

Fatigue Strength of Fiber Reinforced Composite Material

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Abstract— Rotating bending fatigue tests of short carbon-fiber reinforced polyetheretherketone (CFRPEEK) and polyetheretherketone(PEEK) were carried out to investigate the fatigue characteristics of plain and notched specimens at room temperature. The fatigue mechanisms in the matrix and composite were clarified through successive surface observations using the replica method. The results were discussed using linear notch mechanics. In the plain specimen of PEEK, fracture always occurs from defects and the fatigue crack initiation is of the point-initiation type. Furthermore, the fatigue crack growth rate is very high and the fatigue strength is very sensitive to a notch. The fatigue strength of the composite is much more insensitive to a notch than that of PEEK. In general the fatigue crack initiates from near the fiber end, and propagates to the circumferential direction after it grows to some extent along the fiber. The fatigue strength of an arbitrary notched specimen of these two materials will be estimated from the present results rearranged based on “linear notch mechanics”.

1. Introduction

High performance composites are being applied to engineering applications such as space vehicles, aircraft, road transportation, fishing rods, golf club shafts and yachts because of their ease of fabrication, their light weight, and economy [1]. Many studies on the fatigue and static fracture of long

carbon-fiber reinforced composites have been made to keep structures working safely [2]. There are a few studies, however, on short fiber reinforced composites because of their low specific strength, low specific elastic modulus and complexity of fracture phenomena due to the source of stress concentration such as the fiber end or fiber-matrix interface. It is very important to investigate the fatigue mechanism of the composite for the requirement of reliability in the components. However, the fatigue mechanism of the composite seems not to have been clarified. It was reported that the thermoplastic PEEK, which has excellent properties at high temperature, is a semicrystalline polymer which affords good environmental and chemical resistance. In addition, the fatigue properties and fracture toughness of PEEK are superior to those of other matrix resins [3, 4].

There are several reports [5–7] on the fatigue behavior of PEEK and carbon- or glass-fiber reinforced PEEK. For example, Friedrich et al. [5] studied the effect of short fibers on the fatigue crack propagation and fracture of PEEK-matrix composites, and showed that both the fatigue crack propagation and fracture toughness can be correlated to the microstructure of the composites by means of a microstructural efficiency. They also showed that the dominant fiber-related failure mechanism from fractographic analysis is separation along the fiber-matrix interface in regions where the fibers are oriented parallel to the crack direction, and that the crack tends to run along fiber ends if the fibers are oriented perpendicular to the crack front. Dickson et al. [6] investigated the environmental fatigue behavior of carbon-fiber reinforced PEEK, and showed that the improved fatigue resistance is clearly related to the better crack resistance, or ductility, of PEEK. There are few studies, however, on the investigation of the notch sensitivity of PEEK and PEEK-matrix composite.

In this paper, rotating-bending fatigue tests for PEEK and short carbon-fiber reinforced PEEK were carried out to investigate the fatigue characteristics of plain and notched specimens at room temperature. The fatigue

mechanisms were clarified through successive surface observations using the replica method. The results were discussed based on “linear notch mechanics” [8, 9].

In the following, linear notch mechanics will be explained briefly. Linear notch mechanics [8] is useful in general for treating notch problems [10, 11] is useful in general for treating notch problems [10, 11].

2. Linear notch mechanics [8, 9]

Usually a crack appears before any kind of fracture. Therefore, it may be considered that linear fracture mechanics is sufficient in treating all kinds of fracture problems. However, many notch problems cannot be treated correctly without considering the characteristics of notches. Linear notch mechanics provides an engineering method which treats the notch problems by elastic stress fields alone; similarly, linear fracture (crack) mechanics provides an engineering method which treats crack problems by elastic stress fields (stress intensity factors) alone.

In order to predict the strength of a real object from the strength of specimens, we must know the condition causing the same phenomena in both. Comparing the cases of crack and notch, we notice that there are almost the same situations between crack and notch, concerning the conditions for causing the same phenomena in two cracked members or in two notched members. If the external loads are adjusted so that K_1 values are equal in two members with a crack, the elastic-plastic stress distributions in both members become equal to each other even after the materials near the crack tips undergo slight plastic deformations. The same elastic-plastic stress fields in two cracked members should result in the same phenomena.

