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論 文

Nonlinear Interaction of Second Order Stokes Waves and Two-Dimensional Submerged Moored Floating Structure

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2차원잠수계류부체와 2차Stokes파와의 비선형간섭에 관한 연구

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Key Words : Nonlinear Interaction of Structure-Waves(구조물과 파의 비선형간섭), Time-Domain Method(시간영역해석법), Frequency-Domain Method(주파수영역해석법), Air-chamber Flaoting Structure(압축공기주입 부체구조물), Nonlinear Wave Transformation(비선형파랑변형), Nonlinear Dynamic Behaviors(비선형동적응답).

요 지

2차의 섭동법과 경계요소법에 기초한 시간영역해석법은 불규칙파의 파동장에 있어서 파-구조물의 비선형간섭을 해석할 수 있는 해석법이지만, 파와 구조물의 운동이 정상상태에 도달하기까지 시간스텝으로 계산을 수행하여야 하므로 계산시간이 매우 길어지고, 각 성분파와 그에 의한 운동요소를 평가하는 것이 어렵다. 반면에 주파수영역해석법은 계산시간이 짧고, 각 성분요소들의 변화특성을 쉽게 판단할 수 있지만, 불규칙파동장으로의 적용이 현실적으로 어렵다는 단점을 가진다.

본 연구에서는 잠제 등에 대해서 전개되어 있는 주파수영역해석법을 임의형상의 부체구조물에 대해 새롭게 수식의 전개를 수행하고, 압축공기주입 부체구조물에 적용하여 실험 및 이론해석결과로부터 그의 타당성을 확인한다. 이 때 압축공기의 거동은 Boyle법칙을 사용하여 평가한다.

1. INTRODUCTION

A time-domain method(e.g., Isaaction et al., 1991), based on the second order perturbation expansion and boundary integral method, has been developed to study the nonlinear properties of the floating structure-wave interaction. Although the method is suitable to solve the nonlinear wave-structure interaction problems in irregular wave field, it requires long computational time until waves and motions of a floating structure attain a stationary state by a time-stepping procedure.

On the other hand, a frequency-domain method, which is also based on the second order perturbation expansion, is also available to predict the wave-structure nonlinear in-

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teraction problems in case of regular waves, and to investigate the fundamental nonlinear characteristics of the wave-structure interaction. The frequency-domain method can be classified in three groups, that is, the methods based on 1)Green's function (Vada, 1987 ; McIver et al., 1990 ; Palm, 1991 ; Kioka et al., 1993) ; 2)Green theorem (Yoshida et al., 1989) and 3)potential matching method with eigen-function expansion (Massel, 1983 ; Yoshida et al., 1990). These methods have been applied to a fixed submerged structure such as a submerged breakwater and a horizontal circular cylinder.

In this paper, nonlinear theory based on the frequency-domain method is newly developed to evaluate the wave deformation due to a submerged and moored floating structure with arbitrary shape, and the nonlinear dynamic responses of the structure. Theoretical formulation is made by the second order perturbation expansion and boundary integral method. Validity of the present theory is verified by comparing its results with experimental values obtained for the airchamber floating structure. This airchamber structure can be control well the wave transformation and its dynamic behaviors by adjusting the initial air depth in the air chamber. In addition, it can reduce the tensile force acting on the mooring line because the action of air between the structure and the water surface within the structure acts as a buffer (Fig. 2). In the application of the present theory, the second order air pressure variation in the airchamber is newly formulated by using Boyle's law, under the condition of the adiabatic process of ideal gas.

2. THEORETICAL FORMULATION

2.1 Boundary condition

An arbitrary shaped floating structure is considered here in the two-dimensional wave flume of constant water depth h, as shown in Fig. 1. Fictitious open boundaries S_{01} and S_{02} at $x=\pm b$, where evanescent mode waves can be neglected, divide the fluid domain R into three regions, such as R^+, R^- and R^0. The Cartesian coordinate system (x, z) is employed, in which x is measured horizontally in the direction of reflected wave propagation and z is measured vertically upward from the stillwater level. Let η_0 denote the amplitude of the first order incident wave, $k^{(1)}$ the wave number of the first order in the fluid region of water depth h, ν the outward normal direction. $\eta(x;t)$(t is time) the free surface elevation from stillwater level, respectively.

With the assumption of the irrotational motion of incompressible and inviscid fluid, the fluid motion can be described with a velocity potential $\Phi(x, z; t)$ which satisfies the Laplace equation over the fluid domain R:

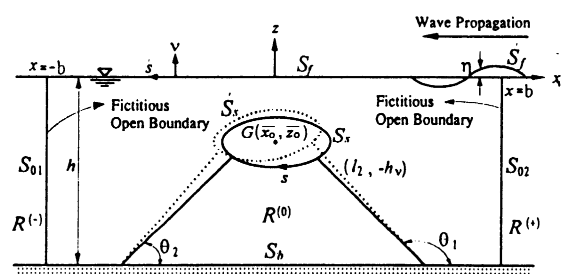


Fig. 1 Definition sketch

$$\nabla^2\Phi=0, \text{ in } R \tag{1}$$

Boundary conditions subjected to Φ on each divided fluid region are:

$$\frac{\partial \eta}{\partial t}+\frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x}-\frac{\partial \Phi}{\partial z}=0, \text{ on } S_f \quad (2)$$

$$\frac{\partial \Phi}{\partial t}+\frac{1}{2}\left\{\left(\frac{\partial \Phi}{\partial x}\right)^2+\left(\frac{\partial \Phi}{\partial z}\right)^2\right\}+g\eta=Q, \text{ on } S_f \quad (3)$$

$$\frac{\partial \Phi}{\partial z}=0, \text{ on } S_b \quad (4)$$

$$\left\{\frac{\partial \Phi}{\partial x}-\left(\frac{\partial x_o}{\partial t}-(\bar{z}-z_o)\frac{\partial \theta_o}{\partial t}\right)\right\}\ell+\left\{\frac{\partial \Phi}{\partial z}+\left(\frac{\partial z_o}{\partial t}-(\bar{x}-x_o)\frac{\partial \theta_o}{\partial t}\right)\right\}m=0, \text{ on } S_s \quad (5)$$

where g the acceleration of gravity, Q the Bernoulli constant, $\bar{x}_o$ and $\bar{z}_o$ the initial positions of the center of gravity, x_o, z_o and θ_o the horizontal (swaying), vertical (heaving) and rotational (rolling) displacements of the structure, respectively, $\ell=dz/ds$, $m=-dx/ds$, $\bar{x}=x_s-\bar{x}_o$, $\bar{z}=z_s-\bar{z}_o$, s the surface line on the structure, and x_s and z_s the coordinate on the surface of the structure.

