Model Reference Adaptive Controller with a Fuzzy Model-based Compensator for Nonlinear Time-varying Systems

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비선형 시변시스템을 위한 퍼지 보상기를 갖는 기준모델 적응제어기

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ABSTRACT

본 논문에서는 비선형 시변특성을 가지는 플랜트를 제어하기 위해 이미 잘 정립된 기준모델 적응제어기와 퍼지제어기법을 결합시키는 방법을 연구하였다.

표준 기준모델 적응제어시스템에서 플랜트에 인가되는 입력과 제어시스템의 출력오차를 이용하여 오차생성자를 퍼지모델링하였으며 오차생성자의 출력이 정상상태에서 0에 수렴하도록 상태궤환을 갖는 퍼지제어기를 설계함으로써 플랜트의 비선형 시변특성으로 인한 정상상태 출력오차를 점근적으로 0에 수렴시킬 수 있는 부가적인 제어입력을 구하였다. 이를 표준기준모델 적응제어기에 의하여 생성된 제어입력과 더하여 비선형 시변 플랜트를 제어하기 위한 제어입력을 구함으로써, 궁극적으로 비선형 시변 플랜트의 출력이 정상상태에서 기준모델의 출력을 추종하도록 하는 제어기법을 제안하였다. 제안된 제어시스템의 안정도를 Lyapunov 안정도 이론을 토대로 증명하였고 예제를 이용하여 시뮬레이션을 수행함으로써 그 타당성을 입증하였다.

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I. INTRODUCTION

In case when a knowledge about plants is deficient or imprecisely known, adaptive control is one of methodologies which can deal with such plants effectively. Especially, model reference adaptive control (MRAC) schemes can completely control unknown plants as long as they are modelled as linear time-invariant systems and satisfy some assumptions required to apply them.

Most of MRAC schemes work successfully in their typical control environments. However, there are many cases where plant dynamics are nonlinear time-varying and they cannot be approximated as linear time-invariant models. In these cases, MRAC schemes may not work properly because of mathematical restrictions.

An alternative approach which solves this problem is based on fuzzy logic[4,5]. Fuzzy logic is a very useful technique which can handle complex plants through the use of an expert's control strategy. However, when plants possess highly nonlinear time-varying characteristics, the fuzzy logic technique alone may be ineffective because it suffers from inherent nonlinear problems.

To solve such a problem, a new methodology of combining a model reference adaptive control scheme with the fuzzy logic technique is developed. First, under model reference adaptive control, a fuzzy model which characterizes the nonlinear time-varying characteristics of the plant, called the error generator, is obtained and its structure and parameters are identified using input/output data. For identification of the fuzzy model, Takaki and Sugeno's method[6] is adopted. Secondly, a fuzzy model-based compensator is designed for correcting the output error, that is, making the output of the error generator converge to zero asymptotically. Finally, the additional control input obtained from the fuzzy compensator is added to the control input generated by model reference adaptive control.

In order to illustrate the effectiveness and robustness of the proposed control scheme, computer simulations are conducted on a nonlinear plant.

Ⅱ. MODEL REFERENCE ADAPTIVE CONTROL

2.1 Structure of the Model Reference Adaptive System

First of all, consider a linear time-invariant plant as

$$P: \quad \dot{x}_p = A_p x_p + b_p u \tag{2.1a}$$

$$y_p = h_p^T x_p \tag{2.1b}$$

where x_p is the $n\times 1$ state vector, u is the scalar input, y_p is the scalar output, A_p is an $n\times n$ matrix, and h_p and b_p are $n\times 1$ matrices. The transfer function $W_p(s)$ of the plant can be represented as

$$W_p(s) = h_p^T(sI - A_p)^{-1} b_p \triangleq k_p \frac{Z_p(s)}{R_p(s)} \tag{2.2}$$

It is assumed that $W_p(s)$ is strictly proper with $Z_p(s)$ a monic Hurwitz polynomial of degree $m\,(\le n-1)$, $R_p(s)$ a monic polynomial of n, and k_p a constant parameter. It is further assumed that m, n and the sign of k_p are *a priori* known.