Accordingly, the same K_1 values under small scale yielding should result in the same phenomena. On the other hand, if the external loads are adjusted so that the elastic maximum stress, σ_{\max} , values are equal to each other in

two members having a notch with the same size notch root radius, ρ , the elastic-plastic stress distributions in both members become equal to each other. This is based on the following two facts, similar to the case of crack problems [8, 9].

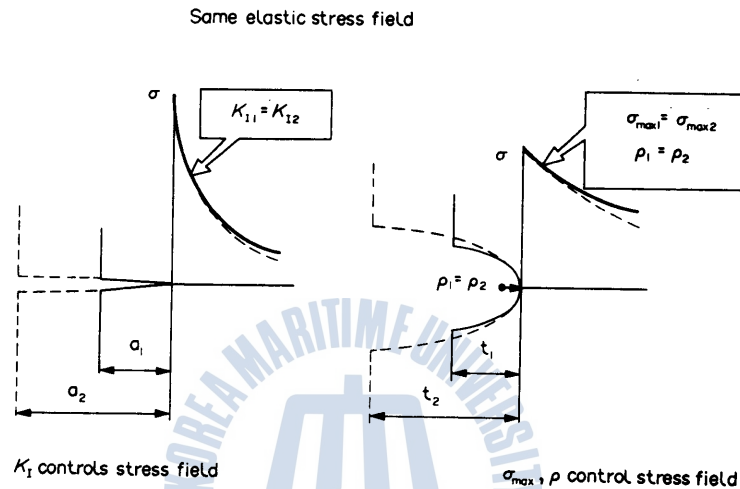
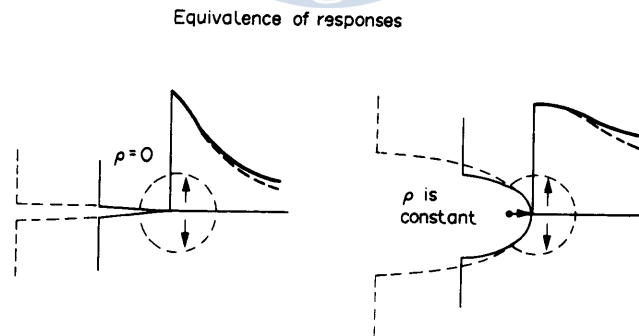


Fig. 1. Conditions for the same elastic stress fields in two cracked members or two notched members. (When the values of K_I are equal in two cracked members, the elastic stress fields near the crack tips are nearly equal to each other. Similarly, when the values of σ_{max} and ρ are equal in two notched members, the elastic stress fields near the notch roots are nearly equal to each other, independent of notch depths or the other geometrical conditions of notched members.)



Geometric condition controls response

Fig. 2. Conditions for the equivalence of responses in two cracked members or two notched members. (In the cases when ρ is constant, the additional stress fields due to a given amount of plastic deformation occurring at a given place near the notch roots are nearly equal to each other.)

- (1) Same elastic stress field: when the values of σ_{\max} and ρ are constant respectively in various notches, the elastic stress distributions near the notch roots are nearly equal to each other, independent of notch depth or other geometrical conditions of notched members. This situation is shown in Fig. 1.

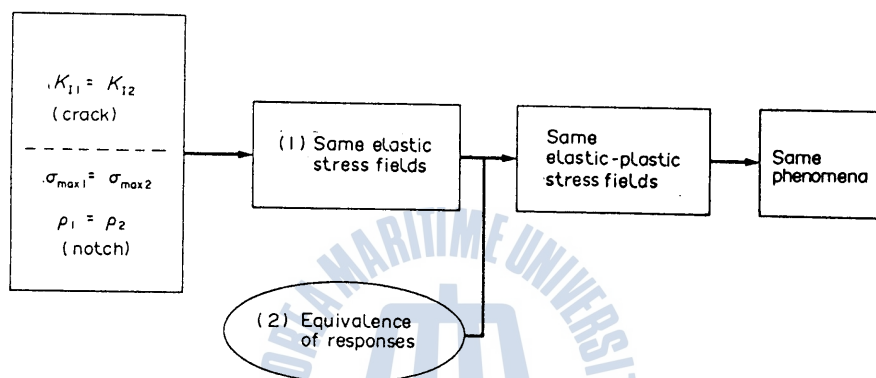


Fig. 3. Conditions for the same phenomena occurring in two cracked members or two notched members. (The same elastic stress fields and equivalence of responses assure the same elastic-plastic stress fields. The same elastic-plastic stress fields assure the same phenomena in two cracked members or two notched members.)