Boundary conditions on the open boundaries S_{01} and S_{02} are given by means of determining analytically the velocity potentials on the divided fluid regions $R^{(+)}$ and $R^{(-)}$. Now it is assumed that the velocity potential $\Phi(x,z;t)$, the free surface elevation $\eta(x;t)$, Bernoulli constant Q and displacements of the floating structure can be expanded into a convergent power series with respect to a small parameter $\varepsilon(\eta_0 k^{(1)})$:

$$\Phi=\varepsilon\phi^{(1)}e^{i\sigma t}+\varepsilon^2\{\phi^{(2)}e^{2i\sigma t}+\phi_0^{(2)}\}+\cdots \quad (6)$$

$$\eta=\varepsilon\eta^{(1)}e^{i\sigma t}+\varepsilon^2\eta^{(2)}e^{2i\sigma t}+\cdots \quad (7)$$

$$Q=\varepsilon Q^{(1)}+\varepsilon^2 Q^{(2)}+\cdots \quad (8)$$

$$x_o=\varepsilon\alpha^{(1)}e^{i\sigma t}+\varepsilon^2\{\alpha^{(2)}e^{2i\sigma t}+\alpha_0^{(2)}\}+\cdots \quad (9)$$

$$z_o=\varepsilon\beta^{(1)}e^{i\sigma t}+\varepsilon^2\{\beta^{(2)}e^{2i\sigma t}+\beta_0^{(2)}\}+\cdots \quad (10)$$

$$\theta_o=\varepsilon\omega^{(1)}e^{i\sigma t}+\varepsilon^2\{\omega^{(2)}e^{2i\sigma t}+\omega_0^{(2)}\}+\cdots \quad (11)$$

where $\phi_0^{(2)}$, $\alpha_0^{(2)}$, $\beta_0^{(2)}$ and $\omega_0^{(2)}$ are the second order velocity potential and amplitudes of three motions which are independent of time.

Substituting the perturbation parameters (Eqs. 6 ~ 11) into Eqs. 1, 4 and Taylor's expansion of Eq. 5 about the center of gravity $G(\bar{x}_0, \bar{z}_0)$ at the initial position, and collecting the terms of same order of ε, the Laplace equation, seabed boundary condition and surface boundary condition of the structure for the first and second orders can be derived. Hence the combined water surface boundary condition also can be obtained by substituting the perturbation relations into Taylor's expansion of Eqs. 2 and 3 about the stillwater level, collecting the terms of the same order of ε, and eliminating $\eta^{(1)}$ and $\eta^{(2)}$ from each same order equation about ε and ε^2. These results are summarized as follows:

First order:

$$\nabla^2\phi^{(1)}=0, \text{ on } R \quad (12)$$

$$\frac{\partial \phi^{(1)}}{\partial z}-\Gamma\phi^{(1)}=0, \text{ on } S_f \quad (13)$$

$$\frac{\partial \phi^{(1)}}{\partial z}=0, \text{ on } S_b \quad (14)$$

$$\frac{\partial \phi^{(1)}}{\partial \nu}=i\sigma\{\ell\alpha^{(1)}+m\beta^{(1)}+(m\bar{x}-\ell\bar{z})\omega^{(1)}\}, \text{ on } S_s \quad (15)$$

Second order:

- time-dependent components

$$\nabla^2\phi^{(2)}=0,\ \text{on}\ R \tag{16}$$

$$\frac{\partial\phi^{(2)}}{\partial z}-4\Gamma\phi^{(2)}=-\frac{i\sigma}{g}\left\{\left(\frac{\partial\phi^{(1)}}{\partial x}\right)^2+\left(\frac{\partial\phi^{(1)}}{\partial z}\right)^2\right\}$$
$$-\frac{\eta^{(1)}}{2}\left\{-\Gamma\frac{\partial\phi^{(1)}}{\partial z}+\frac{\partial^2\phi^{(1)}}{\partial z^2}\right\},\ \text{on}\ S_f \tag{17}$$

$$\frac{\partial\phi^{(2)}}{\partial z}=0,\ \text{on}\ S_b \tag{18}$$

$$\frac{\partial\phi^{(2)}}{\partial\nu}=2i\sigma\{\ell\alpha^{(2)}+m\beta^{(2)}+(m\bar{x}-\ell\bar{z})\omega^{(2)}\}$$
$$+\frac{\alpha^{(1)}}{2}\left(\ell\frac{\partial^2\phi^{(1)}}{\partial s^2}+m\frac{\partial^2\phi^{(1)}}{\partial\nu\partial s}\right)$$
$$-\frac{\beta^{(2)}}{2}\left(\ell\frac{\partial^2\phi^{(1)}}{\partial\nu\partial s}-m\frac{\partial^2\phi^{(1)}}{\partial s^2}\right)$$
$$-\frac{\omega^{(1)}}{2}\left(\frac{\partial\phi^{(1)}}{\partial s}+i\sigma(m\alpha^{(1)}-\ell\beta^{(1)})\right.$$
$$+(\ell\bar{x}+m\bar{z})\frac{\partial^2\phi^{(1)}}{\partial\nu\partial s}$$
$$\left.-(m\bar{x}-\ell\bar{z})\frac{\partial^2\phi^{(1)}}{\partial s^2}\right),\ \text{on}\ S_s \tag{19}$$

- time-independent components

$$\nabla^2\phi_0^{(2)}=0,\ \text{on}\ R \tag{20}$$

$$\frac{\partial\phi_0^{(2)}}{\partial z}=-\frac{\eta^{(1)}}{g}\left\{-\Gamma\frac{\partial\phi_*^{(1)}}{\partial z}+\frac{\partial^2\phi_*^{(1)}}{\partial z^2}\right\},\ \text{on}\ S_f \tag{21}$$

$$\frac{\partial\phi_0^{(2)}}{\partial z}=0,\ \text{on}\ S_b \tag{22}$$

$$\frac{\partial\phi_0^{(2)}}{\partial\nu}=\frac{\alpha_*^{(1)}}{2}\left(\ell\frac{\partial^2\phi^{(1)}}{\partial s^2}+m\frac{\partial^2\phi^{(1)}}{\partial\nu\partial s}\right)$$
$$-\frac{\beta_*^{(2)}}{2}\left(\ell\frac{\partial^2\phi^{(1)}}{\partial\nu\partial s}-m\frac{\partial^2\phi^{(1)}}{\partial s^2}\right)$$
$$-\frac{\omega_*^{(1)}}{2}\left(\frac{\partial\phi^{(1)}}{\partial s}+i\sigma(m\alpha^{(1)}-\ell\beta^{(1)})\right.$$
$$+(\ell\bar{x}+m\bar{z})\frac{\partial^2\phi^{(1)}}{\partial\nu\partial s}$$
$$\left.-(m\bar{x}-\ell\bar{z})\frac{\partial^2\phi^{(1)}}{\partial s^2}\right\},\ \text{on}\ S_s \tag{23}$$

where $\phi_*^{(1)}$, $\alpha_*^{(1)}$, $\beta_*^{(1)}$ and $\omega_*^{(1)}$ are the conjugate complex numbers of $\phi^{(1)}$, $\alpha^{(1)}$, $\beta^{(1)}$ and $\omega^{(1)}$ respectively, σ the angular frequency, $i=\sqrt{-1}$ and $\Gamma=\sigma^2/g$.

