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Linear and Integer Linear Programming Models for Container Liner Fleet Routing Strategy

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Abstract

This paper provides two optimization models for routing strategies of container liner fleets. A linear programming model for profit maximization, and a mixed integer programming model for cost minimization are presented. The models provide optimal routing mix over the planning horizon and also optimal capital investment alternatives related to chartering or ship building. For the formulation process, the concept of 'flow-route incidence matrix' has been devised and used. This links demands of cargo to route utilization in a simple and systematic way, and is expected to serve as a generally useful modelling tool for ship routing or scheduling problems.

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1. Introduction

It is well known that the gross revenue of liner shipping amounts to more than 50% of the total shipping industry. We can therefore expect that the shipping companies with average size of liner fleet can benefit a lot from improving routing or scheduling by systematic methods. The objective of this paper is to suggest easy and practical optimization models of routing strategies for container liner fleets.

Many useful routing and scheduling problems have been studied for vehicles. A comprehensive survey of vehicle routing and scheduling problems can be found in [1, 7]. As for the ship scheduling or routing problems, relatively less effort has been devoted, in spite of the fact that sea transportation involves very large capital and operating costs. The reason of this less research and a survey of many relevant studies are found in the paper of Ronen [14].

Of course, there have been studies on optimization models for routing or scheduling problems in sea transportation. But the majority of them have been on industrial carriers, bulk carriers, or tankers. On liner fleet management, some heuristic approaches rather than analytic optimization models have been dominant. For example, Boffey *et al.* [2] developed a heuristic optimization model and an interactive decision support system for scheduling containerships on the North Atlantic route. Olson, Sorenson and Sullivan [8] used a simulation model to obtain regular schedules for a fleet of cargo ships involved in a liner trade. But it should be mentioned that, unlike the exact optimization methods, the heuristic methods or simulation models can pick up a best solution only among a finite set of alternatives. Only quite recently, a few analytic optimization models have been attempted to routing and scheduling problems for liner fleet. Perakis and Jaramillo [6, 11] developed a simple linear programming model for a routing strategy to minimize total operation cost and lay-up cost during a planning horizon. They assumed several predetermined routes (sequences of ports of call) and made a model to assign each ship to some mix of the predetermined routes. Rana and Vickson [12, 13] presented nonlinear programming models. They tried to maximize total profit by finding out an optimal sequence of ports of call for each ship. For solution methods, they used Lagrangean relaxation [5] and decomposition methods.

Since the route of a containership, once determined, is hard to be altered for a certain period of time, the initial routing decision should be made cautiously. It is also highly desirable to rearrange the whole system of routes by some analytic methods, periodically, to adjust to changing shipping environments such as changes in main stream cargo demands, in freight rates, or in international regulations. A slight improvement of routes could yield enormous additional profits or cost savings, for the involved costs or revenues in liner operation are, in general, very large and occurs continuously.

The optimization models developed so far seem to be limited in real applications for some reasons. One [13] is made for only one containership, or time charter decision making. The other model of Rana and Vickson [12] puts on complicating shape because of its nonlinearity in both objective function and constraints. The model of Perakis and Jaramillo [6, 11] is simpler one for practical purpose, but it does not properly take into consideration the cargo demand forecasts that arise between pairs of ports within the model.

The models, developed in this paper, are aimed to be simple for practical purposes, and to systematically reflect the future cargo demand forecasts. For ease of the models, the nonlinearity has been avoided, and the models have been developed only within the area of linear programming and mixed integer programming with 0–1 variables. This would make it possible to get the solution of the models of practical sizes with the aid of well known computer packages. To systematically connect the cargo demand forecasts with the routes under consideration, we have devised the concept of *flow–route incidence matrix* and used it in the models. This is expected to be a general formulation tool for many optimization models for ship routing or scheduling problems.

In section 2, the routing problem treated and the assumptions are discussed. Section 3 introduces the concept of flow–route incidence matrix and gives some illustrations and potential uses. In sections 4 – 5, two optimization models for strategic routing problems of container liner fleet are provided. One is a linear programming model for profit maximization, the other is a mixed integer programming model for cost minimization. Brief discussions of some specific considerations for the solution methods are also added.