- (2) Equivalence of responses: in the cases where ρ is constant, the additional stress fields due to a given amount of plastic deformation occurring at a given place near the notch roots are nearly equal to each other, independent of notch depth or other geometrical conditions of notched members, This situation is shown in Fig. 2.

The same elastic-plastic stress fields should result in the same phenomena. That is, in notch problems the load necessary for causing fracture or a given size of plastic zone is controlled by σ_{\max} and ρ alone, if the condition of small scale yielding is satisfied. This is the basic concept of linear notch mechanics. Linear notch mechanics treats notch problems by considering the

elastic stress fields alone as in the case of linear fracture mechanics. These situations are shown in Fig.3.

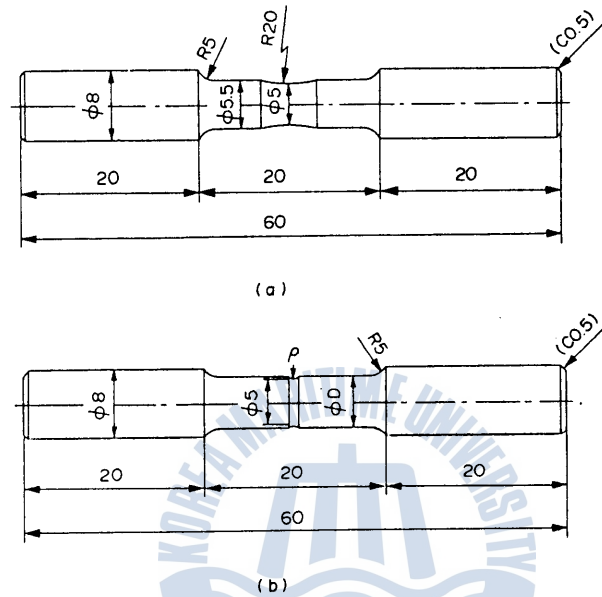


Fig. 5. Shapes and dimensions. (a) Plain specimen. (b) Notched specimen.

3. Materials and experimental procedure

The materials used are polyetheretherketone(PEEK, manufactured by Mitsui Toatsu Co. Ltd., Japan, melting temperature $T_m = 334^\circ\text{C}$, glass transition temperature $T_g = 143^\circ\text{C}$) and short carbon-fiber reinforced polyetheretherketone (CFRPEEK) where the short carbon-fibers having a diameter of $7\ \mu\text{m}$ and a fiber mean length of $150\ \mu\text{m}$ were randomly combined in a PEEK thermoplastic resin matrix with a fiber content of 30 %(by weight).

Figure 4 shows the typical appearance of a polished cross-section of CFRPEEK. The specimens used for rotating-bending fatigue tests were from

cylindrical bars with a diameter of 10 mm. The specimens were machined

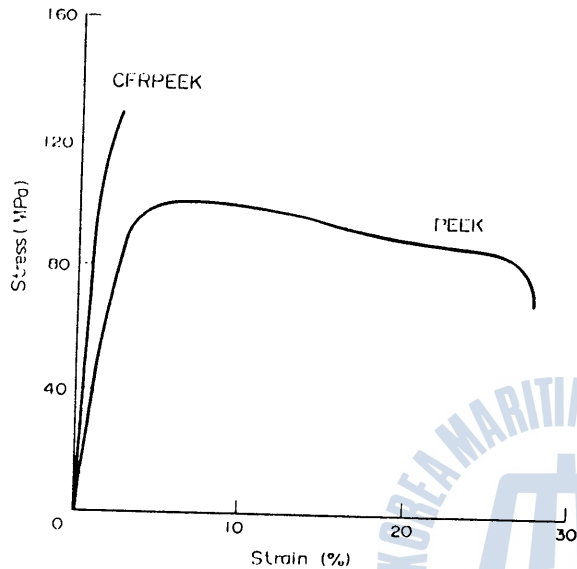


Fig. 6. Stress-strain curves.