Here $\phi_0^{(2)}$ is out of discussion because $\phi_0^{(2)}$ does not have any contribution to the water pressure and free surface fluctuation up to the second order.

The first order velocity potential $\phi^{(1)}$ in the region $R^{(+)}$, which satisfies Eqs. 12 ~ 14, is:

$$\phi^{(1)}=\frac{g}{k^{(1)}\sigma}(e^{ik^{(1)}x}+B^{(1)}e^{-ik^{(1)}x})Z(k^{(1)}z) \tag{24}$$

where $B^{(1)}$ is the unknown corresponding to the reflection coefficient, $k^{(1)}$ the wave number determined by the dispersion relationship $\sigma^2/g=k^{(1)}\tanh k^{(1)}h$, and $Z(k^{(1)}z)=\cosh k^{(1)}(z+h)/\cosh k^{(1)}h$. Substitution of Eq. 24 into Eq. 17 yields the second order water surface boundary condition given by:

$$\frac{\partial\phi^{(2)}}{\partial z}-4.0\Gamma\phi^{(2)}=$$
$$-\frac{3g(\Gamma^2-k^{(1)^2})}{2k^{(1)^2}\sigma}\{i(e^{2ik^{(1)}x}+B^{(1)^2}e^{-2ik^{(1)}x})\}$$
$$-\frac{g\{3\Gamma^2+k^{(1)^2}\}}{k^{(1)^2}\sigma}(iB^{(1)}) \tag{25}$$

Considering Eqs. 16, 18 and 25, the second order velocity potential $\phi^{(2)}$ in the region $R^{(+)}$ can be assumed as:

$$\phi^{(2)} = \frac{g}{k^{(1)}\sigma} B^{(2)} e^{-ik^{(2)}x} Z(k^{(2)}z) + ia_s(e^{2ik^{(1)}x} + B^{(1)^2} e^{-2ik^{(1)}x}) Z(2k^{(1)}z) + ib_s B^{(1)} \tag{26}$$

where the term of $e^{2ik^{(1)}x}$ presents the second order velocity potential of the incident wave component, well-known from a Stokes second order wave theory, $B^{(2)}$ the coefficient of the second order free wave, which satisfies the two-dimensional Sommerfeld radiation condition, and $k^{(2)}$ the wave number of second order free wave. The coefficients a_s, b_s and the dispersion relationship for the second order free wave are obtained by substituting Eq. 26 into Eq. 25.

$$a_s = -\frac{3\sigma \cosh 2k^{(1)}h}{gk^{(1)^2} \sinh^4 k^{(1)}h}$$

$$b_s = \frac{g(3\Gamma^2 + k^{(1)^2})}{4k^{(1)^2}\sigma\Gamma}$$

$$4\Gamma = k^{(2)} \tanh k^{(2)}h$$

The first and second order velocity potentials in the region $R^{(-)}$ can be obtained in the similar way:

$$\phi^{(1)} = \frac{g}{k^{(1)}\sigma} I^{(1)} e^{ik^{(1)}x} Z(k^{(1)}z) \tag{27}$$

$$\phi^{(2)} = \frac{g}{k^{(1)}\sigma} I^{(2)} e^{ik^{(2)}x} Z(k^{(2)}z) + ia_s I^{(1)^2} e^{2ik^{(1)}x} Z(2k^{(1)}z) \tag{28}$$

where $I^{(1)}$ is the unknown corresponding to the transmission coefficient, $I^{(2)}$ the coefficient of the second order free wave. In Eq. 28, the term of $I^{(2)}$ satisfies the Sommerfeld radiation condition.

The open boundary condition at $x = \pm b$ are given by calculating the velocity potentials and the corresponding normal derivatives of the first and second orders, $\phi^{(1)}_{x=\pm b}$, $\phi^{(2)}_{x=\pm b}$, $\partial\phi^{(1)}/\partial\nu_{x=\pm b}$ and $\partial\phi^{(2)}/\partial\nu_{x=\pm b}$, from Eqs. 24, 26, 27 and 28.

Open boundary conditions at $x = b$

$$\phi^{(1)} = \frac{g}{k^{(1)}\sigma}(e^{ik^{(1)}b} + B^{(1)} e^{-ik^{(1)}b}) Z(k^{(1)}z) \tag{29}$$

$$\frac{\partial\phi^{(1)}}{\partial\nu} = \frac{ig}{\sigma}(e^{ik^{(1)}b} - B^{(1)} e^{-ik^{(1)}b}) Z(k^{(1)}z) \tag{30}$$

$$\phi^{(2)} = \frac{gB^{(2)}}{k^{(1)}\sigma} e^{-ik^{(2)}b} Z(k^{(2)}z) + ia_s(e^{2ik^{(1)}b} + B^{(1)^2} e^{-2ik^{(1)}b}) Z(2k^{(1)}z) + ib_s B^{(1)} \tag{31}$$

$$\frac{\partial\phi^{(2)}}{\partial\nu} = -\frac{igk^{(2)}B^{(2)}}{k^{(1)}\sigma} e^{-ik^{(2)}b} Z(k^{(2)}z) - 2k^{(1)} a_s(e^{2ik^{(1)}b} - B^{(1)^2} e^{-2ik^{(1)}b}) Z(2k^{(1)}z) \tag{32}$$

Open boundary conditions at $x = -b$

$$\phi^{(1)} = \frac{gI^{(1)}}{k^{(1)}\sigma} e^{-ik^{(1)}b} Z(k^{(1)}z) \tag{33}$$

$$\frac{\partial\phi^{(1)}}{\partial\nu} = -\frac{igI^{(1)}}{\sigma} e^{-ik^{(1)}b} Z(k^{(1)}z) \tag{34}$$

$$\phi^{(2)} = \frac{gI^{(2)}}{k^{(1)}\sigma} e^{-k^{(2)}b} Z(k^{(2)}z) + ia_s I^{(1)^2} e^{-2ik^{(1)}b} Z(2k^{(1)}z) \tag{35}$$

$$\frac{\partial\phi^{(2)}}{\partial\nu} = -\frac{igk^{(2)}I^{(2)}}{k^{(1)}\sigma} e^{-ik^{(2)}b} Z(k^{(2)}z) + 2k^{(1)} a_s I^{(1)^2} e^{-ik^{(1)}b} Z(2k^{(1)}z)$$

2.2 Procedure of Numerical Calculation

First and second order velocity potentials at an arbitrary point X on the boundary surface of the closed fluid region $R^{(0)}$ can be expressed by following integral equation:

$$\Phi(X)=\int_{s'}\left\{\phi(X_b)\frac{\partial G(r)}{\partial\nu}-G(r)\frac{\partial\phi(X_b)}{\partial\nu}\right\}ds' \tag{37}$$

$$G(r)=\frac{1}{\pi}\log r$$

where r is the distance between the points X and X_b, $G(r)$ a Green's function and s' the surface line on the boundary in the region $R^{(0)}$.

