

# The State Estimator for Underwater Vehicle Systems

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**ABSTRACT :** *In this paper, an estimation method for estimating the states of underwater vehicle systems with external unknown disturbance is proposed. First, the dynamics of underwater vehicle are induced by Taylor series expansion in the vertical plane and horizontal plane, respectively. For constructing the vehicle model, the external efforts, i.e., the sea surface disturbances, the current, wave and etc., are regarded as external unknown disturbances. Thus the disturbance is added as external input into state-space form of underwater vehicle systems. To estimate the state of systems with unknown disturbance, a disturbance observer which does not effected the external unknown input is proposed, and the existence condition for the observer is given. Finally, the effectiveness of the proposed disturbance observer for underwater vehicle systems is verified by using numerical simulation*

**KEY WORDS :** *Estimation, Underwater Vehicle, External Disturbance*

## 1. Introduction

Recently, the use of underwater vehicle continues to increase the needs in commercial and military which interest in searching and exploring the oceans and coastal plains (Třebi-Ollennu and White 1996, You 1997, You and Chai 1998, Giuseppe and Andrea 1998, Lee et.al. 1999, Zoran et.al. 1999). The underwater vehicle is generally classified into ROV (Remotely Operated Vehicle) and AUV (Autonomous Underwater Vehicle), and the design of autopilot for the underwater vehicle is of interest both the view of motion stabilization as well as maneuvering and tracking performance.

In order to achieve the given control performance, the control system must have stability to make effective the use of all information. For this, several literatures are proposed to design the robust control algorithm which concerns the model uncertainty and modeling error for thruster dynamics (Třebi-Ollennu and White 1996, You 1997). Also other control case, sliding mode control has been successfully applied to underwater vehicle (Lee, 1999).

To performs the given control tasks, the underwater vehicle requires not only a robust controller but also variable sensor which helps measuring the vehicle's position and attitude by using inertial measurement unit, gyros, Doppler sonar etc.. To ensure the reliability and autonomy of the vehicle,

additional sensors are required. But, this additional sensors need power consumption and make a heavy production. To avoid the problem, generally observer and estimator working with the sensor are designed to estimate the states of unmeasured state signals. However, the sea surface disturbance, the current, wave and etc. effects to the underwater vehicle directly. When the observer technique is used to estimate the states of system, the above efforts should be considered.

Recently, in observer design fields, the disturbance observer and PI observer are generally used to estimate the state of system with step disturbance (Kim, 1996) and applied to real plane. But in case of unknown external disturbance, observer design methods are still needs and verified in the real plant.

In this paper, an estimation method for estimating the states of underwater vehicle systems with external unknown disturbance is proposed. First, the dynamics of underwater vehicle are induced by Taylor series expansion in the vertical plane and horizontal plane, respectively. To construct the vehicle model, the external efforts, i.e., the sea surface disturbances, the current, wave and etc., are regarded as external unknown disturbances. Thus the disturbance is added as external input into state-space form of underwater vehicle systems.

To estimate the state of systems with unknown disturbance, a disturbance observer which does not effected the external unknown input is proposed, and the existence condition for

the observer is given. Finally, the effectiveness of the proposed disturbance observer for underwater vehicle systems is verified by using numerical simulation.

## 2. Underwater Vehicle Modeling

### 2.1 Vehicle kinematics and dynamics

Generally, the motion analysis of underwater vehicle is considered with vehicle's dynamics in which vertical motion and horizontal motion.

Fig. 1 shows two orthonormal coordinate systems for the diving plane:  $(O-X, Y, Z)$  is the initial reference frame which fixed on sea surface and  $(o-x, y, z)$  is the body-fixed frame which fixed on vehicle's center of gravity(CG), where  $\delta_b$  and  $\delta_s$  denote the bow hydroplane angle and stern hydroplane angle, respectively. Using this two hydroplane angles, the underwater vehicle is controlled its diving plane guidance(depth/pitch).

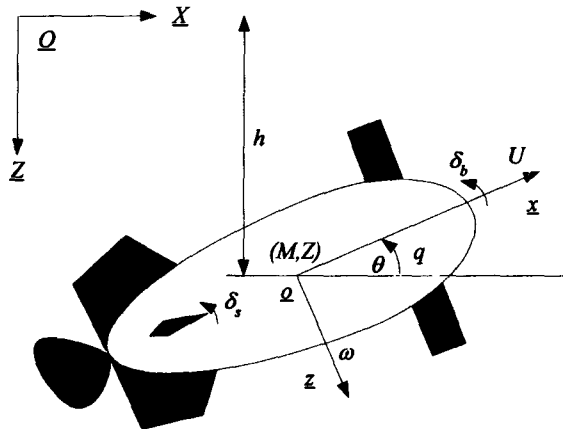


Fig. 1 Underwater vehicle system configuration

For linearization of the vehicle, Taylor series expansion was be taken into partial derivatives of forces or moments terms.