Table 1. The values of stress concentration factor at the notch root radius

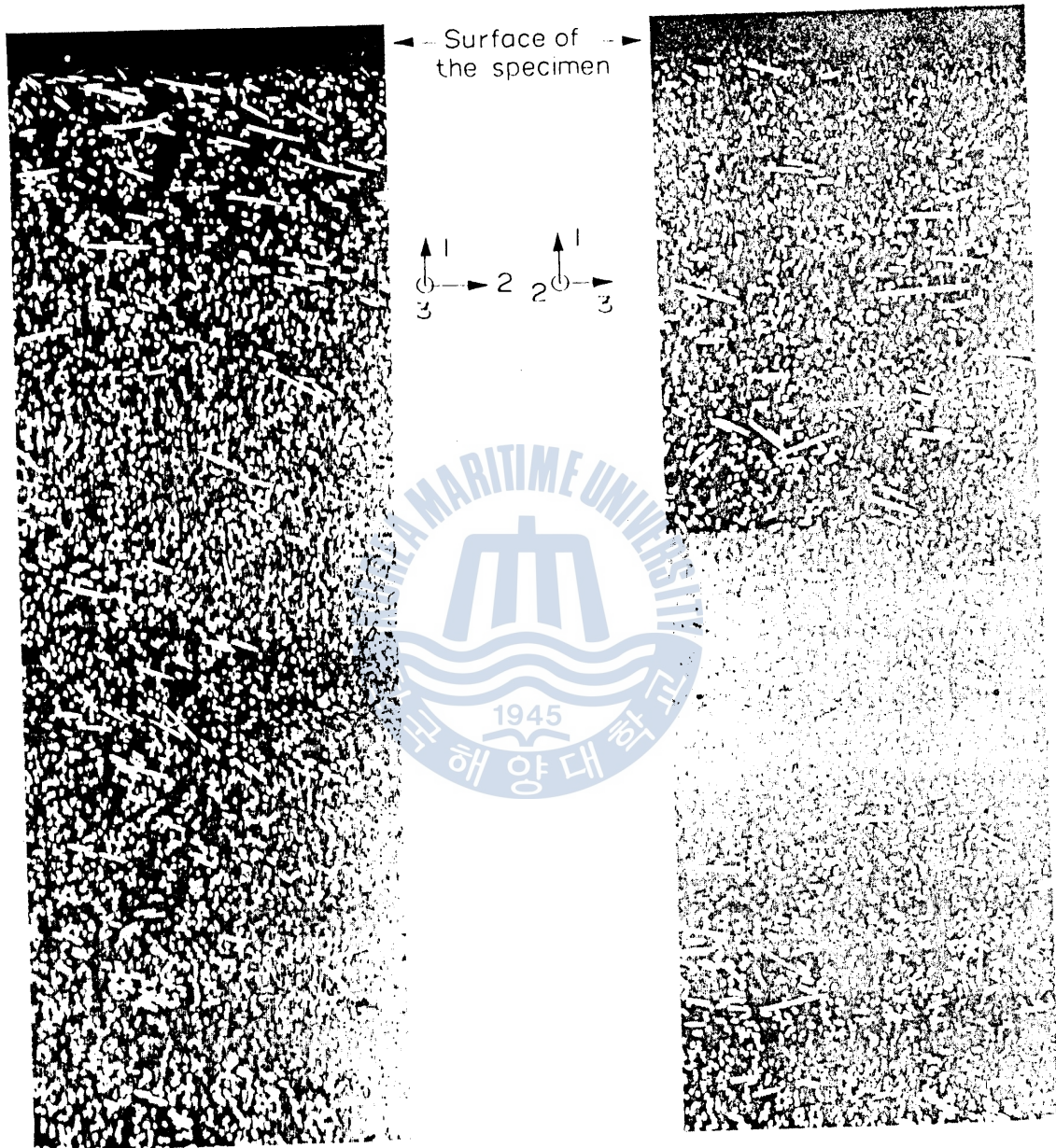
D	t	ρ	K_t
5.4	0.2	1.0	1.49
		0.3	2.13
		0.1	3.20
6	0.5	1.0	1.60
		0.3	2.48
		0.1	3.95
		0.05	4.48
9	2.0	1.0	1.63
		0.3	2.62
		0.1	4.20

t , notch depth (mm); ρ , notch root radius (mm); K_t , stress concentration factor.

from the bars after annealing at 200°C for 2 h [10]. The shape and dimensions of the specimens used are shown in Fig. 5.

Rotating-bending fatigue tests of the notched specimens for PEEK and CFRPEEK were carried out for a wide range of notch root radii and for four different notch depths under the conditions of constant specimen diameter of the minimum section ($d=5$ mm). Stress concentration factors at a notch root were calculated by the body force method [13].