The boundary surface consists of the water surface S_f, the seabed S_b, the structure surface S_s, and the open boundaries S_{01} and S_{02} at $x=\pm b$, as shown in Fig. 1. Eq. 37 can be solved by means of numerical calculation using the obtained boundary conditions, in which the boundary surfaces S_f, S_{01}, S_{02}, S_b and S_s are discretized into the finite numbers of small segments. Integration is performed in the order of S_f, S_{01}, S_b, S_{02} and S_s.

Substitution the boundary conditions into Eq. 37, the following set of simultaneous equations is obtained for each order:

$$\sum^{S_f,S_b,S_s}A_{ij}^{(k)}(\phi_{ij}^{(k)})+\sum^{S_{01}}B_{ij}^{(k)}(I^{(k)})+\sum^{S_{02}}B_{ij}^{(k)}(B^{(k)})$$
$$+\sum^{S_s}C_{ij}^{(k)}\{\ell\alpha^{(k)}+m\beta^{(k)}+(m\bar{x}-\ell\bar{z})\omega^{(k)}\}=D_{ij}^{(k)} \tag{38}$$

where $i=1,2,\ldots,N$ and N is the total number of segments on the whole boundary surfaces and j indicates the number of the segment on the boundary. The coefficients $A_{ij}^{(k)}$, $B_{ij}^{(k)}$ and $C_{ij}^{(k)}$ depend on the integration of Green's function and its normal derivative, which can be evaluated numerically or analytically. $D_{ij}^{(1)}$ is given by the velocity potential component of the first order incident wave, and $D_{ij}^{(2)}$ is given by the known functions such as $\phi^{(1)}$ on the boundary surfaces S_f, S_b and S_s. The values of $B^{(1)}$, $I^{(1)}$, $\alpha^{(1)}$, $\beta^{(1)}$ and $\omega^{(1)}$ are obtained from the first order solution. It should be noted that Eq. 38 must be solved simultaneously with the equation of the motion of the structure, in order to obtain the displacements of the structure.

2.3 Equation of Motion

The equation of motion is constructed with the wave exciting force and reaction force of the mooring lines against the motion of the structure, that is:

$$M\frac{d^2x_o}{dt^2}=P_x+F_x \tag{39}$$

$$M\frac{d^2z_o}{dt^2}=P_z+F_z \tag{40}$$

$$I\frac{d^2\theta_o}{dt^2}=T_r+M_r \tag{41}$$

in which

$$P_x=\int_{s'}\tilde{P}\ell\, ds' \tag{42}$$

$$P_z=\int_{s'}\tilde{P}m\, ds' \tag{43}$$

$$T_r=\int_{s'}\tilde{P}\{(\bar{x}-x_o)m-(\bar{z}-z_o)\ell\}ds' \tag{44}$$

$$\frac{\tilde{P}}{\rho}=Q-\frac{\partial\Phi}{\partial t}-\frac{1}{2}\left\{\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial z}\right)^2\right\}-gz \tag{45}$$

where M is the mass of the structure, I the inertia moment with respect to $G(\overline{x_o}, \overline{z_o})$, ρ the water density, P_x and P_z the horizontal and vertical exciting forces, T_r the moment due to P_x and P_z with respect to $G(\overline{x_o}, \overline{z_o})$, F_x and F_z the horizontal and vertical resistant forces of on-offshore side mooring lines, and M_r the moment due to F_x and F_z with respect to $G(\overline{x_o}, \overline{z_o})$.

Let the initial length of the offshore side mooring line and the angle of the mooring line to the seabed be m_o and θ_1, and their variation under wave action be Δm_o and $\Delta\theta_1$, respectively, then the reaction forces and moment F_{x1}, F_{z1} and M_{r1} are expressed as follows:

$$F_{x1} = F_o\cos\theta_1 - F_w\cos\theta_2 \tag{46}$$

$$F_{z1} = F_o\sin\theta_1 - F_w\sin\theta_2 \tag{47}$$

$$\begin{aligned} M_{r1} = \ & F_w\{(\ell_x\cos\theta_o + \ell_z\sin\theta_o)\sin\theta_2 \\ & + (\ell_x\sin\theta_o - \ell_z\cos\theta_o)\cos\theta_2\} \\ & - F_o(\ell_x\sin\theta_1 - \ell_z\cos\theta_1) \end{aligned} \tag{48}$$

where F_o is the initial tensile force of the offshore side mooring line, and K the spring constant of the mooring line, $\ell_x = \overline{x_o} - \ell_2$, $\ell_z = \overline{z_o} + h_v$, $\theta_2 = \theta_1 + \Delta\theta_1$ and $F_w = F_o + K\Delta m_o$.

The reaction forces and moment of the onshore side mooring line can be determined in the same manner, and therefore the total reaction forces and moment of the mooring lines are given by the sum of them. It is assumed in this paper that the shape of the floating structure and the mooring system are symmetric with respect to z-axis, and that the resistant forces of the mooring line obeys Hook's law. The first and second order equations of swaying, heaving and rolling motions can be derived by substituting Eqs. 6 ~11 into Taylor's expansions of Eqs. 39-41 about its initial position. Arranging the resultant equations by the order of ε, the following equation are obtained.

First order:

$$\begin{aligned} -\sigma^2 M\alpha^{(1)} = & -\rho\sigma\int_s \phi^{(1)}\ell\, ds + K_{x,x}\alpha^{(1)} \\ & + K_{x,z}\beta^{(1)} + K_{x,r}\omega^{(1)} \end{aligned} \tag{49}$$

$$\begin{aligned} -\sigma^2 M\beta^{(1)} = & -\rho\sigma\int_s \phi^{(1)} m\, ds + K_{z,x}\alpha^{(1)} \\ & + K_{z,z}\beta^{(1)} + K_{z,r}\omega^{(1)} \end{aligned} \tag{50}$$

$$\begin{aligned} -\sigma^2 I\omega^{(1)} = & -\rho\sigma\int_s \phi^{(1)}(\overline{x}m - \overline{z}\ell)\, ds \\ & + K_{r,x}\alpha^{(1)} + K_{r,z}\beta^{(1)} + K_{r,r}\omega^{(1)} \end{aligned} \tag{51}$$

Second order:

- Time-dependent

$$\begin{aligned} -4\sigma^2 M\alpha^{(2)} = & -\rho\int_s\{[C^{(2)}]\ \ell - \frac{i\sigma}{2} \\ & \phi^{(1)}\omega^{(1)} m\} ds + [R_p^{(2)}]\ |_{p=x} \end{aligned} \tag{52}$$

$$\begin{aligned} -4\sigma^2 M\beta^{(2)} = & -\rho\int_s\{[C^{(2)}]\ m + \frac{i\sigma}{2} \\ & \phi^{(1)}\omega^{(1)}\ell\} ds + [R_p^{(2)}]\ |_{p=z} \end{aligned} \tag{53}$$

$$\begin{aligned} -4\sigma^2 I\omega^{(2)} = & -\rho\int_s [C^{(2)}]\ (\overline{x}m - \overline{z}\ell)\, ds \\ & + [R_p^{(2)}]\ |_{p=r} \end{aligned} \tag{54}$$

