller is robust because the controller itself compensates the feedforward control. However this approach may introduce overshoots when the weighting matrix  $Q$  is increased for a rapid response. Therefore, the problem to reduce overshoots may be desired.

There are three well known approaches, set point filtering, model following, and integral reset, which attempt to reduce the large overshoot problem happened in the practical control applications. A very simple method to lessen overshoot is to reset the integral action in equation (10) to zero when the output  $y$  first reaches 90 per cent of the set point  $y_r$ . Since the integral action until the output  $y$  reaches the reference input  $y_r$  still

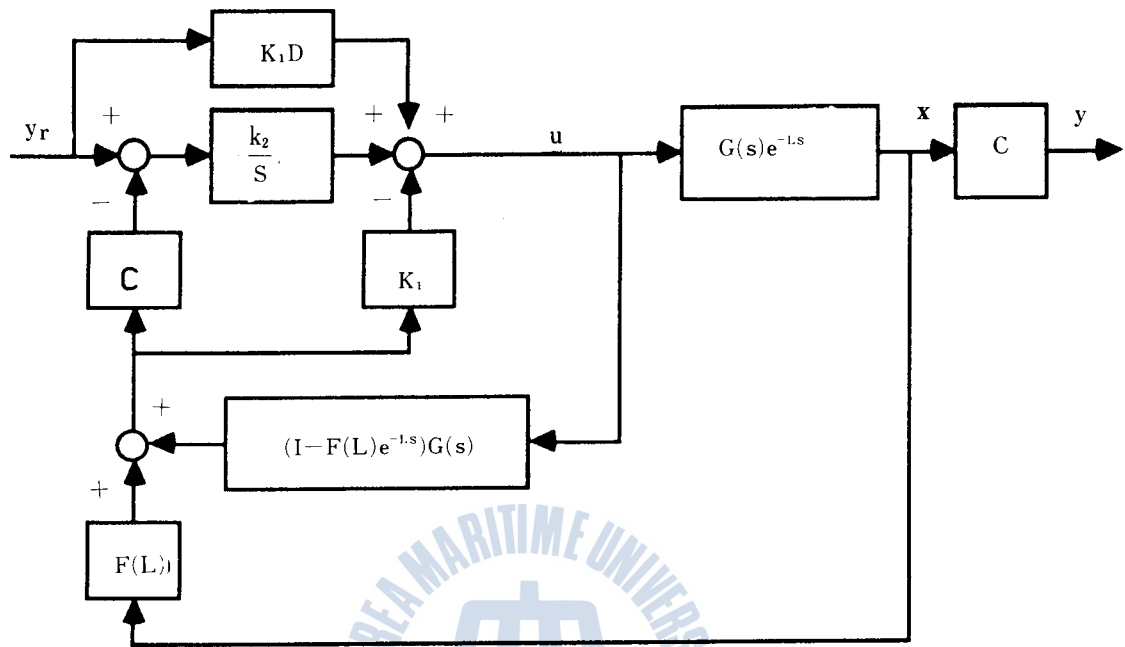


Fig. 4. Error feedback control system with delay in the control

influences  $y$  after  $y$  does  $y_r$ , this approach is to set the size of the integral term zero. Moreover, this method makes full use of the integral control action initially and the extra effort required for implementation is just to reset the variable  $z(t)$ . But this reset scheme is experimentally found to be not applicable for the system with a slower response and small overshoot. We should not use this scheme such a system.

## V. State Observer Design

The synthesis of the designed control law requires all the state variables. In practical situations, the state  $x(t)$  may not be accessible directly. Therefore, the state vector must be estimated from the input and output of the controlled plant in the presence of noise. As the pair  $(A, C)$  is assumed to be observable, we can estimate the state by means of the Luenberger observer<sup>13</sup> given by



$$\dot{\hat{\mathbf{x}}}(t) = A\hat{\mathbf{x}}(t) + Bu(t-L) + G[y(t) - C\hat{\mathbf{x}}(t)] \quad (11)$$

where the observer feedback matrix  $G$  can be obtained by the pole assignment such that  $\hat{\mathbf{x}}(t)$  approaches  $\mathbf{x}(t)$  asymptotically as  $t \rightarrow \infty$ . If the eigenvalues of  $(A-GC)$  are put further into the left half-plane so that the approach of  $\hat{\mathbf{x}}(t)$  to  $\mathbf{x}(t)$  is as rapid as possible, the effect of the noise increases since the effective bandwidth of the estimator increases. It is necessary to trade off speed of estimation against minimization of the effect of the noise.