The transfer function $W_m(s)$ of a reference model used here can be represented by

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$$M : \ W_m(s) = k_m \frac{Z_m(s)}{R_m(s)} \tag{2.3}$$

where $Z_m(s)$ and $R_m(s)$, respectively, are monic Hurwitz polynomials of degree $n-1$ and n and k_m is a positive constant.

The deviation of the system from the desired behavior is measured by the absolute value of the error between the plant and reference model outputs as

$$e_1(t) \triangleq | y_p(t) - y_m(t) | . \tag{2.4}$$

The objective is to determine suitable control law $u(t)$ to the plant so that all signals in the system remain bounded and

$$\lim_{t\to\infty} | e_1(t) | = \lim_{t\to\infty} | y_p(t) - y_m(t) | = 0. \tag{2.5}$$

The controller consists of a gain $k(t)$, an input feedback control loop with the parameter vector $\theta_1(t)$ and an output feedback control loop with the parameters $\theta_0(t)$ and $\theta_2(t)$. It is described by the following equations

$$u(t) = \theta^T(t)\omega(t) \tag{2.6a}$$

$$\omega(t) \triangleq [r(t), \omega_1^T(t), y_p(t), \omega_2^T(t)]^T \tag{2.6b}$$

$$\theta(t) \triangleq [k(t), \theta_1^T(t), \theta_0(t), \theta_2^T(t)]^T \tag{2.6c}$$

$$\dot{\omega}_1(t) = \Lambda\omega_1(t) + \ell u(t) \tag{2.6d}$$

$$\dot{\omega}_2(t) = \Lambda\omega_2(t) + \ell y_p(t) \tag{2.6e}$$

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where $\omega_1, \omega_2 \in \mathbb{R}^{n-1}$, $\theta_1, \theta_2 \in \mathbb{R}^{n-1}$, $\theta_0, k \in \mathbb{R}$, Λ is an $(n-1)\times(n-1)$ stable matrix, ℓ is a scalar and $det[sI-\Lambda] = \lambda(s)$.

When the relative degree $n^* = 1$, the parameter error vector $\phi(t)$ is updated according to control law

$$\dot{\phi} = \dot{\theta} = -sgn(k_p)\, e_1(t)\, \omega(t). \tag{2.7}$$

By lemma given in Narendra and Valavani's work[2], the state error $e(t)$ and the parameter error $\phi(t)$ are bounded. Since e_1 as well as the output of the reference model is bounded, y_p is bounded and $\omega(t)$ is bounded so that $e(t)\to 0$ as $t\to\infty$ or $|e_1(t)|\to 0$ as $t\to\infty$.

Ⅲ. DESIGN OF MODEL REFERENCE ADAPTIVE CONTROLLER WITH A FUZZY MODEL-BASED COMPENSATOR

When the plant is linear time-invariant as described in the previous chapter the model reference adaptive control scheme is sufficient to achieve good control performance. However, there are some situation, as in many real control environments, where the plant evolves nonlinear time-varying characteristics during its operation. In these cases, this adaptive control scheme is not applicable because some assumptions are no longer satisfied. Intuitively, it can be assumed that the output error would be generated due to the time-varying characteristics of the plant, denoted as the error generator. In order to eliminate the output error, the adaptive controller must be compensated while plant dynamics change. This motivates the developement of a further enhanced control scheme.

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3.1 Fuzzy Modelling of the Error Generator

If the behavior of the error generator is analyzed, it could be achieved to obtain an additional control input so that the output of the error generator converges to zero in the steady state. To do this, first a fuzzy model characterizing the error generator is obtained. Then, a fuzzy model-based compensator is designed in such a way that the output of the identified fuzzy error generator is made as small as possible and resultantly an additional control input is added to the control input produced by the MRAC.