2. Problem description and assumptions

Managers of shipping companies can manage to adapt to environmental changes by using their insight acquired through many years of experiences in their business. In particular, if they have only a few ships available for their operation, they might do well without any help of analytic models for ship routing or scheduling problems. But as their fleet size and involved shipping routes increase, the decision making problems should consider more and more complex factors that human brain cannot process simultaneously, and the number of feasible alternatives also increase beyond their best insights. The important thing is that a slightly better alternative than their best insights that would be suggested by analytic models could get them the larger amount of additional benefits, the larger and the more complicated the problem at hand is. The optimization models in this paper are developed to help the managers of shipping companies under this situation to make their decisions better in liner fleet routing problems.

Since the route of a liner, once determined, cannot be changed for a certain period of time in practice, the routing problem of a container liner fleet is something like the strategic production planning problem of a manufacturing company. Therefore, we need, as important preliminary data for the routing problems, the demand forecasts of cargo between any two ports the shipping company plans to serve. In this light, we assume that the shipping company has all the required demand forecasts, d_{ij} (the demand forecast of cargo from port i to port j), for the planning horizon. With this data the decision maker (manager) wants to assign each ship (or each type of ship) k, $(k = 1, \cdots, K)$ to proper routes r ($r = 1, \cdots, R$) among a finite set of candidate routes considered by the shipping company to optimize his decision criteria. At the same time, he wants to determine approximate frequencies of the liner service on each route. Additionally he also might want to decide which ship to add to available fleet, i.e., which ship to charter in for the planning horizon, which ship to build or purchase among finite set of the capital investment options. This kind of problem corresponds to the capital investment planning problem of a manufacturing company, and the mixed integer programming model in Section 5 will be of help.

In this paper the following assumptions are imposed for the models and the following notations are used:

Assumptions

(a) The demand of cargo (expected number of containers) from port i to port j, d_{ij}, over the planning horizon is deterministic, known, and occurs uniformly during the planning horizon.

(b) A ship can be used on more than one route if needed.

(c) The managers of the shipping company can suggest a finite set of candidate routes for their liner fleet, old or new, derived from common sense, their past experience, or their future view of main cargo flows. We could also think of a model which determines the sequences of ports of call, from the very start, without any *a priori* set of routes just like the model in [12]. But it seems more practical to assume that the managers have their own sense of the routes, and the routes determined through the analytic models should not be far different from this.

Notations

(a) d_{ij}: Cargo demands (number of containers) from port i to port j; we simply call it the ***flow*** from port i to port j.

(b) m_{ij}: Minimum required number of trips from port i to port j to satisfy the flow d_{ij}.

(c) c_{rk}: Expected operation cost of ship k on the route r per a round trip.

(d) π_{rk}: Expected profit from a round trip on route r by ship k.

(e) t_{rk}: Total travel time for ship k on route r per a round trip, roughly the sum of sailing time and the time spent at ports on route r.

(f) t_k: Maximum time ship k is available during the time horizon.

(g) h_k: Lay-up (idle) cost of ship k per unit time.

(h) f_k: Fixed cost of ship k during the planning horizon; total charter rate for a newly chartered ship, or total capital cost incurred for a new ship built or purchased.

(i) $a_{ij,rk}$: A component of the augmented flow–route incidence matrix.

(j) x_{rk}: Number of round trips of ship k on route r.

(k) y_k: Lay-up time of ship k during the planning horizon.

(l) $(i, j) \in r$: Route r contains a path from port i to port j either in outbound direction or in inbound direction.

3. Flow–route incidence matrix

For the models suggested in the next two sections, this section introduces a matrix which is expected to serve as a generally useful formulation tool for routing or scheduling problems of ships. We call this matrix the *flow–route incidence matrix* since this links the cargo demands to candidate routes.