Table 1 shows the values of stress concentration factors at a notch root. In the case of a plain specimen, a specimen with a small hole was also used to examine the crack initiation and propagation behavior. All specimens were polished with fine emery papers, alumina and diamond paste. Moreover, the



- 1 Radial direction
- 2 Circumferential direction
- 3 Axial direction

200 μm

Fig. 4. Microstructure of polished CFRPEEK.

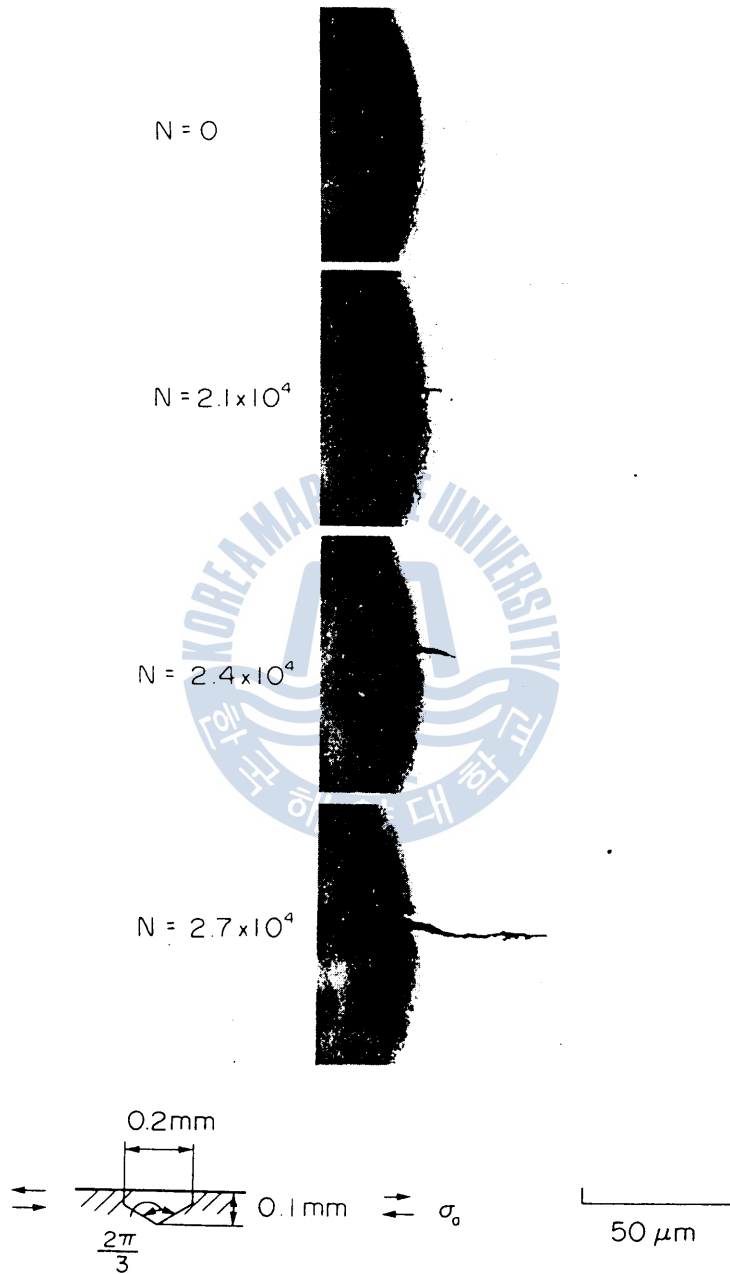


Fig. 9. The change in the surface state of PEEK with a small hole.



Fig. 10(a).

[Fig. 10(b) and caption--*overleaf*]

(b)