Figure 5 shows the tracking control system incorporating the observer.

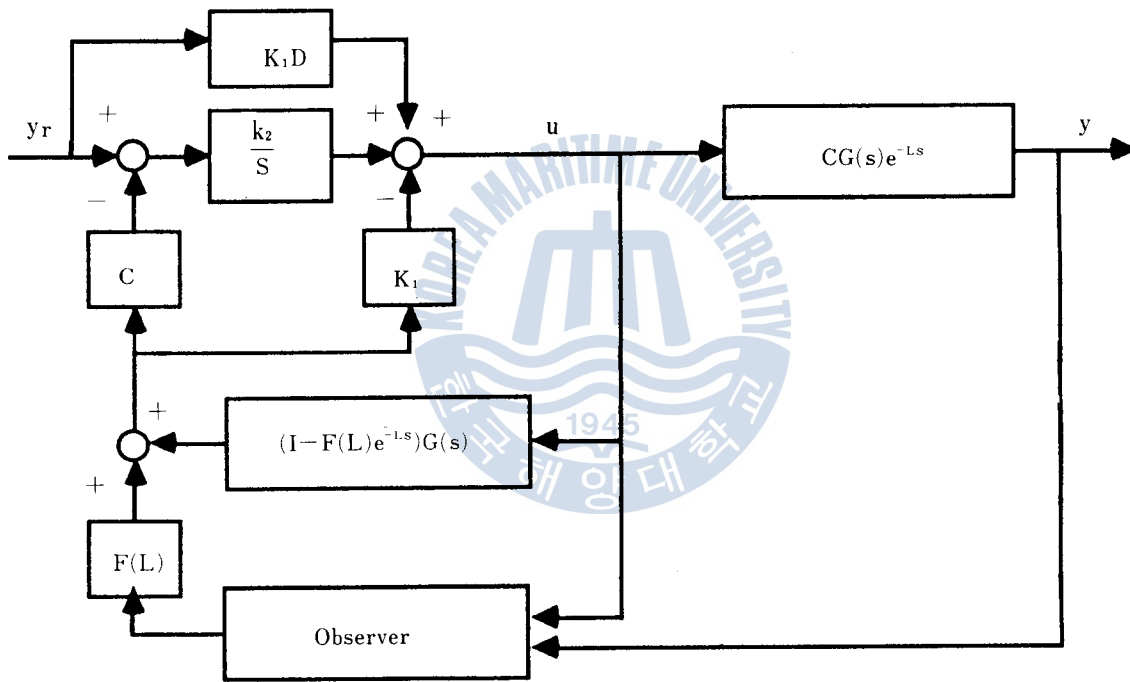


Fig. 5. Joint state tracking controller-observer system.

## VI. Numerical Examples

Example 1 :

Consider a temperature control system described by

$$\dot{x} = -0.4x + u(t-2), \quad x(0) = 0.1$$

$$y=x,$$

$Q=\text{diag}(100,2000)$  and  $r=1$  for the cost function are chosen such that the 2% settling time for the step reference input is less than 0.8 seconds. The corresponding optimal control law is numerically obtained by

$$u(t) = -13,370x(t+L) - 44,721 \int_0^t [y(\tau+L) - y_r] d\tau + 13,370y_r \quad (12)$$

The Smith predictor method, the Donoghue method, and the proposed method are calculated for the comparison as shown in figure 6, where the initial condition of the plant model is set to zero.