Let us define stability derivative coefficients  $Z_q$  and  $M_q$

$$Z_q \approx \left[ \frac{\partial Z}{\partial q} \right]_{q_0}$$

$$M_q \approx \left[ \frac{\partial M}{\partial q} \right]_{q_0}$$

Considering only the diving plane motions with constant forward speed  $U$ , the coupled differential equations for vehicle motions are written in the body frame as follows:

○ Heave motion:

$$\begin{aligned} m_v(\dot{w} - Uq\dot{q}) &= \frac{\rho}{2}l^2[Z_wUw + U^2(Z_{\delta_b}\delta_b + Z_{\delta_s}\delta_s)] \\ &+ \frac{\rho}{2}l^3(Z_qUq + Z_w w) + \frac{\rho}{2}l^4Z_qq + (\dot{W}_w - \dot{W}_B)\cos\theta \\ &+ g_F \end{aligned} \quad (1)$$

□ Pitch motion:

$$\begin{aligned} I_y\dot{q} - m_v[x_G(\dot{w} - Uq)] &= \frac{\rho}{2}l^3[M_wUw + U^2(M_{\delta_b}\delta_b + \\ &M_{\delta_s}\delta_s)] + \frac{\rho}{2}l^4(M_qUq + M_w\dot{w}) + \frac{\rho}{2}l^5M_q\dot{q} - (x_GW_w \\ &- x_BW_B)\cos\theta - (z_GW_w - z_BW_B)\sin\theta + g_M \end{aligned} \quad (2)$$

In eqns. (1) and (2), the hydro-dynamics and external forces are shown. Also, the underwater vehicle system is effected by sea surface disturbance, the current, wave, other external efforts and etc.. To simplify the vehicle model, we assumed that these various sea conditions change slowly and affect the forces and moments of underwater vehicle systems. From this, we have  $g_F = b_1w_1$  and  $g_M = b_2w_2$  (You 1996).

### 2.2 State-space model

In above section, the kinematic relations which determine the CG path relative to the inertial reference frame are obtained as

$$\dot{\theta} = q \quad (3)$$

$$\dot{h} = w\cos\theta - U\sin\theta + w\cos\theta \approx w - U\theta \quad (4)$$

where the pitch angle  $\theta$  is small (approximately  $\sin\theta \approx \theta$  and  $\cos\theta \approx 1$ ).

Without loss of generality, the following assumptions are defined for generating the state-space form:

- The rudders are locked for the pitch plane
- The underwater vehicle is a rigid with having a port/starport symmetric structure
- The metacentric height  $z_{GB} (= z_G - z_B > 0)$  is equal to  $z_G$  with  $z_B = 0$
- The underwater vehicle is neutrally buoyant, or  $W_w = W_B$
- The body  $x$ -frame of CG is equal to that of buoyancy,

$$\text{or } \underline{x}_G = \underline{x}_B$$

Let us define a state vector, input vector and disturbance vector as

$$x = \begin{bmatrix} \delta w \\ \delta q \\ \delta h \\ \delta \theta \end{bmatrix}, u = \begin{bmatrix} \delta_b \\ \delta_s \end{bmatrix}, w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad (5)$$

where the state variables  $\delta$ , denote the variation at the steady state operating point ( $x_0 = 0$ ) ( i.e.,  $\delta w = w_0 - w$ ,  $\delta q = q_0 - q$ ,  $\delta h = h_0 - h$ , and  $\delta \theta = \theta_0 - \theta$ ).