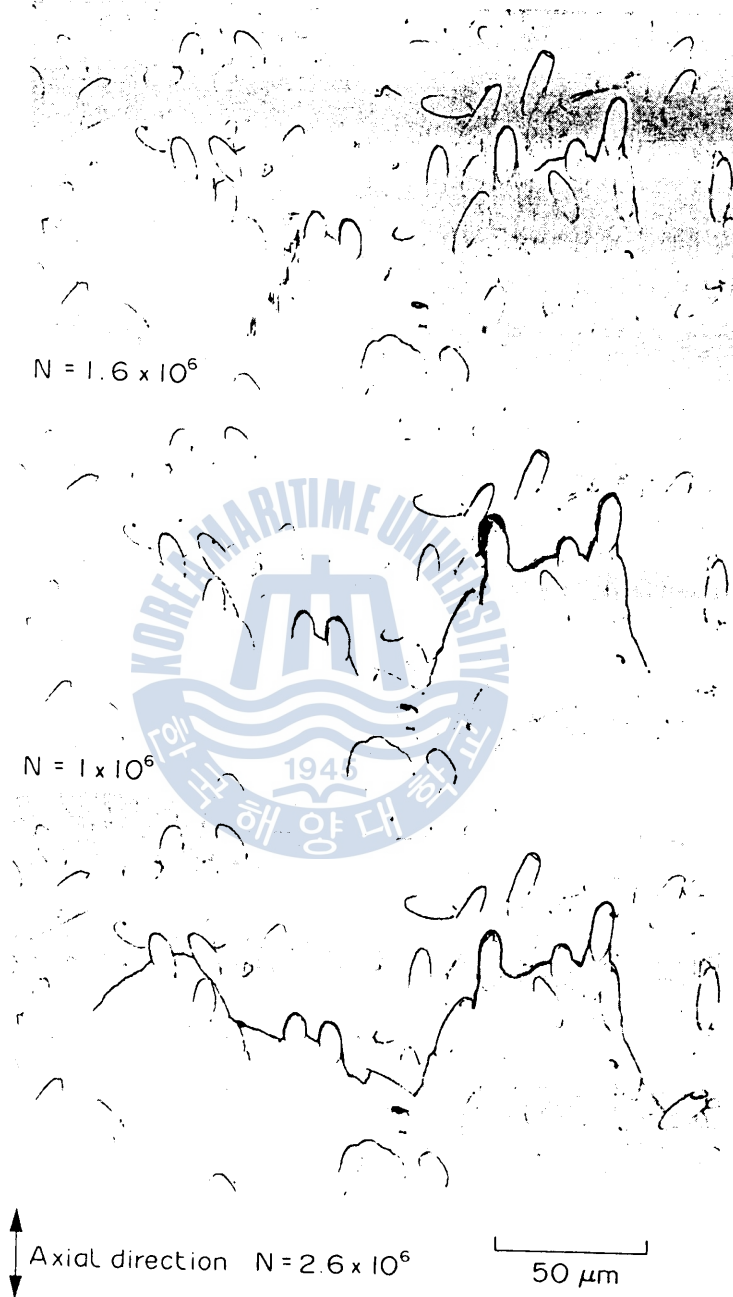


Fig. 10(b).

Fig. 10. Crack initiation and propagation processes of CFRPEEK ($\sigma_a = 64 \text{ MPa}$, $N_f = 4.2 \times 10^7$ cycles).



50 μ m

$\rho = 0.1$ mm
 $l = 0.2$ mm
 $\sigma_0 = 34.3$ MPa

$\rho = 0.1$ mm
 $l = 0.5$ mm
 $\sigma_0 = 24.5$ MPa

$\rho = 0.1$ mm
 $l = 2$ mm
 $\sigma_0 = 22.1$ MPa

Fig. 14. The surface state at the notch root for PEEK.