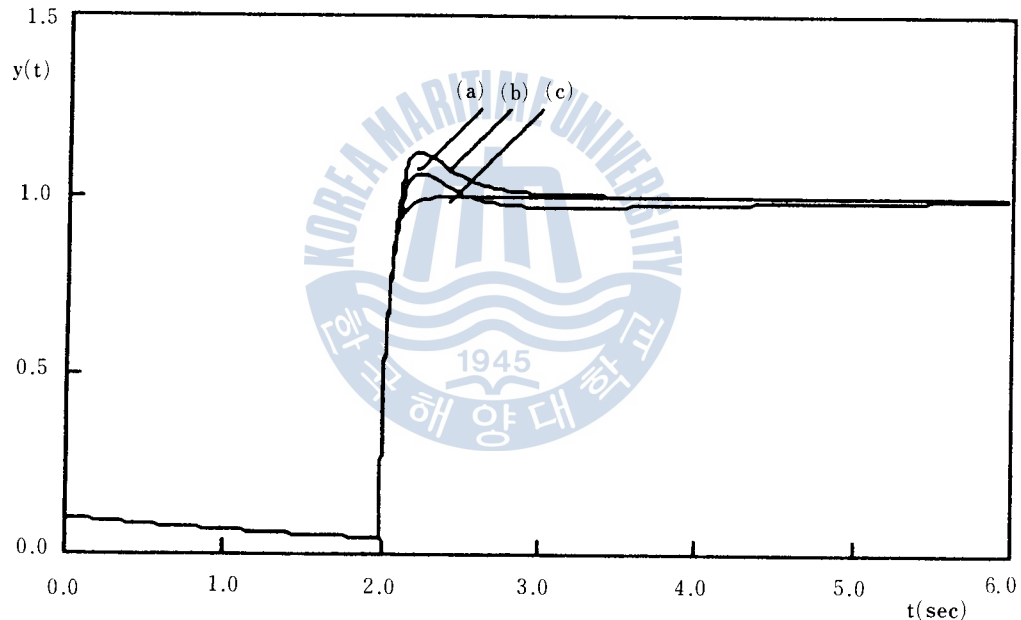


Fig. 6. Step responses

Curve (a) in figure 6 shows the step response of the Smith predictor control system. The result of curve (a) is possible by using the structure depicted in figure 1 with the proportional gain  $K_p=13,370$  and the integral time  $T_1=0,299$ . Curve (b) shows the step response of the augmented system proposed by Donoghue with  $k_1=13,370$  and  $k_2=44,721$ . Applying the controller structure of figure 4 with the same gains, an improved response is achieved like curve (c). Obviously, the tracking behaviour is considerably improved by the proposed scheme.

Example 2 :

Consider an unstable system given by

$$\dot{x} = 0.4x + u(t-2), \quad x(0) = 0.1$$

$$y = x$$

If the same Q and r are chosen, the corresponding optimal control becomes

$$u(t) = -14.169x(t+L) - 44.721 \int_0^t [y(\tau+L) - y_r] d\tau + 14.169y_r \quad (13)$$

It is seen that, in figure 7, both the Donoghue method(b) and the proposed method(c) can track the step input but the Smith predictor method(a) cannot, that is, gives the unstable response as mentioned already.

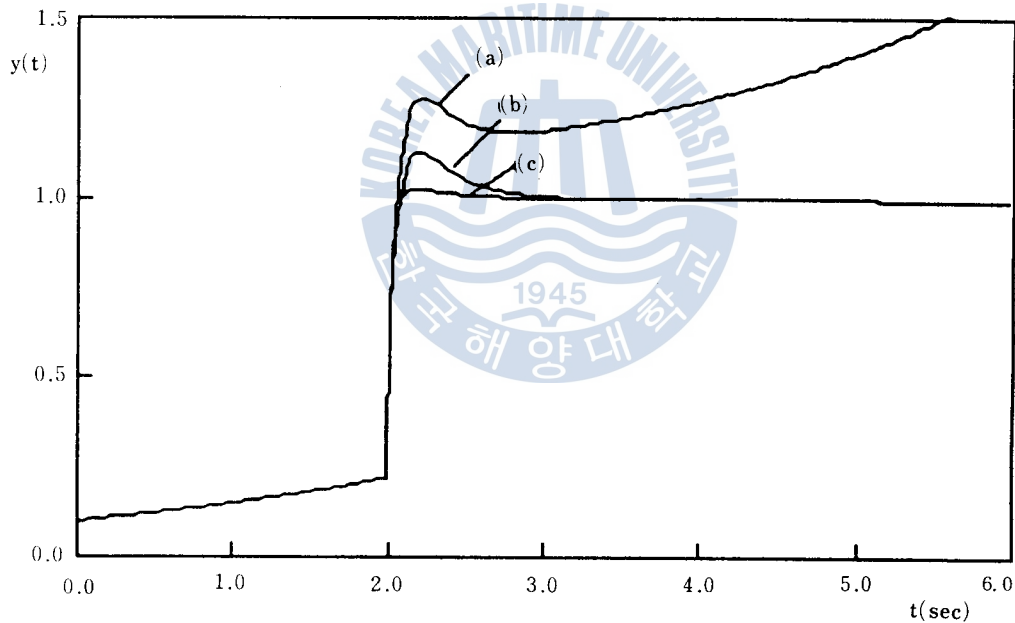


Fig. 7. Step responses

Example 3 :