- Time-independent

$$-K_{x,x}\alpha_o^{(2)}-K_{x,z}\beta_o^{(2)}-K_{x,r}\omega_o^{(2)}=$$

$$-\rho\int_s\left\{[\;C_o^{(2)}]\;\ell+\frac{i\sigma}{2}\phi_*^{(1)}\omega^{(1)}m\right\}ds \tag{55}$$

$$+[\;R_{p,o}^{(2)}]\;|_{p=x}$$

$$-K_{z,x}\alpha_o^{(2)}-K_{z,z}\beta_o^{(2)}-K_{z,r}\omega_o^{(2)}$$

$$=-\rho\int_s\left\{[\;C_o^{(2)}]\;m-\frac{i\sigma}{2}\phi_*^{(1)}\omega^{(1)}\ell\right\}ds \tag{56}$$

$$+[\;R_{p,o}^{(2)}]\;|_{p=z}$$

$$-K_{r,x}\alpha_o^{(2)}-K_{r,z}\beta_o^{(2)}-K_{r,r}\omega_o^{(2)}$$

$$=-\rho\int_s[\;C_o^{(2)}]\;(\bar{x}m-\bar{z}\ell)ds \tag{57}$$

$$+[\;R_{p,o}^{(2)}]\;|_{p=r}$$

in which

$$[\;C^{(2)}]\;=\;2i\sigma\phi^{(2)}+\frac{1}{4}\left\{\left(\frac{\partial\phi^{(1)}}{\partial s}\right)^2+\left(\frac{\partial\phi^{(1)}}{\partial\nu}\right)^2\right\}$$

$$+i\sigma\left\{\alpha^{(1)}\left(\ell\frac{\partial\phi^{(1)}}{\partial\nu}-m\frac{\partial\phi^{(1)}}{\partial s}\right)\right.$$

$$+\beta^{(1)}\left(m\frac{\partial\phi^{(1)}}{\partial\nu}+\ell\frac{\partial\phi^{(1)}}{\partial s}\right)$$

$$+\omega^{(1)}\left\{(\bar{x}m-\bar{z}\ell)\frac{\partial\phi^{(1)}}{\partial\nu}\right.$$

$$\left.\left.+(\bar{x}\ell+\bar{z}m)\frac{\partial\phi^{(1)}}{\partial s}\right\}\right\}$$

$$[\;C_o^{(2)}]\;=\frac{1}{4}\left\{\left|\frac{\partial\phi^{(1)}}{\partial s}\right|^2+\left|\frac{\partial\phi^{(1)}}{\partial\nu}\right|^2\right\}$$

$$[\;R_p^{(2)}]\;|_{p=x,z,r}=K_{p,x}\alpha^{(2)}+K_{p,z}\beta^{(2)}+K_{p,r}\omega^{(2)}$$

$$+\frac{1}{2}(K_{p,xx}\alpha^{(1)^2}+K_{p,zz}\beta^{(1)^2}+K_{p,rr}\omega^{(1)^2}$$

$$+K_{p,xz}\alpha^{(1)}\beta^{(1)}+K_{p,zr}\beta^{(1)}\omega^{(1)}$$

$$+K_{p,rx}\omega^{(1)}\alpha^{(1)})$$

$$[\;R_{p,o}^{(2)}]\;|_{p=x,z,r}=\;\frac{1}{2}(K_{p,xx}|\alpha^{(1)}|^2$$

$$+K_{p,zz}|\beta^{(1)}|^2+K_{p,rr}|\omega^{(1)}|^2$$

$$+K_{p,xz}\alpha_*^{(1)}\beta^{(1)}+K_{p,zr}\beta_*^{(1)}\omega^{(1)}$$

$$+K_{p,rx}\omega_*^{(1)}\alpha^{(1)})$$

$$K_{x,x}=-2K\cos^2\theta_2-2f_o\sin^2\theta_2$$

$$K_{x,z}=K_{z,x}=K_{z,r}=K_{r,z}=0$$

$$K_{x,r}=K_{r,x}=-2Kh_q\cos^2\theta_2-2f_oh_p\sin^2\theta_2$$

$$K_{z,z}=-2K\sin^2\theta-2f_o\cos^2\theta_2$$

$$K_{r,r}=\;-2Kh_q^2\cos^2\theta_2-2f_oh_p^2\sin^2\theta_2$$

$$-2f_om_oh_p\sin\theta_2$$

$$K_{x,xx}=K_{x,zz}=K_{x,rr}=K_{x,rx}=0$$

$$K_{x,xz}=-2[\;K]\;\sin\theta_2(3\sin^3\theta_2-2)/m_o$$

$$K_{x,zr}=\;[\;K]\;(h_q\cos\theta_2\sin2\theta_2$$

$$+2h_p\sin\theta_2\cos2\theta_2)/m_o$$

$$K_{z,xx}=0.5K_{x,xz}$$

$$K_{z,zz}=-3[\;K]\;\sin\theta_2\cos^2\theta_2/m_o$$

$$K_{z,rr}=\;-Kh_p\sin^2\theta_2-f_oh_q\cos^2\theta_2$$

$$-[\;K]\;h_p^2\sin^3\theta_2/m_o$$

$$+2[\;K]\;h_qh_p\sin\theta_2\cos^2\theta_2/m_o$$

$K_{z,rx} = K_{x,zr}$

$K_{r,xz} = K_{x,zr}$

$K_{r,zr} = 2K_{z,rr}$

$K_{z,xr} = K_{z,zr} = K_{r,xx} = K_{r,zz} = K_{r,rx} = K_{r,rr} = 0$

$h_p = \ell_2 + \ell_2 \cot\theta_2$

$h_q = \ell_2 - \ell_2 \tan\theta_2$

$f_o = F_o / m_o$

$[K] = K - f_o$

where the first and second order equations of motion must be solved simultaneously with Eq. 38. Hence the second order time-independent motions of the structure can be calculated using Eqs. 55 ~57, associated with the determined first order solution.

2.4 Tensile Force of Mooring Line

The tensile force $\tilde{F}(t)$ acting on the offshore side mooring line is evaluated with Hook's law:

$$\tilde{F} = K \Delta m_o \tag{58}$$

$\tilde{F}$ may be expanded into power series of parameter ε in the similar way as the velocity potential Φ:

$$\tilde{F} = F_o + \varepsilon F^{(1)} e^{i\sigma t} + \varepsilon^2 (F^{(2)} e^{2i\sigma t} + F_o^{(2)}) + \cdots \tag{59}$$

where $F_o^{(2)}$ is the time-independent second order tensile force.