Under the above assumptions, the linearized state-space form for underwater vehicle can be given as:

$$E_M \dot{x}(t) = E_A x(t) + E_B u(t) + E_D w(t) \quad (6)$$

where,

$$E_A = \begin{bmatrix} \mu_1 Z_w U & (m_v + \mu_2 Z_q) U & 0 & 0 \\ \mu_3 M_w U & -(m_v \underline{x}_G - \mu_3 M_q) U & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$E_B = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, E_D = \begin{bmatrix} \mu_1 Z_{\delta b} U^2 & \mu_1 Z_{\delta s} U^2 \\ \mu_2 Z_{\delta b} U^2 & \mu_2 Z_{\delta s} U^2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mu_1 = (\rho/2)l^2, \mu_2 = (\rho/2)l^3, \mu_3 = (\rho/2)l^4, \text{ and } \mu_4 = (\rho/2)l^5.$$

By using transformation of matrix  $E_M$ , the above state space equation can be rewritten as

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dw(t) \\ y(t) = Cx(t) \end{cases} \quad (7)$$

where  $A = E_M^{-1} E_A$ ,  $B = E_M^{-1} E_B$ , and  $D = E_M^{-1} E_D$ . And the dimension of these matrices are given as  $A \in R^n$ ,  $B \in R^r$ ,  $C \in R^p$ , and  $D \in R^m$ .

The purpose of this paper is to estimate the states of underwater vehicle system with external disturbance by using observation techniques. And the detail observation techniques will represent in next section.

### 3. State Estimation by using Unknown Disturbance Observer

In this section, we construct the state estimator for estimating the states of underwater vehicle system by using unknown disturbance observer.

In underwater vehicle system eq. (7), firstly we assume that  $p \geq m$  and without loss of generality, rank  $D = m$  and rank  $C = p$ .

Consider a full-order observer described by

$$\Sigma_{UD} : \begin{cases} \dot{z} = \hat{A}z + \hat{B}y + \hat{J}u \\ \hat{x} = \hat{C}z + \hat{D}y \end{cases} \quad (8)$$

where  $z \in R^n$  and  $\hat{x} \in R^n$  denote a transformed estimated vector and an estimated state vector, respectively.  $\hat{A}$ ,  $\hat{B}$ ,  $\hat{C}$ ,  $\hat{D}$ , and  $\hat{J}$  are unknown matrices of appropriate dimensions.

**Definition 1 :** The system  $\Sigma_{UD}$  is said to be an unknown disturbance full order observer for the linear systems  $\Sigma$  if and only if

$$\lim_{t \rightarrow \infty} e(t) = 0, \forall x(0_-), z(0_-), u(\cdot)$$

where  $e(t) = \hat{x}(t) - x(t)$  represents the observer error.  $\square$

Define an estimation error as

$$\xi = z - Ux \quad (9)$$

Then, the dynamics of observation error is given by

$$\begin{aligned} \dot{\xi} &= \dot{z} + U\dot{x} \\ &= \hat{A}\xi + (\hat{A}U + \hat{B}C - UA)x + (\hat{J} - UB)u - UDv \end{aligned} \quad (10)$$

The states of system can be estimated as

$$\hat{x} = \hat{C}\xi + (\hat{C}U - \hat{D}C)x \quad (11)$$

If there exists a matrix  $U$  which satisfies the following conditions:

$$\begin{aligned} \hat{A}U + \hat{D}C &= UA \\ \hat{J} &= UB \\ UD &= 0 \\ \hat{C}U - \hat{D}C &= I_n \end{aligned}$$

then, (10) and (11) are rewritten as

$$\dot{\xi} = \hat{A}\xi \tag{12}$$

$$\dot{x} = \hat{C}\xi + x \tag{13}$$

From the above equations, if  $\hat{A}$  is stable, then  $\xi \rightarrow 0 (t \rightarrow \infty)$  and  $x - \hat{x} = 0$ .

Thus, the system  $\Sigma_{VD}$  is an unknown disturbance full order observer for the system  $\Sigma$  with unknown disturbance vector. From the above statements, we obtain the following theorem.

**Theorem 1** (Darouach, 1994) : The system  $\Sigma_{VD}$  is an unknown disturbance observer for the system  $\Sigma$  with unknown disturbance vector, if  $\hat{A}$  is stable and there exists a matrix  $U \in R^n$  which satisfies the following conditions:

$$\hat{A}U + \hat{D}C = UA \tag{14}$$

$$\hat{J} = UB \tag{15}$$

$$UD = 0 \tag{16}$$

$$\hat{C}U + \hat{D}C = I_n \tag{17}$$

□

Here let  $\hat{C} = I_n$  for simplicity. Then, from (17), we obtain

$$U = I_n - \hat{D}C \tag{18}$$

By substitution of (18) into (14), we have

$$\hat{A} = UA - KC \tag{19}$$

$$\hat{B} = \hat{A}\hat{D} + K \tag{20}$$

where  $K = \hat{B} - \hat{A}\hat{D}$ .