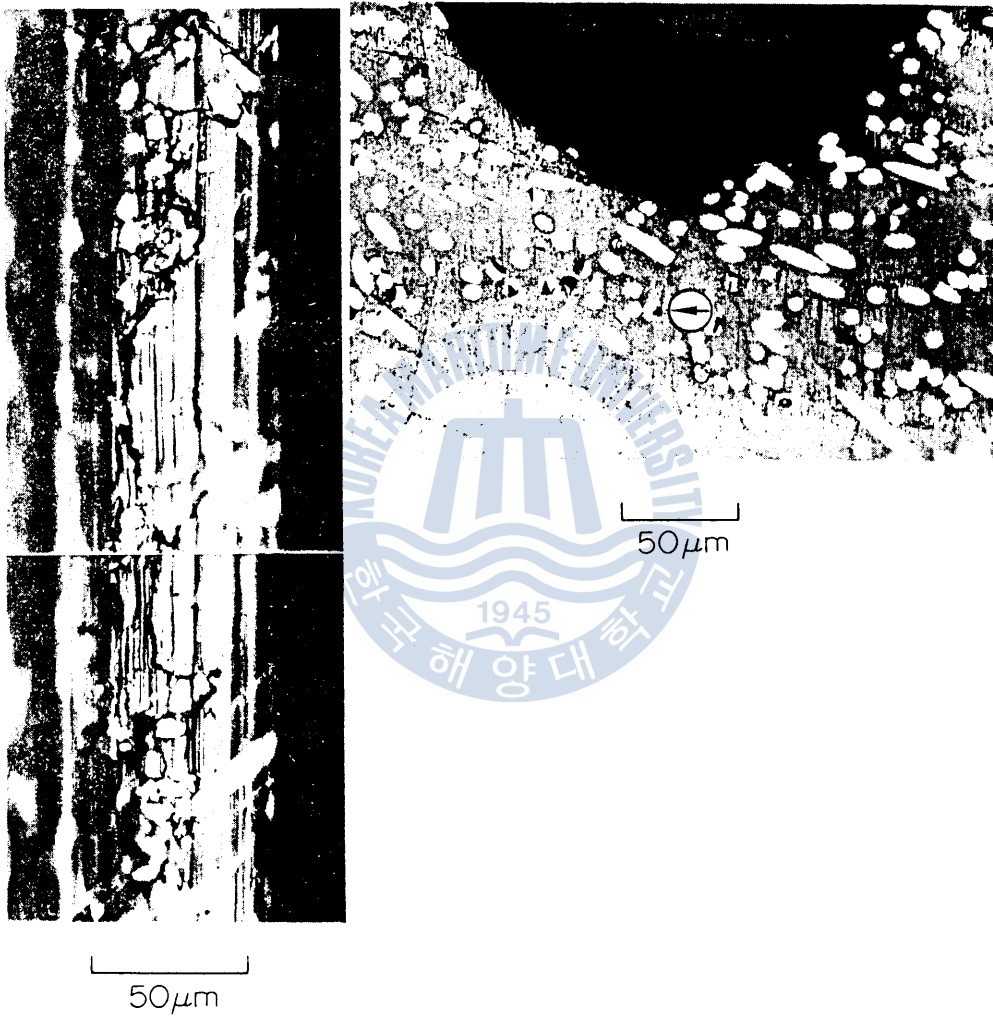


Fig. 16. The non-propagating crack at the notch root ($\rho = 0.1 \text{ mm}$, $\sigma_n = 34.3 \text{ MPa}$).



Fig. 18. Typical fracture appearance. (a) Near the fracture origin. (b) Near the final fracture

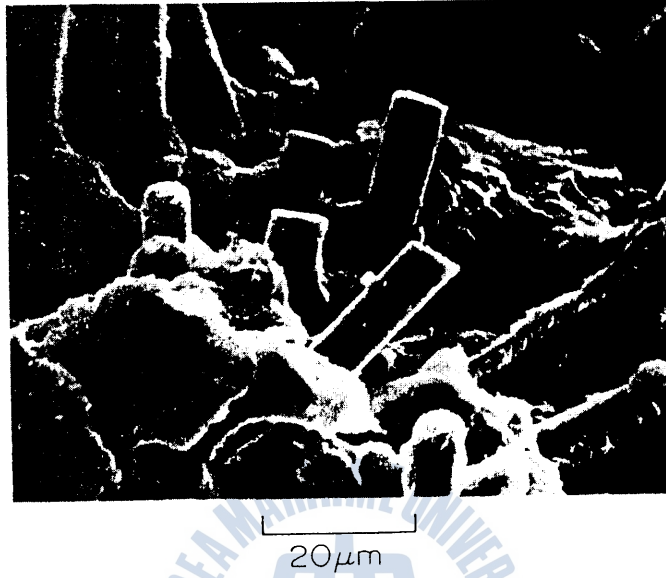


Fig. 23. Fracture appearance near the center of the tensile specimen for CFRPEEK.