Employing the same method used for the equation of motion, the tensile force for first and second orders is derived as follows from Eq. 58:

First order

$$\frac{F^{(1)}}{K} = \alpha^{(1)} f_x + \beta^{(1)} f_z + \omega^{(1)} f_r \tag{60}$$

Second order

- time-dependent component

$$\frac{F^{(2)}}{K} = \alpha^{(2)} f_x + \beta^{(2)} f_z + \omega^{(2)} f_r + [L^{(2)}] \tag{61}$$

- time-independent component

$$\frac{F_o^{(2)}}{K} = \alpha_o^{(2)} f_x + \beta_o^{(2)} f_z + \omega_o^{(2)} f_r + [L_o^{(2)}] \tag{62}$$

in which

$$[L^{(2)}] = \frac{1}{4}\{\alpha^{(1)2} f_{xx} + \beta^{(1)2} f_{zz} + \omega^{(1)2} f_{rr}\} + \frac{1}{2}\{\alpha^{(1)}\beta^{(1)} f_{xz} + \beta^{(1)}\omega^{(1)} f_{zr} + \omega^{(1)}\alpha^{(1)} f_{rx}\}$$

$$[L_o^{(2)}] = \frac{1}{4}\{|\alpha^{(1)}|^2 f_{xx} + |\beta^{(1)}|^2 f_{zz} + |\omega^{(1)}|^2 f_{rr}\} + \frac{1}{2}\{\alpha_*^{(1)}\beta^{(1)} f_{xz} + \beta_*^{(1)}\omega^{(1)} f_{zr} + \omega_*^{(1)}\alpha^{(1)} f_{rx}\}$$

$f_x = -\cos\theta_2$

$f_z = -\sin\theta_2$

$f_r = -\ell_2 \sin\theta_2 - \ell_z \cos\theta_2$

$f_{xx} = \sin^2\theta_2 / m_o$

$f_{zz} = \cos^2\theta_2 / m_o$

$$f_{rr} = -\ell_2 \cos\theta_2 + \ell_z \sin\theta_2 + (-\ell_2 \cos\theta_2 + \ell_z \sin\theta_2)^2 / m_o$$

$$f_{xz} = -\sin\theta_2 \cos\theta_2 / m_o$$

$$f_{zr} = -(-\ell_2 \cos\theta_2 + \ell_z \sin\theta_2)\cos\theta_2 / m_o$$

$$f_{rx} = (-\ell_2 \cos\theta_2 + \ell_z \sin\theta_2)\sin\theta_2 / m_o$$

$$\ell_z = h_v + \overline{z_o}$$

3. APPLICATION TO AIR-CHAMBER STRUCTURE

The theory developed in this paper is applied to the fully submerged air-chamber structure as shown in Fig. 2. The combined water surface boundary condition in the airchamber can be derived easily from the dynamic and kinematic boundary conditions, including the variation of the air pressure in the airchamber, by using the same procedure as in the case of the stillwater surface boundary S_f. On the other hand, the first and second boundary conditions on the open boundary, free water surface, seabed and structure can be applied to the air-chamber structure without any modifications.

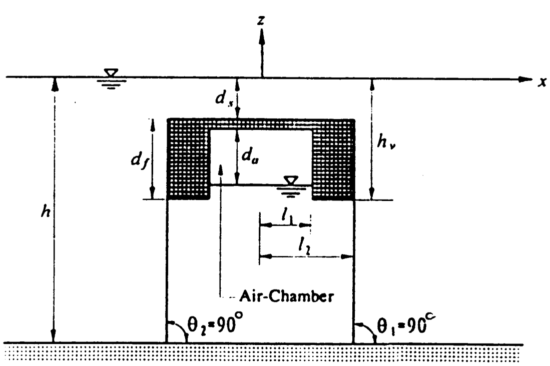


Fig. 2 Definition sketch of air-chamber structure

The time variation of the air volume $\Delta V_a(t)$ in the airchamber under wave action, which causes the air pressure variation, is given by:

$$\Delta V_a = V_a - V_a^{(0)} = V_f + V_m \tag{63}$$

In Eq. 63, ΔV_a is given by the sum of V_f and V_m, in which V_f and V_m the air volume change by the water surface variation in the airchamber (ξ) and motions of structure, respectively. V_a the total air volume under wave action, and $V_a^{(0)}$ the initial air volume.

The air pressure $\widetilde{P}(t)$ is assumed to be expanded into power series of ε.

$$\widetilde{P}_a = P_a^{(0)} + \varepsilon P_a^{(1)} e^{i\sigma t} + \varepsilon^2 (P_a^{(2)} e^{2i\sigma t} + P_{a,o}^{(2)}) + \cdots \tag{64}$$

where $P_a^{(0)}$ is the initial air pressure, and $P_{a,o}^{(2)}$ the time-independent air pressure of second order.

The air pressure variation by ΔV_a can be evaluated with Boyle's law:

$$\widetilde{P}_a = P_a^{(0)} + \Delta P_a = P_a^{(0)} \left\{ \frac{V_a^{(0)}}{V_a^{(0)} + \Delta V_a} \right\}^{\gamma} - P_a^{(0)} \tag{65}$$

in which, ΔP_a is the air pressure change and, $\gamma = 1.4$ when the air compression is the adiabatic process of ideal gas.

Expanding Eq. 54 into Taylor's expansion about the initial state yields:

$$\Delta P_a = -\gamma P_a^{(0)} \left(\frac{\Delta V_a}{V_a^{(0)}} \right) + \frac{1}{2} \gamma (1+\gamma) \left(\frac{\Delta V_a}{V_a^{(0)}} \right)^2 + \cdots \tag{66}$$

The air pressure variation of the first and second orders is obtained by means of the following procedure: 1)formulating V_f and V_m in Eq. 63 as the function of ξ, x_o, z_o and θ_o; 2)expanding the result of 1) into Taylor's expansion about the initial state; 3)substituting the result of 2)into Eq. 66; 4)substituting the perturbation relations into the result of 3); and 5)collecting the terms of the same order of ϵ. Effect of air pressure to the motion of structure can be evaluated in the same way as in the case of the tensile force acting on the mooring line.

4. RESULTS AND DISCUSSION

4.1 Water Surface

The nondimensional water surface profiles η/η_o measured at $x/L \fallingdotseq 0.113$, $x/L=0$ and $x/L=-0.113$ above the structure in the case of $d_s/h=0.11$, $d_a/h=0.2$, $2\eta_o/L=0.0143$ and $2\ell_2/L=0.45$, are plotted versus t/T in Fig.3, in which L is the incident wavelength, T the wave period, d_s the submerged water depth of structure's crown, and d_a the initial air depth in the airchamber (Fig. 2). Also, in Fig. 3, the calculated values together with the first and second order surface variation η_1 and η_2 are presented. According to Fig. 3, the second order component $\eta^{(2)}$ becomes larger with wave propagation (from Figs. 3a to c)above the structure. This means that the nonlinear wave-structure interaction grows with wave propagation above the crown of the structure. An abrupt change of the water depth at the offshore side of the structure, above which lots of wave energy is transmitted, may enlarge the second order component. The phase of the second order surface profile is, in general, different from that of the first order component, as shown in Figs. 3a, b and c; thus, the total profile η depends largely on the phase lag between the first and second orders. It is also found that the amplitude of the first order component $\eta^{(1)}$ is varied according to the location on the structure. This arises from wave reflection due to rapid change of water depth at the onshore side of the structure.