Suppose there are N pairs of ports (i, j) where flow, d_{ij} $(>)0$, occurs from port i to port j over the planning horizon, and there are R candidate routes considered. By $(i, j) \in r$, we indicate that route r contains a path from port i to port j either on its outbound direction or on inbound direction, i.e., a flow from port i to port j can be fulfilled by a finite number of round trips on route r.

Definition 3.1. A matrix $A = \left(a_{ij,r}\right)_{N\times R}$ is called the **flow–route incidence matrix** with N–flows and R–routes, if,

$$a_{ij,r} = \begin{cases} 1, & \text{if } (i,j) \in r \text{ ;} \\ 0, & \text{otherwise.} \end{cases}$$

■

The following example gives a simple illustration of the flow–route incidence matrix.

Example 3.1. Suppose there are flows d_{12}, d_{13}, d_{24}, d_{41} and two candidate routes 1 – 2 – 4 – 1, and 1 – 3 – 4 – 1 where port 1 and 4 are the common end ports of both routes. Then, the corresponding flow–route incidence matrix becomes as follows,

$$A = \begin{array}{c} \\ (1,2) \\ (1,3) \\ (2,4) \\ (4,1) \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \end{array}.$$

■

The above flow–route incidence matrix seems to have various potential usefulness for modelling strategic routing problems or operational scheduling problems of ships. For an instance, the following simple 0–1 integer programming model, in fact an instance of a *set covering problem* [9], is used to find a way of minimum number of routes to satisfy the future cargo demands (flows):

$$\min \{ x_1 + \cdots + x_R \mid Ax \geq 1, \ x_j \in \{0,1\} \} \cdot$$

where $x = (x_1, \cdots, x_R)^T$ and 1 is the column vector every component of which is 1. For another example, we can think of a problem to find a feasible set of ship schedules to satisfy a set of given temporary cargo demands. Let c_r indicate the cost involved to run the schedule r, then the best set of schedules can be found by solving the following similar problem:

$$\min \{ cx \mid Ax \geq 1, \ x_j \in \{0,1\} \}$$

where c is the row vector of the costs c_r, and each column of A should represent each candidate schedule rather than a route. This application is actually found in a bulk cargo ship scheduling model [4], and in a tanker scheduling model [3, 10].

The next two sections, in fact, use an extended modification of the above matrix defined as below. Other formulation also could lead to different variations. For each route r, let K_r denote the set of available ships during the planning horizon and which can be assigned to route r. In practice, we may also start by setting $K_r = K$ for any route r, if we have no *a priori* idea about what K_r is like.

Definition 3.2. By an **augmented flow–route incidence matrix** we mean a matrix $\bar{A} = \left(a_{ij,rk}\right)$ ($a_{ij,rk}$ is the component in the row corresponding to the flow of (i,j) and in the column corresponding to route r and ship k with $k \in K_r$) where

$$a_{ij,rk} = \begin{cases} 1 & \text{if } (i,j) \in r; \\ 0 & \text{otherwise} \end{cases}.$$

■

Example 3.2. Continuing with Example 3.1, suppose there are two ships 1, 2 available during the planning horizon, and ship 1 can be assigned to both routes while ship 2 cannot be assigned to route 2 because of a certain government regulation. Then the resulting augmented flow–route incidence matrix becomes

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

■

4. A linear programming model

In this section we present a linear programming model for routing strategy of a liner fleet by using the augmented flow–route incidence matrix $\bar{A}$. Unlike the model in the next section it is assumed that all the ships available during the planning horizon is known and fixed; the shipping company has already decided about what additional ships to add to the existing fleet and what ship to delete for the planning horizon, i.e., the shipping company has already got a fixed plan about what ships to charter in, what to charter out, and what to build or purchase.

Given the capital investment plan for the time horizon, given demand forecasts, the decision making problem of this section is to find a routing mix of the liner fleet to candidate routes to maximize the expected profit from liner operation during the time horizon.