Furthermore substituting (18) into (16), we have

$$\hat{D}CD = D \tag{21}$$

In order to guarantee the matrix  $\hat{D}$  satisfying (21), the following condition should be hold,

$$\text{rank}CD = \text{rank}D = m \tag{22}$$

The condition (22) requires that  $p \geq m$ , i.e., the number of measured output must be greater than or equal to that of the

external disturbance input.

The general solution of (21) can be obtained as

$$\hat{D} = D(CD)^+ + G(I_p - CD(CD)^+) \tag{23}$$

where the superscript + indicates the generalized inversion and  $G$  is an arbitrary matrix.

By substituting (23) into (18), we can get

$$U = (I_n - GC)\{I_n - D(CD)^+C\} \tag{24}$$

From the above equation, there exists a matrix  $G$ , which makes  $(I_n - GC)$  nonsingular, and then the rank  $U = n - m$ .

Since  $\text{rank} D = m$ , there exists the left-inverse of matrix  $D$ , i.e.,  $D^+D = I_m$ .

Under the condition of  $\text{rank} U = n - m$ , we have  $\text{Ker} U \cap \text{Ker} D^+ = 0$ , i.e.,

$$\text{rank} \begin{bmatrix} U \\ D^+ \end{bmatrix} = n \tag{25}$$

Then, we have the following relation

$$\text{rank} \begin{bmatrix} sI_n - A & D \\ C & 0 \end{bmatrix} = \text{rank} \begin{bmatrix} sI_n - UA \\ C \end{bmatrix} + m \tag{26}$$

Consequently, for  $\forall s \in \mathbb{C}$ , where  $\mathbb{C}$  denotes the complex space,

$$\text{rank} \begin{bmatrix} sI_n - A & D \\ C & 0 \end{bmatrix} = n + m, \forall s \in \mathbb{C} \tag{27}$$

which means that the invariant zeros of the system  $(A, D, C, 0)$  must be stable.

From the above statements, we summarize the following theorem.

**Theorem 2** : The unknown disturbance full order observer  $\Sigma_{VD}$  for the system  $\Sigma$  can be realized if

$$(i) \text{rank} CD = \text{rank} D = m \tag{28}$$

$$(ii) \text{rank} \begin{bmatrix} sI_n - A & D \\ C & 0 \end{bmatrix} = n + m, \forall s \in \mathbb{C} \tag{29}$$

For constructing an unknown disturbance observer, based on Theorem 2, the underwater vehicle system requires that the number of sensor output should be great or equal to that of disturbance input and the invariant zeros of system should be

stable.

#### 4. Simulation Results

To show the effectiveness of the proposed state estimator, we consider a model parameters for a standard underwater vehicle in Table 1 (You and Chai, 1998).

Table 1. Parameters of underwater vehicle

|                    |                         |
|--------------------|-------------------------|
| $U = 3.065 [m/s]$  | $W_B = 53,400 [N]$      |
| $W_B = 53,400 [N]$ | $\rho = 1,000 [kg/m^3]$ |
| $l = 5.3 [m]$      | $x_G = 0.0 [m]$         |
| $x_B = 0.0 [m]$    | $z_G = 6.1 [m]$         |
| $z_B = 0.0 [m]$    | $J_x = 13,587 [kgm^2]$  |
| $b_1 = 1$          | $b_2 = 10$              |
| $g = 9.81 [m/s^2]$ |                         |

And the non-dimensional hydrodynamic coefficients for the underwater vehicle are referred in You and Chai(1998).

By using above parameters, the matrices of underwater vehicle eq. (7) are obtained as

$$A = \begin{bmatrix} -0.8935 & -4.9294 & 0 & 8.1423 \\ 0.2949 & -1.4044 & 0 & -7.0743 \\ 0.1 & 0 & 0 & -3.0650 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -2.3679 & -0.6110 \\ 0.5819 & -3.0593 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0.2078 \\ 0.1922 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where the disturbance property is assumed as  $w = w_1 = w_2$ .