In this paper, the Takaki and Sugeno's method[7] is used to build a fuzzy model which is of the following discrete-time form. This model is composed of fuzzy "if-then" rules which represent locally linear input/output relations whose consequent part is denoted as "subsystem of the error generator". The rules are expressed as:

$$L^1:\ IF\ x_1(k)\ is\ A_1^1\ \text{and} \cdots \text{and}\ x_p(k)\ is\ A_p^1$$
$$THEN\ \ x^1(k+1) = a_{11}x_1(k) + a_{21}x_2(k) + \cdots + a_{p1}x_p(k) + b_1 u_c(k) \tag{3.1a}$$
$$\vdots$$
$$L^l:\ IF\ x_1(k)\ is\ A_1^l\ \text{and} \cdots \text{and}\ x_p(k)\ is\ A_p^l$$
$$THEN\ \ x^l(k+1) = a_{1l}x_1(k) + a_{2l}x_2(k) + \cdots + a_{pl}x_p(k) + b_l u_c(k) \tag{3.1b}$$

where $L^i\ (i=1,2,\cdots,l)$ is the i-th implication, l is the implication number, $x_j(k)\ (j=1,2,\cdots,p)$ is the j-th linguistic variable representing a state variable of the fuzzy system, $x^i(k+1)\ (i=1,\cdots,l)$ is the output from the i-th rule,

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a_{ji} and b_i are consequent parameters, and A_{ji} are fuzzy sets whose membership functions are represented by continuous piecewise-polynomial functions.

Given a set of the input($x_1, \cdots, x_p$, u_c), the output $x(k+1)$ of the fuzzy system is obtained by

$$x(k+1) = \frac{\sum_{i=1}^{l}(A_1^i(x_1)*, \cdots, *A_p^i(x_p))\ x^i(k+1)}{\sum_{i=1}^{l}(A_1^i(x_1)*, \cdots, *A_p^i(x_p))} \tag{3.2}$$

where the symbol $*$ denotes a triangular norm. In this work, algebraic product is employed.

Defining $\beta_i = \dfrac{\prod_{j=1}^{p} A_j^i(x_j)}{\sum_{i=1}^{l}\prod_{j=1}^{p} A_j^i(x_j)}$ yields

$$x(k+1) = \sum_{i=1}^{l} \beta_i(a_{1i}x_1(k) + a_{2i}x_2(k) + \cdot\ \cdot\ \cdot + a_{pi}x_p(k) + b_i u_c(k))$$

$$= \sum_{i=1}^{l}(a_{1i}x_1(k)\beta_i + a_{2i}x_2(k)\beta_i + \cdots + a_{pi}x_p(k)\beta_i + b_i u_c(k)\beta_i). \tag{3.3}$$

Once the fuzzy model for the error generator has been constructed, its parameters can be obtained using input/output data of the model reference adaptive system in the steady state. The parameters of the fuzzy model are identified according to the identification procedure suggested by Sugeno and Kang[7].

3.2 Design of a Fuzzy Model-based Compensator

To design a fuzzy compensator, the consequence part of the identified fuzzy model is rewritten in a vector-matrix form as:

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$$L^i: \textit{IF} \ x_1(k) \ \textit{is} \ A_1^i \ \text{and} \ \cdots \ \text{and} \ x_p(k) \ \textit{is} \ A_p^i$$
$$\textit{THEN} \ \boldsymbol{x}^i(k+1) = A_i\boldsymbol{x}(k) + B_i u_c(k) \quad (i=1,2,\cdots,l) \tag{3.4}$$

where the state vector $\boldsymbol{x}^T(k) = [x_1(k), \cdots, x_p(k)]$ and A_i, B_i are constant matrices. When a pair of $\{\boldsymbol{x}(k), u_c(k)\}$ is given, the final output of the fuzzy system is inferred by

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{l} \omega_i(k)\,[A_i\boldsymbol{x}(k) + B_i u_c(k)]}{\sum_{i=1}^{l} \omega_i(k)} \tag{3.5}$$

with $\omega_i(k) = \prod_{j=1}^{p} A_j^i(x_j(k))$, where $A_j^i(x_j(k))$ is the membership grade of the fuzzy set A_j^i at $x_j(k)$. It is assumed that $\sum_{i=1}^{l} \omega_i(k) > 0$ and $\omega_i(k) \geq 0$ $(i=1,2,\cdots,l)$ for all k.