4.1. Objective function

Let e_{rk} be the expected revenue per voyage on route r by ship k, which could be estimated from past experience data of operation, or from the managers' hunch in case r is a newly suggested route. Let c_{rk} is the estimated operation cost of the ship k per voyage on route r. This could be calculated by such methods as suggested in [11]. Then an estimated profit per voyage on route r by ship k, π_{rk}, is determined by

$$\pi_{rk} = e_{rk} - c_{rk}.$$

A natural objective function appears as follows.

$$\max \quad \sum_{r=1}^{R} \sum_{k \in K_r} \pi_{rk} x_{rk} \tag{1}$$

4.2. Constraints

The augmented flow–route incidence matrix is used to make a set of constraints that the selected routes and service frequencies should be enough to satisfy all the required flows d_{ij}. Let w_{ij} be the average amount of cargo (average number of containers) that has been transported per voyage from port i to port j from past experience data. (In case d_{ij} is a newly required flow, we could compute w_{ij} from other sources such as data from other companies or just from decision maker's hunch.) Let x_{rk} be the decision variable denoting the number of voyages of ship k on route r during the planning horizon, and $a_{ij,rk}$ as defined in Definition 3.2. To serve all the cargo demands over the planning horizon the following set of constraints should be satisfied.

$$w_{ij}\left(\sum_{r=1}^{R} \sum_{k \in K_r} a_{ij,rk} x_{rk}\right) \geq d_{ij} \qquad \forall (i,j)$$

Again, if we set $m_{ij} = d_{ij}/w_{ij}$, the above constraints can be expressed equivalenltly as follows.

$$\sum_{r=1}^{R} \sum_{k \in K_r} a_{ij,rk} x_{rk} \geq m_{ij} \qquad \forall (i,j) \tag{2}$$

From the derivation, the above constraints (2) can be regarded as *cargo demand constraints*, and the right hand side m_{ij} can be interpreted as an estimation of the minimum number of voyages on the whole network of routes required to satisfy the estimated demand flow d_{ij}.

The second group of constraints depicts the *time constraints* or *ship capacity constraints*. Let t_k be the total time that ship k is available for operation for the planning, and t_{rk} the total travel time for ship k on route r. We also can refer to [11] for practical computation of them. If we are to keep the same speed for any ship assigned to a route for the regularity of service we can simply set a common number t_r to t_{rk}, i.e., $t_{rk} = t_r$ for any k. For each ship k, let R_k be the set of routes r that ship k can be assigned to for the planning horizon, i.e., $R_k = \{ r \mid k \in K_r \}$. Then the following set of constraints should be satisfied.

$$\sum_{r \in R_k} t_{rk} x_{rk} \leq t_k \qquad \forall k = 1, \cdots, K \tag{3}$$

The objective function (1), and the constraints (2), (3), with the nonnegativity constraints $x_{rk} \geq 0$ together, compose a linear programming model for the routing strategy.

4.3. Completed model and comments

The completed models can be represented in matrix form as follows,

$$\text{(P1)} \qquad \max \ \pi x \tag{4}$$

$$\text{subject to} \ \ \bar{A}x \geq m \tag{5}$$

$$Tx \leq t \tag{6}$$

$$x \geq 0$$

where (4), (5), (6) are the matrix representations of (1), (2), (3) respectively with consistent dimensions of matrices. The above (P1) has no more than $R \times K$ variables, and $N + K$ constraints, and can be solved by standard linear programming packages. The larger part of the constraints can be completely determined by the augmented flow–route incidence matrix.

With the optimal solution, $x^* = (x^*_{rk})$, obtained, we come to know which candidate routes we should choose and which we should not. The optimal service frequencies on each route r can be found as $\sum_k x^*_{rK}$. As for the utilization of each ship, x^*_{rk} shows how many round trips should be made on each route r by ship k. The shipping company should operate the ship k for $\sum_r t_{rk} x^*_{rk}$ time units during the planning horizon. The resulting lay-up time for ship k is $t_k - \sum_r t_{rk} x^*_{rk}$.

Of course we can incorporate this lay-up (idle) time and lay-up (idle) cost to the above model by extending (1) to

$$\max \ \sum_{r=1}^{R} \sum_{k \in K_r} \pi_{rk} x_{rk} - \sum_{k=1}^{K} h_k y_k \tag{1'}$$

and by modifying (3) into

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k = t_k \qquad \forall k = 1, \cdots, K \tag{3'}$$

where h_k is the lay-up cost of ship k per unit time and y_k is the decision variable indicating the lay-up time of ship k.

