The Doppler Shift Attenuation Factor for the Nuclear Lifetime Measurement

Woon Hyuk Chung

〈초 록〉

^{53}Mn 핵수명을 측정하는데 필요한 Doppler 변이 감쇄율 $F(\tau)$를 $^{53}Cr(p,n\gamma)^{53}Mn$에 대해서 큰 각도 산란 및 핵 저지능의 효과를 고려하여 계산하였다.

Ⅰ. Introduction

The average (centroid) energy of the de-excitation gamma-rays emitted by an ensemble of recoil nuclei slowed down and stopped in a solid is given by

$$\bar{E}(\theta)=E_o\left[1+F(\tau)\frac{V_o}{c}\cos\theta\right],$$

where V_o is the initial velocity of the recoil nucleus, and $F(\tau)$ is the attenuation factor related to the nuclear lifetime τ and slowing-down properties of the recoil nuclei in the stopping material.

In essence, the general problem involved in obtaining nuclear liftimes τ based on the Doppler-shift attenuation method is the determination of the attenuation factor, $F(\tau)$, theoretically and experimentally.

For any given lifetime, the value of $F(\tau)$ can be calculated if the slowing-down properties of the ion are known[1]

$$F(\tau)=\frac{\int_0^\infty V(t)\ \overline{\cos\phi(t)}\ \exp(-t/\tau)dt}{\int_0^\infty V_o\exp(-t/\tau)dt}$$

$$=\frac{1}{V_o\tau}\int_0^\infty V(t)\ \overline{\cos\phi(t)}\ \exp(-t/\tau)dt,$$

where $\phi(t)$ is the angle between the recoil direction at time t and the initial recoil direction, $\overline{\cos\phi(t)}$ denotes an ensemble average over the observed nuclei which are scattered at the angle $\phi(t)$ relative to their initial direction.

The experimental attenuation factor, $F(\tau)_{exp}$, is the ratio of the observed (average) Doppler shift to the maximum possible shift and can be expressed by[2]

$$F(\tau)_{exp} = \frac{\text{observed average shift}}{\text{maximum possible shift}}$$

$$= \frac{\triangle E}{E_o \frac{V_o}{c}(\cos\theta_1 - \cos\theta_2)}$$

where θ_1 and θ_2 are two different gamma-ray detector angles, and $\triangle E = \overline{E}(\theta_1) - \overline{E}(\theta_2)$.

Thus, comparison of the experimentally determined value of $F(\tau)$ with the calculated values will yield the nuclear lifetimes, τ.

Ⅱ. The Attenuation Factor, $F(\tau)$

At the high velocities of the recoil ion, only the electronic stopping power plays a significant role in the energy loss process, and at the low velocities the nuclear stopping power is dominant, but both may play significant roles. At high velocities, the Bethe-Bloch formula may by applied in the electronic stopping power. At low velocities, Lindhard, Scharff and Schiott have derived the electronic and nuclear stopping power formulae using the Thomas-Fermi model, which give a good over-all fit to experimental data.

Fig. 1 shows nuclear and electronic stopping powers derived by *LSS* in the low velocity region. The solid curve represents the nuclear stopping power and the broken lines are the electronic stopping powers for $k=0.15$ and $k=1.5$.

Starting from the expression for $\left(\frac{d\varepsilon}{d\rho}\right)_e$ and $\left(\frac{d\varepsilon}{d\rho}\right)_n$ derived by Lindhard et.al., Blaugrund has calculated the velocity of a recoil ion in a stopping medium as a function of time, *i.e.* Vct), and has first derived the average scattering angle resulting from multiple nuclear collisions, i.e. $\overline{\cos\phi(t)}$. By use of these effects, Blaugrund has given the average Doppler shift and the attenuation factor $F(\tau)$ in an analytical form.

For low recoil velocities, the electronic stopping power is given in $\varepsilon-\rho$ units by.[3]

$$\left(\frac{d\varepsilon}{d\rho}\right)_e = K\varepsilon^{1/2}$$

where

$$K = Z_1^{1/6}\frac{0.0793\, Z_1^{1/2} Z_2^{1/2} (A_1+A_2)^{3/2}}{(Z_1^{2/3}+Z_2^{2/3})^{3/4} A_1^{3/2} A_2^{1/2}}$$

$\approx 0.1 \sim 0.2$: normal case

> 1 : $Z_1 \ll Z_2$

$= 0.133 \sim Z_2^{2/3} A_2^{-1/2}$: $Z_1 = Z_2,\ A_1 = A_2$

$$\varepsilon = \frac{aM_2}{Z_1 Z_2 e^2 (M_1+M_2)} E$$

$$\rho = 4\pi a^2 N \frac{M_1 M_2}{(M_1+M_2)^2} R$$

$$a = 0.8853 \frac{1}{(Z_1^{2/3} + Z_2^{2/3})^{1/2}} \cdot \frac{h^2}{4\pi^2 m_e e^2}$$

Z is the atomic number, M the atomic mass,
N the number of scattering atoms per unit volume,
E the kinetic energy of the moving atom,
R the distance traveled along its path, and
a the screening parameter in the Thomas-Fermi potential.
The nuclear stopping power is given by[4]

$$\left(\frac{d\varepsilon}{d\rho}\right)_n = 0.4\ \varepsilon^{-\frac{1}{3}} \qquad : 1.2 < \varepsilon < 20$$