Thus, if a fuzzy controller is designed through a knowledge of the fuzzy model and the regulation input can be obtained so that the output $\boldsymbol{x}(k+1)$ converges to zero in the steady state, the overall control system could be controlled in a stable fashion. The i-th control rule has the following form and acts only on the i-th rule of the fuzzy system.

$$\textit{Control rule } i: \ \textit{IF} \ x_1(k) \ \textit{is} \ A_1^i \ \text{and} \ \cdots \ \text{and} \ x_p(k) \ \textit{is} \ A_p^i$$
$$\textit{THEN} \ u_c^i(k) = -K_i\,\boldsymbol{x}(k) \quad (i=1,2,\cdots,l) \tag{3.6}$$

where K_i $(i=1,\cdots,l)$ denotes a feedback gain which is appropriately selected to ensure satisfactory responses and stability. The final output of the fuzzy compensator is given by

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$$u_c(k) = -\frac{\sum_{i=1}^{l} \omega_i(k) K_i \boldsymbol{x}(k)}{\sum_{i=1}^{l} \omega_i(k)} \tag{3.7}$$

Thus the i-th subsystem of the fuzzy control system is expressed as

$$\begin{aligned} &\textit{Control subsystem } i\text{: } \textit{IF } x_1(k) \textit{ is } A_1^i \text{ and } \cdots \text{ and } x_p(k) \textit{ is } A_p^i \\ &\qquad \textit{THEN} \quad \boldsymbol{x}^i(k+1) = (A_i - B_i K_i)\,\boldsymbol{x}(k) \end{aligned} \tag{3.8}$$

and the overall output of the fuzzy control system can be obtained as.

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(k)\omega_j(k)\,(A_i - B_i K_j)\,\boldsymbol{x}(k)}{\sum_{i=1}^{l} \sum_{j=1}^{l} \omega_i(k)\omega_j(k)} \tag{3.9}$$

Therefore, the total control input $u(t)$ used to control nonlinear time-varying plant is obtained by adding the additional control input $u_c(t)$ from the fuzzy compensator to the control input from the MRAC. Fig. 1 depicts the overall control system which combines the MRAC system with the fuzzy control system.

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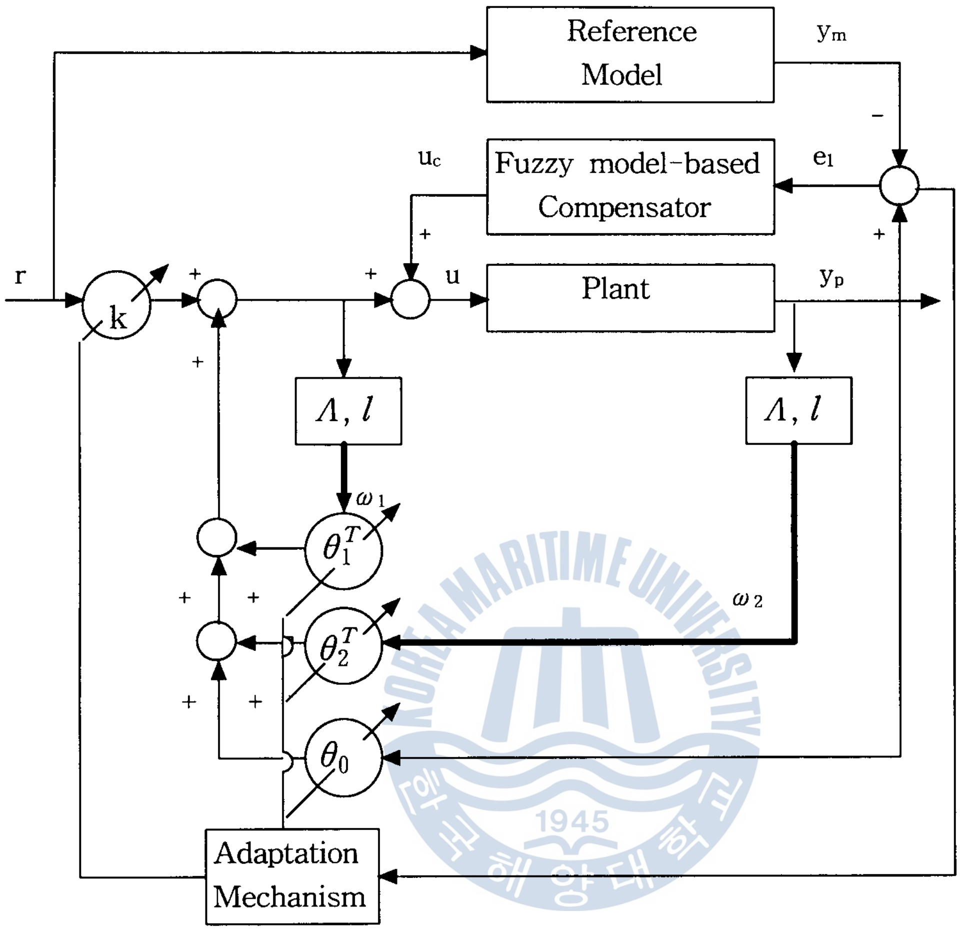