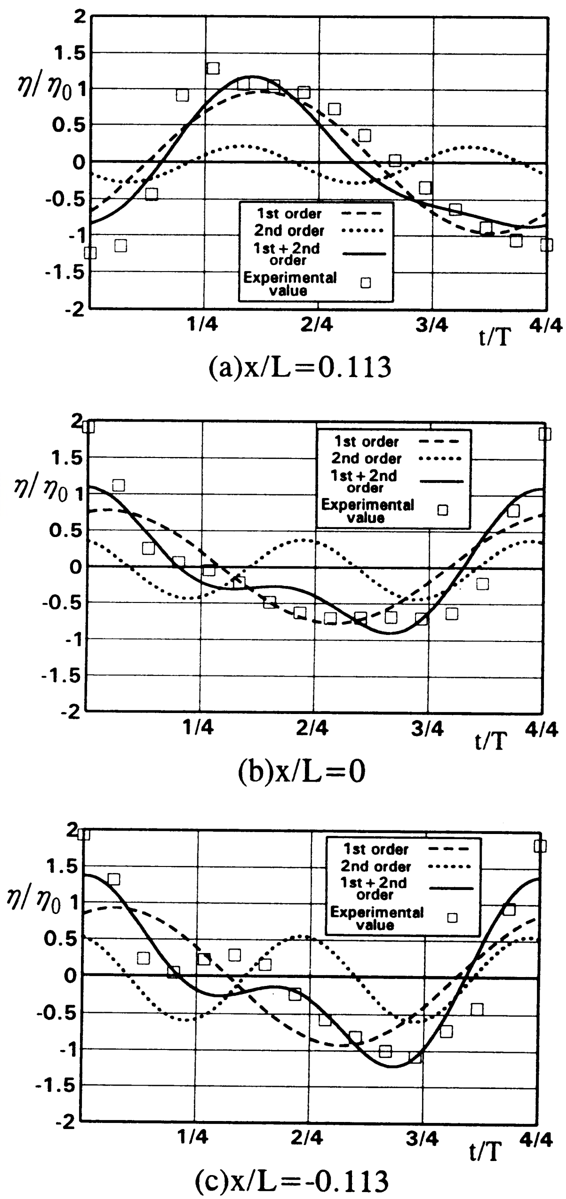


Fig. 3 Time variation of water surface on structure

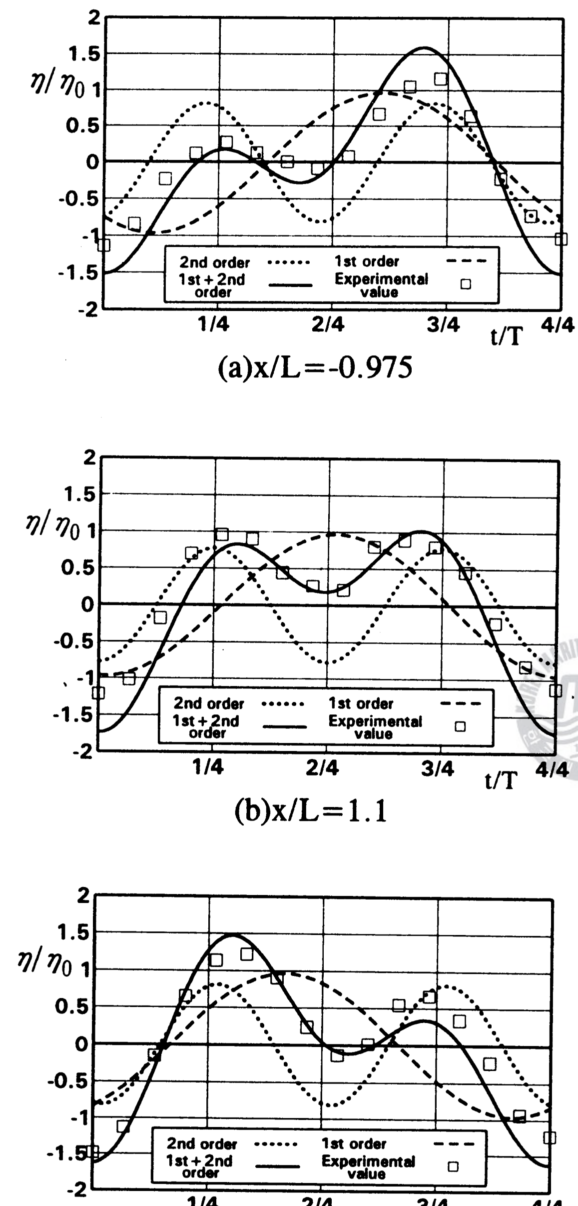


Fig. 4 Time variation of water surface on onshore side region of structure

Nondimensional water surface profiles η/η_o at $x/L=-0.975$, $x/L=-1.100$, and $x/L=-1.225$ in the onshore region of the structure under the same wave conditions as Fig. 3 are shown in Fig. 4. The amplitudes of the first and second order components are seen to be unchangeable. Hence, the second order values in the onshore region of the structure are mainly dominated by those formed above the structure. The total wave profile η has the same characteristics as those in Fig. 3, in the sense that the total wave profile η is largely changed by the phase lag between the first and second order components.

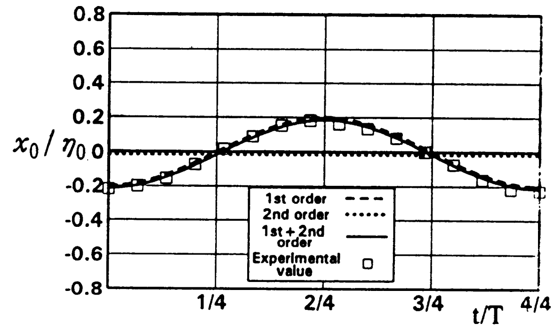
Appearance of the nonlinearity in the wave profile due to the submerged moored air-chamber floating structure is very similar to that in the case of a fixed and submerged breakwater (Yoshida et al., 1989). Although the results are not presented graphically here, it is proved that the effect of the initial air pressure in the airchamber on the water surface profile is not so significant. The reason is that most of the wave energy is transmitted on the upper fluid region of the structure crown. Therefore, the nonlinearity appearing in the water surface profile on the onshore side of the structure seems to be dominated largely by the submerged water depth of the structure's crown.