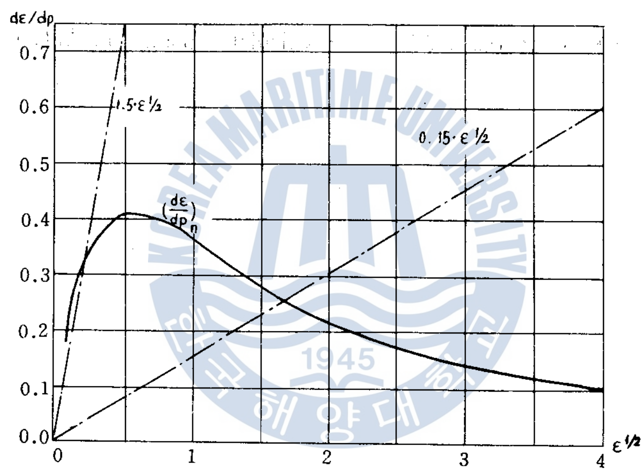

Fig. 1 Nuclear and electronic stopping powers

For the particular case of the scattering law given by *LSS*, $\overline{\cos\phi}$ is given by[1]

$$\overline{\cos\phi} = \exp\left[-\frac{1}{2}\frac{A_2}{A_1}G(r)I\right]$$

where
$$I = \int_{\varepsilon}^{\varepsilon_0} \frac{\left(\frac{d\varepsilon}{d\rho}\right)_n}{\varepsilon\left(\frac{d\varepsilon}{d\rho}\right)} d\varepsilon$$

$$G(r) = 1 + \frac{2}{3}r - \frac{7}{15}r^2 + 8\sum_{n=3}^{\infty}\frac{(-r)^n}{(2n+1)(2n-1)(2n-3)} \quad : r < 1$$

$$= \frac{2}{3} + \frac{8}{15}\,\frac{1}{r} - 8\sum_{n=3}^{\infty}\frac{\left(-\frac{1}{r}\right)^{n-1}}{(2n+1)(2n-1((2n-3)} \quad : r > 1$$

$$r = \frac{A_1}{A_2}$$

In the region $1.2<\varepsilon<20$, $\left(\frac{d\varepsilon}{d\rho}\right)$ is fairly well approximated by[1]

$$\frac{d\varepsilon}{d\rho}=0.4\ \varepsilon^{-\frac{1}{2}}+K\ \varepsilon^{\frac{1}{2}}$$

and $\overline{\cos\phi}$ is given by[1]

$$\overline{\cos\ \phi}=\left|\frac{1+\dfrac{0.4}{k\varepsilon}}{1+\dfrac{0.4}{k\varepsilon_0}}\right|^{-G/2r}$$

Once $V(t)$ and $\overline{\cos\phi}$ are known as a function of the time t, the calculation of the attenuation factor, $F(\tau)$, is straight forward.

Ⅲ. Result

The theoretical attenuation factors, $F(\tau)$, were calculated using an ALGOL program OXDS

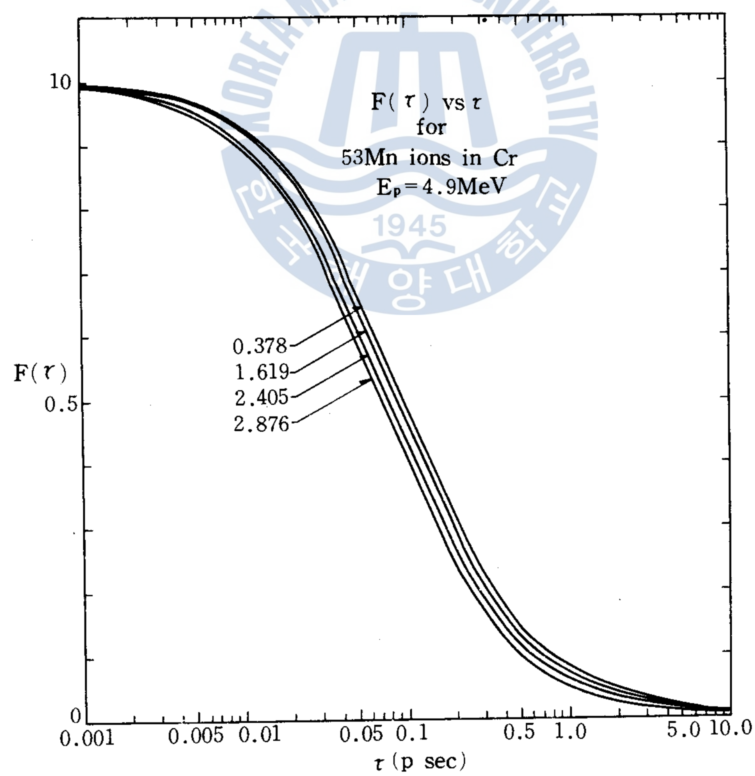

Fig. 2 $F(\tau)$ curves corresponding to ^{53}Mn ions recoiling with various initial velocities. Numbers indicate energies (in MeV) of excited states in ^{53}Mn.

which includes the effects of the large-angle scattering and the nuclear stopping power, as well as the electronic stopping power based on the Lindhard theory.

The results agree with Blaugrund's calculations to within 10% over most of the range of $F(\tau)$. Typical $F(\tau)$ versus τ curves [5] are shown in Fig. 2.

References

1) A. E. Blaugrund, Nucl. Phys. 88(1966), 501.
2) M. A. Eswaran et al., Nucl. Phys. 66(1965). 401.
3) J. Lindhard. M. Scharff and H. E. Schiott, Mat. Fys. Medd. Dan. Vid., Selsk. 33(1963) No. 14.
4) B. C. Robertson, Ph. D. Thesis, Oxford University, 1969.
5) W. H. Chung, Ph. D. Thesis, University of Alberta. 1972.