Fig. 1. The proposed control system

Ⅳ. SIMULATION STUDIES

Consider the following nonlinear time-varying plant with a bounded disturbance.

plant : $\ddot{y} = \dot{y} + (2.0 + \cos(t))y^2 + \dot{u} + u + v$

disturbance : $v(t) = 0.5\sin(t) + \cos(2t)e + 0.5e^2\cos(t)$,

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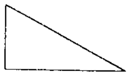
The following reference model was used.

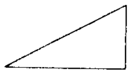
reference model : $\dot{y} = -y + r$

The model reference adpative control loop developed in Chapter 2 was constructed and a fuzzy compensator was incorporated. To build the fuzzy compensator, a fuzzy model was identified from input-output data of the model reference adaptive system with a sinusoidal input, $r(t) = \cos(t) + 5\cos(5t)$.

The sampling time was set as T=0.01. Through the identification procedure using a set of observed data, $\{ e_1, \Delta e_1, u_c(k)$, and $e_1(k+1)\}$, the fuzzy model together with the root mean square value was obtained in the following form.

IF $\Delta e_1(k)$ is  THEN $e_1(k+1)$=1.0021 $e_1(k)$+0.009 $\Delta e_1(k)$+0.0005 $u_c(k)$

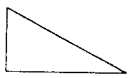
-3.5 8.0

IF $\Delta e_1(k)$ is  THEN $e_1(k+1)$=1.0009 $e_1(k)$+0.01 $\Delta e_1(k)$-0.0002 $u_c(k)$

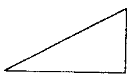
-3.5 8.0

Performance index= 0.0229

The type-B connection[8] of this model and the control rule with a stable feedback gain K= [200 -1] yielded:

S^1 : IF $\Delta e_1(k)$ is  THEN $e_1(k+1)$=0.9362 $e_1(k)$-0.009 $e_1(k-1)$

-3.5 8.0

S^2 : IF $\Delta e_1(k)$ is  THEN $e_1(k+1)$=1.0107 $e_1(k)$-0.01 $e_1(k-1)$

-3.5 8.0

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4.1 Tracking performance

The step response of the overall control system with the selected model is given in Fig. 2. It can be seen that the proposed system exibits good tracking and steady-state performance behaviors even if the plant has highly nonlinear characteristics.

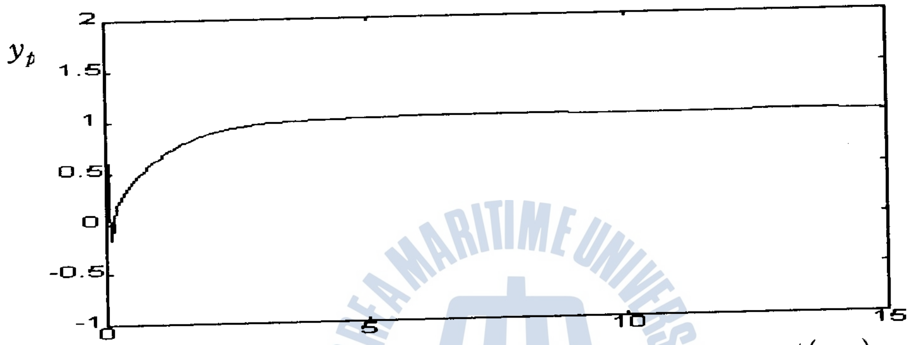

Fig. 2 Step response of the proposed system

4.2 Responses to disturbance change and parameter variations

Fig. 3-4 show the robustness of adaptive fuzzy control scheme through the disturbance change and parameter modification.

Disturbance change : $v(t) = \sin(0.2t) + \tan(0.3t)$ (t>5).