Since our routing problem is strategic and so concerns a relatively long planning horizon, it is inevitable that the coefficients used in (P1) have a certain degree of uncertainty involved. However, we can easily treat these various uncertainties with the help of the well established post-optimal procedures of the linear programming. Many slight fluctuations in operating costs, for example in fuel costs, or uncertainties of future freight rates can be treated in the framework of objective function sensitivity analysis. Difficulties caused by uncertainties in the estimation of cargo demands d_{ij} can be alleviated by the aid of such method as the right hand side sensitivity analysis.

4.4. Implementation under insufficient information of $\bar{A}$

Suppose the shipping company cannot determine, at first, the right structure of $\bar{A}$, say, for example, the shipping company is not sure if the ship k can be operated well on a newly suggested route r. By many practical reasons such as insufficient ship capacity, or government regulations against ships of some flags, a ship k might be incompatible with a route r, i.e., $k \notin K_r$. But without complete preliminary consideration, i.e., without exact idea of what K_r is like, (P1) cannot be solved at once. In this case the decision maker, at first, may be confronted with the linear program (P1) with the largest $\bar{A}$, i.e., the one with the objective function (a slight extension of (1)),

$$\max \quad \sum_{r=1}^{R} \sum_{k=1}^{K} \pi_{rk} x_{rk} \qquad (1'')$$

with constraints (3), and with the following extension of (2)

$$\sum_{r=1}^{R} \sum_{k=1}^{K} a_{ij,rk} x_{rk} \geq m_{ij} \qquad \forall (i,j) \qquad (2')$$

where $a_{ij,rk} = 1$ if $(i,j) \in r$, and 0 otherwise. However, in order to get practical solutions, the decision maker, in addition, should consider the following implicit constraint that cannot be identified at first.

$$\sum_{r=1}^{R} \sum_{k \notin K_r} x_{rk} = 0 \qquad (7)$$

The fact is that the decision maker should solve the linear programming problem of $(1'')$, $(2')$, (3) and the implicit constraint (7). But since the implicit constraint (7) is not known at first, we can think of some repeated approaches that gradually expose it. A natural procedure, outlined below, is similar to a concept of gradual column deletions:

> At first, we solve the linear program with $(1'')$, $(2')$ and (3). If the optimal solutions obtained are found to satisfy (7), i.e., if the solution does not have any value $x^*_{rk} > 0$ where $k \notin K_r$, by some intuitive tests of the decision maker, the obtained solution may well be (supposed to

be) a real optimal solution for (P1). Otherwise, i.e., the solution has some values $x^*_{rk} > 0$ where $k \notin K_r$, we can delete the corresponding variables or equivalently the columns from the initial model and resolve the resulting reduced linear program. This step would be performed usually very easily, by a well known post–optimal analysis in linear programming. We can repeat the similar procedure until we are sure that the obtained solutions satisfy the implicit condition (7).

The above procedure ends in finite repetition for we have finite number of variables, and the usual number of repetitions are not likely to be so large.

5. A mixed integer programming model

Unlike the linear programming model developed in the last section, this section assumes that the shipping company also has to make capital investment decisions for the planning horizon. To meet the expected increasing future cargo demands, the shipping company may consider some options for fleet capacity expansion such as building or purchasing new ships and chartering some other ships. Since this kind of decision making problem usually requires a longer time planning horizon than the one in the last section, a desirable suggestion for decision criterion would be one not likely to be affected by growing uncertainties due to the longer planning horizon. Hence the objective of the model adopted in this section is to minimize total cost incurred from operations over the planning horizon.

Using the model in this section, the shipping company can get help in decision making problems, in addition to what has been mentioned in the last section, about what types of ships to build, to purchase, or to charter in to add to the existing fleet. To reflect these decision making concerns 0–1 integer variables have been used in the model.