Consider a mixture concentration control system given by

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ 0 & -0.4 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t-1.2), \quad \mathbf{x}(0) = [0.1 \ 0]^T$$

$$y = [1 \ 0] \mathbf{x}$$

With  $Q = \text{diag}(1 \times 10^4, 5 \times 10^2, 5 \times 10^4)$  and  $r = 1$ , the optimal control law becomes

$$u(t) = -105.528x_1(t+L) - 27.906x_2(t+L) - 223.607 \int_0^t [y(\tau + L) - y_r] d\tau + 150.528y_r \quad (14)$$

Similar comparisons such as figure 6 are not shown in figure 8 because the structures of the three control systems are quite different if the order of the open-loop system is greater than or equal to 2.

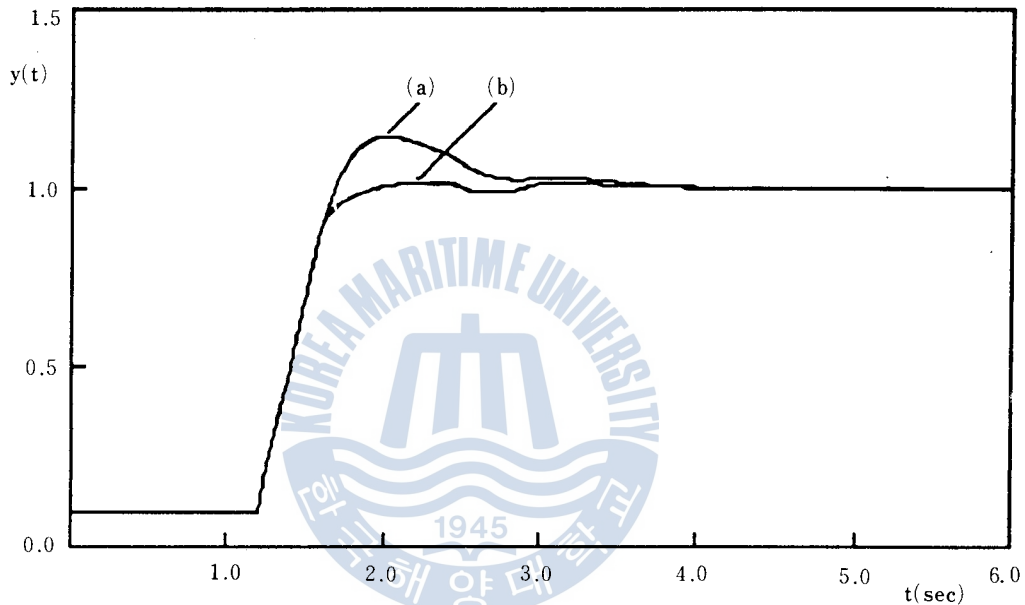


Fig. 8. Step response with observer.

The state variables are reconstructed by the observer and the gain are chosen such that the poles of the observer are assigned to  $s = -10$  and  $-20$ . Figure 8 shows the step responses of the proposed control system with and without the integral reset scheme incorporating the observer, where the initial condition of the observer is set to zero. It is apparent that overshoot of the curve(a) can be completely eliminated by the integral reset scheme as the curve(b).

## VII. Conclusion

A tracking controller has been designed and implemented for a single input/output

system with time delay in the control. The proposed controller gives fast response without producing large overshoot and maintains zero steady-state error for step reference inputs. The proposed approach is based on obtaining the optimal control of the augmented system with one additional integrator at the forward path. Then, the error feedback method followed by the integral reset scheme is applied to speed up the responses without producing large overshoots. In practical situations, as the entire state may not be accessible, an observer is combined to synthesize the controller.



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