4.2 Motion of Floating Structure

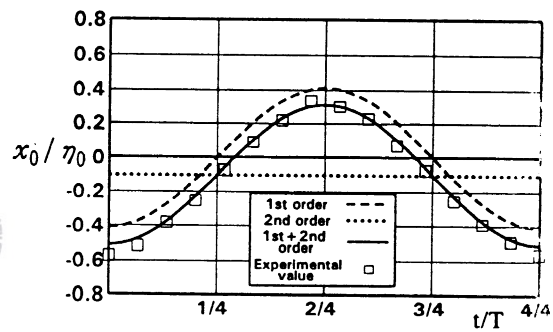
The mooring line, which are made of chain, have a very large spring constant, and they are set vertically on the seabed in the experiments. The vertical and rotational motions are very small due to the very high resistant force of mooring lines, compared to the horizontal motion. Therefore, only the horizontal motion is discussed here.

Fig. 5 shows the relationship between the nondimensional horizontal motion x_o/η_o and t/T. Figs. 5a and b show results of x_o/η_o for two different wave periods under the same initial air pressure, while Figs. 5a and c present the results of x_o/η_o for two different initial air pressure of d_a/h under the same wave period. The nonlinear time-dependent horizontal motion in Fig. 5 is very small, compared to the time-independent one given by the dotted line. This comes from the very high nonlinear resistant force of the mooring lines and the large mass of the structure against the horizontal motion, while the time-independent component has no correlation with the mass of the structure. On the other hand, the linear horizontal motion $\alpha^{(1)}$ becomes large with the increment of wave period, as shown in Figs. 5a and b. This enlarges the nonlinear interaction for other linear motions $\beta^{(1)}$ and $\omega^{(1)}$, as presented in Eqs. 55-57. Therefore, the time-independent motion may increase with increasing wave period, as shown in Figs. 5a and b.

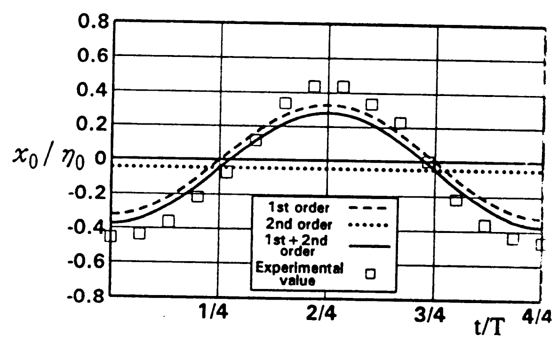
Comparing Figs. 5a with c, it can be seen that the time-independent component decreases with decreasing initial air pressure. The linear horizontal motion increases with an increment of the initial air pressure in the airchamber, as the mass of the fluid moving with the structure in the airchamber, which acts as a resistant against the horizontal motion, becomes smaller. Thus, it seems that the time-independent component, as shown in Figs. 5a and c, becomes large by an increment of the nonlinear interaction between the increasing linear horizontal motion $\alpha^{(1)}$ and other linear motions $\beta^{(1)}$ and other linear motions $\beta^{(1)}$ and $\omega^{(1)}$, as mentioned above.



(a) $d_s/h=0.11, d_a/h=0.10, 2\eta_0/L=0.0189, 2l_2/L=0.68, T=0.8s$



(b) $d_s/h=0.11, d_a/h=0.10, 2\eta_0/L=0.0143, 2l_2/L=0.45, T=1.0s$

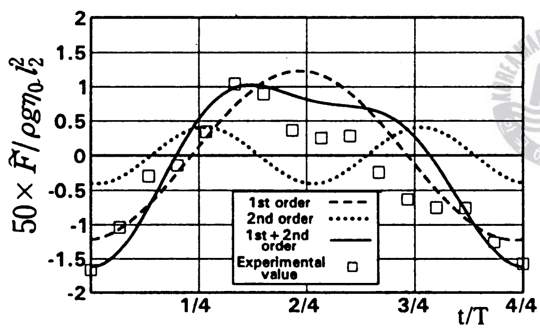


(c) $d_s/h=0.11, d_a/h=0.20, 2\eta_0/L=0.0195, 2l_2/L=0.68, T=0.8s$

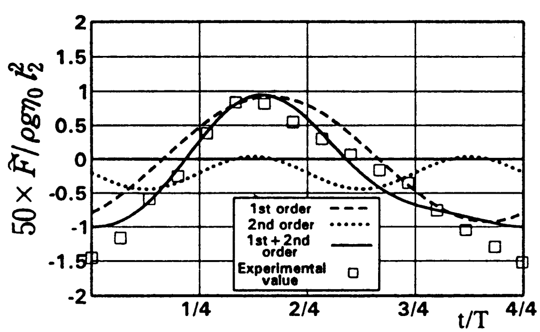
Fig. 5 Time variation of horizontal motion of structure

4.3 Tensile Force of Mooring Line

Fig. 6 shows the time variations of the computational and experimental values of the tensile forces of offshore side mooring line normalized by $\rho g \eta_0 \ell_2^2$. Figs. 6a and b show the results in the case of different wave periods for the same initial air pressure. Figures indicate that the time-dependent component of second order shows larger value than that in the horizontal motion shown in Fig. 5. Although the tensile force acting on the mooring line is caused by the motion of the structure, the vertical motions have direct effect on the tensile force, as given in Eqs. 55 ~57, especially under the vertical mooring condition. And the dynamic water pressure of the second order corresponding to the water surface profile η_2 in Fig. 3 gives rise to the nonlinear vertical motion. Thus it seems that the time-dependent tensile force is given by the vertical motion of the structure. This becomes larger with an increment of the incident wave length, while the time-independent component has the reverse tendency. The results of the second order components are different from those of the second order horizontal motion. The total amplitude of the first and second orders, however, can be seen to increase as the wave length becomes longer.



(a)$d_s/h=0.11, d_a/h=0.20, 2\eta_0/L=0.0106,$ $2l_2/L=0.33$



(b)$d_s/h=0.11, d_a/h=0.20, 2\eta_0/L=0.0143,$ $2l_2/L=0.45$

Fig. 6 Time variation of tensile force acting on offshore side mooring line

5. CONCLUSION

In this study, the nonlinear theoretical analysis, based on the frequency-domain method, has been newly developed to evaluate nonlinear characteristics of a submerged moored floating structure under regular wave train. Comparison between theory and experiments has been made to illustrate the validity of the present theory for the case of the airchamber structure moored vertically. Main conclusions obtained in this study are summarized as follows:

(1) The numerically calculated values are in general in good agreement with experimental ones, and the validity of the proposed theory is confirmed.

(2) The amplitude of the nonlinear wave grows very large by the nonlinear wave-structure interaction immediately after wave propagation on the structure, and the total wave profile of the first and second orders depends largely on their phase lag.

(3) The contribution of the time-independent component to the second order horizontal motion is predominant, while the contribution of

the time-dependent component is very small. The time-independent component tends to increase with an increment of the wave period and the initial air pressure in the airchamber.

(4) The contribution of the time-dependent component to the second order tensile force is larger than that of the horizontal motion, which is caused by the vertical motion of structure. The time-dependent component becomes larger with an increment of the incident wave length, while the time-independent component tend to decrease.

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