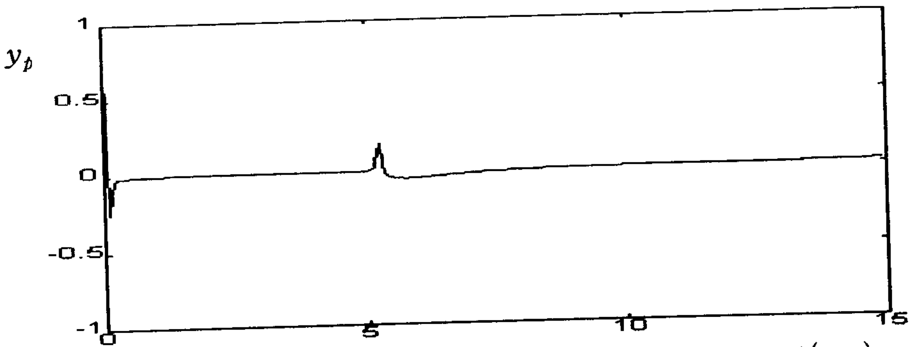

Fig. 3 Output response of the proposed system to a disturbance change (y_p=0)

Plant parameter modification : $\ddot{y} = \dot{y} + (4.0 + 2.0\cos(t))y^2 + \dot{u} + u + v$

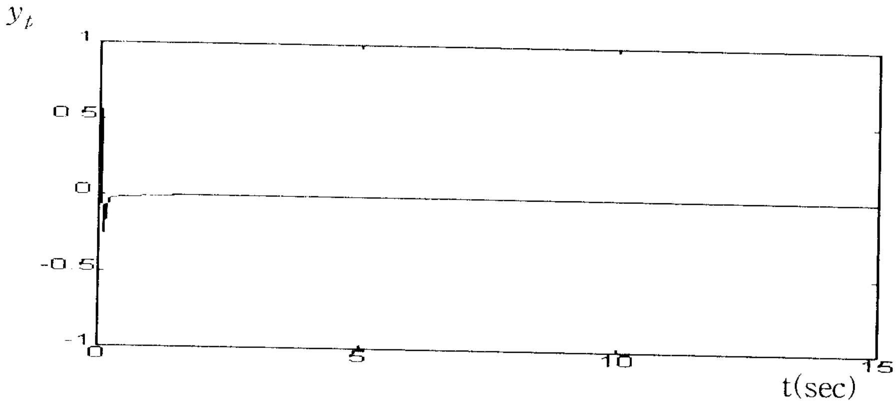

Fig. 4 Output response of the proposed system to a parameter change

It is obvious that the proposed control system maintains the steady-state output in spite of the disturbance and parameter changes.

4.3 Stability

Among all the problems regarding the proposed control system, its stability is obviously one of major concerns. To check the stability of the overall system, it is sufficient to demonstrate that the fuzzy control system is stable. For the subsystems S^1 and S^2 of Model 2-2, two matrices were obtained as

$$A_1 = \begin{bmatrix} 0.9362 & -0.0091 \\ 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.0171 & -0.01 \\ 1 & 0 \end{bmatrix}.$$

The choice of a positive definite matrix $P = \begin{bmatrix} 3.5 & -1 \\ -1 & 1 \end{bmatrix}$ satisfied the condition $A_i^T P A_i - P < 0$ for all $i \in \{1, 2\}$. This means that the fuzzy control system can be asymptotically stabilized by selecting a stable feedback gain.

Ⅴ. CONCLUSION

In this work, a new methodology of combining a model reference adaptive control scheme with the fuzzy logic technique was developed for controlling nonlinear time-varying systems. First, a fuzzy model representing the nonlinear time-varying characteristics of a plant, denoted as the error generator, was obtained and its parameters were identified. Secondly, a fuzzy model-based compensator was designed for correcting the output error. Finally, the additional control input obtained from the fuzzy compensator was added to the control input generated by the MRAC.

The effectivenesss of the proposed control system has been demonstrated through computer simulations on a nonlinear plant. The results have shown that the proposed control system is capable of successfully tracking the set-point and regulating itself to new environments with improved responses. Moreover, the results have demonstrated that it has robustness to disturbance changes and parameter variations of the plant.

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