5.1. Objective function

The total cost of the objective function is taken as the sum of the operating cost, the lay-up (or idle) cost, and the (fixed) capital cost incurred during the planning horizon. Let K^0 be the subset of ships that the shipping company considers for adding to the existing fleet. The resulting objective function appears as follows,

$$\min \quad \sum_{r=1}^{R} \sum_{k \in K_r} c_{rk} x_{rk} + \sum_{k=1}^{K} h_k y_k + \sum_{k \in K^0} f_k z_k \tag{8}$$

where f_k denotes the fixed capital cost incurred by adding ship k to the existing fleet, and the variable z_k is a binary variable to decide whether to add ship k ($z_k = 1$) or not ($z_k = 0$). The capital costs due to the existing fleet should be omitted in the model because they are not relevant to the decision making problem at hand.

5.2. Constraints

The cargo demand constraints are the same as in (2) or (5). The time constraints or ship capacity constraints are a little different from (3), and can be split into the following two groups.

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k = t_k \qquad \forall k \notin K^0 \tag{9}$$

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k - t_k z_k = 0 \qquad \forall k \in K^0 \tag{10}$$

Constraints (9) are the usual time constraints that the operating time of a ship cannot exceed the total available time for that ship. Constraints (10) also maintain this, and additionally, require that any ship that is to be operated must be added to the existing fleet.

5.3. Completed model and solution methods

Now the completed cost minimization model appears as follow.

$$\text{(P2)} \qquad \min \quad \sum_{r=1}^{R} \sum_{k \in K_r} c_{rk} x_{rk} + \sum_{k=1}^{K} h_k y_k + \sum_{k \in K^0} f_k z_k \tag{8}$$

$$\text{subject to} \quad \sum_{r=1}^{R} \sum_{k \in K_r} a_{ij,rk} x_{rk} \geq m_{ij} \qquad \forall (i,j) \tag{2}$$

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k = t_k \qquad \forall k \notin K^0 \tag{9}$$

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k - t_k z_k = 0 \qquad \forall k \in K^0 \tag{10}$$

$$x_{rk} \geq 0, \; y_k \geq 0, \; z_k \in \{0, 1\}$$

The above model (P2) is a mixed integer programming model with no more than K binary variables. The continuous variables are no more than $K \times (R+1)$, and the number of constraints is $N + K$. The integer part of an optimal solution, (z_k^*), gives capital investment suggestions for fleet capacity expansion about what ship to build, to purchase, or to charter.

To solve the above problems in practice, we will have no trouble to resort to standard computer packages, since the number of integer variables are expected to be relatively small in general. But as the problem size goes larger and the the number of integer variables increases, we can also consider using some heuristic solution methods. In particular, it seems that the Lagrangean relaxation approach [5] will go well with (P2); if we relax the constraints (10) with Lagrangean multipliers, the resulting Lagrangean subproblem is split into a linear program with constraints (2) and (9), and a trivial 0–1 integer program that can be solved by simple inspection.

6. Conclusions

This paper has suggested two optimization models that can be applied to shipping companies in general with liner fleet. One model is a linear programming model of profit maximization which provides optimal way of routing mix for each ship available and optimal service frequencies for each candidate route. The other model is a mixed integer programming model with binary variables which not only provides optimal routing mixes and service frequencies but also best capital investment alternatives to expand fleet capacity. The latter model is a cost minimization model.

While formulating the two models we have suggested and used the concept of flow–route incidence matrix and discussed its general usefulness for similar routing and scheduling problems that might arise in shipping industry. The most important merit of using this flow–route incidence matrix was that it linked various cargo demands to route utilization in a systematic way, and in a simple way.

The selected routes found as optimal solutions of the suggested models are seldom likely to come out embarrassing, for the models choose the best combinations of routes among candidate routes from the existing ones and suggestions from experienced managers. By doing this, the suggested models can help to improve the existing network of routes or service frequencies in a generally acceptable way.

As for ease of solution methods, the two models are usually supposed to be implemented well with the standard linear or integer programming packages. More efficient solution methods that would fit better the specific structure of these models, and implementations with real data of shipping companies would be major future research subjects.

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