The matrices of disturbance observer are designed as

$$\hat{A} = \begin{bmatrix} -0.9983 & 0 & 0 & -0.0577 \\ 0.6842 & 3.1549 & 0 & -35.7491 \\ 0.2179 & -1.0000 & 0 & 3.8055 \\ -0.0577 & 1.0000 & 0 & -6.5791 \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} 0 & 0.0577 \\ 4.0394 & 21.1438 \\ -0.8249 & -6.8705 \\ 0.9249 & 6.5791 \end{bmatrix}, \hat{D} = \begin{bmatrix} 1.0000 & 0 \\ 0.9249 & 0 \\ 0 & 1.0000 \\ 0 & 0 \end{bmatrix},$$

$$U = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.9249 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & -1.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix},$$

$$\hat{J} = \begin{bmatrix} 0 & 0 \\ 2.7720 & -2.4942 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \hat{C} = I_4$$

From the above matrices, we can verify  $UD = 0$ . Thus, the designed disturbance observer can estimate the state of underwater vehicle systems without information of external disturbance.

In simulation, two kinds of external disturbance are assumed: step disturbance  $w = 0.5$  and sinusoidal disturbance  $w = 0.5\sin(0.3t)$ .

The simulation results are given in Fig. 2 - 5, respectively, where solid line denotes the real state values and dotted line denotes the estimated state values by disturbance observer.

In case of step disturbance, the estimated state variables and its real value are shown in Fig. 2 and 3.

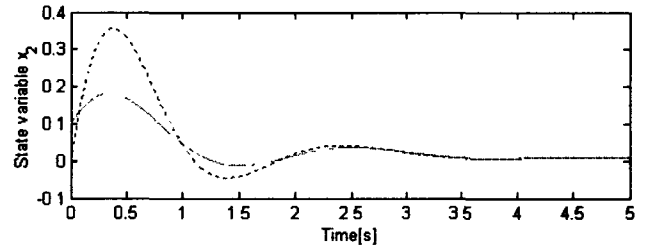


Fig. 2 Estimated variable  $\hat{x}_2$  and real value  $x_2$  with step disturbance

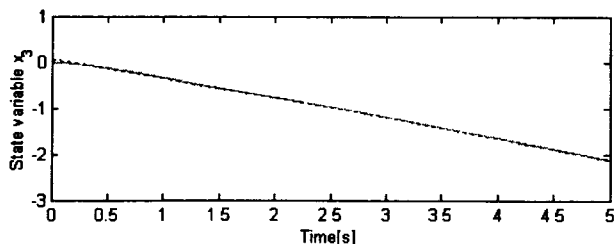


Fig. 3 Estimated variable  $\hat{x}_3$  and real value  $x_3$  with step disturbance

In case of sinusoidal disturbance, the estimated state variables and its real value are shown in Fig. 4 and 5.

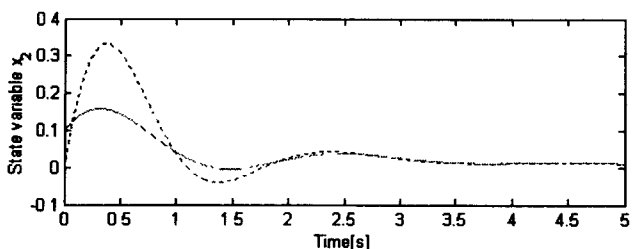


Fig. 4 Estimated variable  $\hat{x}_2$  and real value  $x_2$  with sinusoidal disturbance

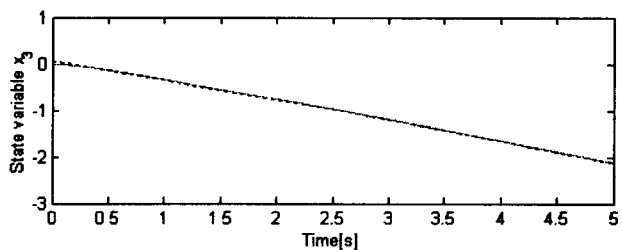


Fig. 5 Estimated variable  $\hat{x}_3$  and real value  $x_3$  with sinusoidal disturbance

From these simulation results, we can verify the effectiveness of proposed observation techniques which does not effected by unknown external disturbance.

In practically, the sea surface disturbance, the current, wave and other external efforts can be regarded as a unknown external disturbances.

### 5. Conclusion

In this paper, an estimation method for estimating the states

of underwater vehicle system with external unknown disturbance have been proposed.

Considering two planes, the vertical plane and horizontal plane, the dynamics of underwater vehicle are induced by Taylor series expansion. To construct the vehicle model for control, the sea surface disturbances, the current, wave and etc. are regarded as external unknown disturbances, and it is inserted into state-space form of underwater vehicle system as a external disturbances.

To estimate the state of systems with such unknown disturbances, a disturbance observer is proposed, and the existence condition for the observer is given. Finally, the effectiveness of the proposed disturbance observer for underwater vehicle system is verified by using numerical example.

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