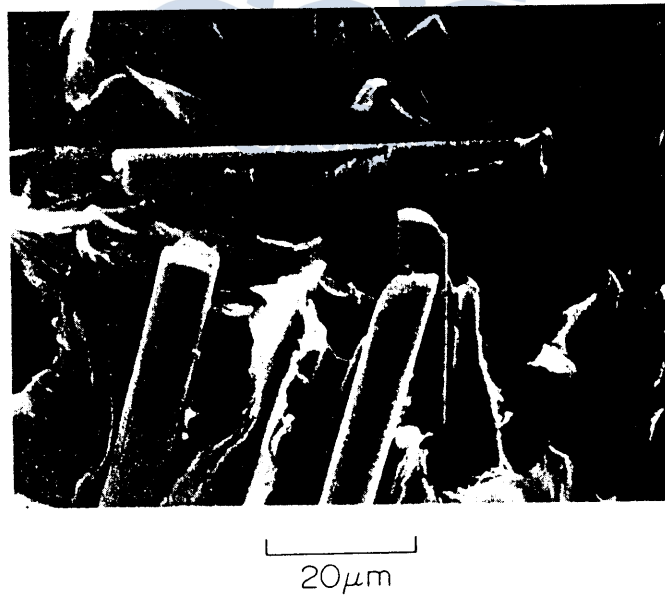


Fig. 24. Fracture appearance near the surface of the tensile specimen for CFRPEEK.

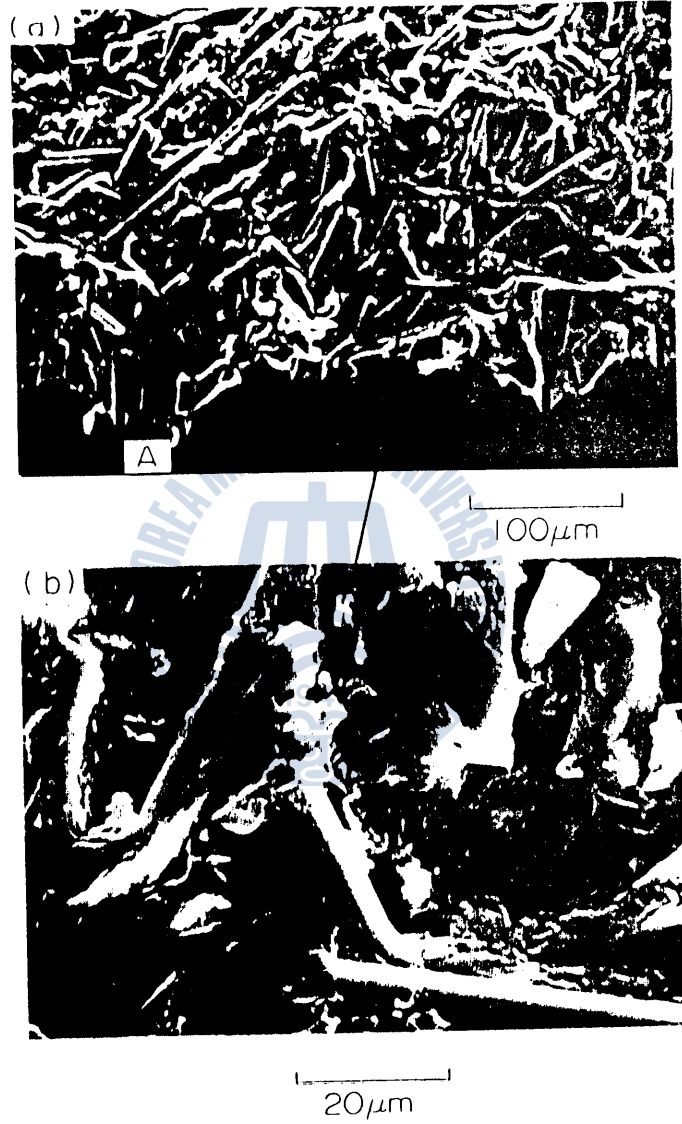


Fig. 25. Fracture appearance of a fatigue part for the fatigue specimen.

Yun Hae Kim



20 μm

Fig. 26. Fracture appearance of a part fractured in a brittle manner for the fatigue specimen.

specimens were annealed at 200°C for 2 h for the purpose of removing the influence of work-hardening.

A rotating-bending fatigue testing machine of uniform bending moment type was used. The stress used in this paper is the nominal stress at the minimum section. the crack initiation and propagation processes for a plain specimen were confirmed from the results of successive observations.

4. Experimental results and discussion

4.1. Tensile properties

Figure 6 shows the stress-strain curves in tensile tests of PEEK and CFRPEEK. The data obtained from the tensile tests are given in Table 2. By reinforcing due to the short carbon-fiber, the tensile strength and the elastic modulus of CFRPEEK become about 1.3 and 2.4 times those of PEEK, respectively, but the fracture elongation decreases.

4.2. Rotating-bending fatigue of a plain specimen

Figure 7 shows $S - N$ curves of plain specimens. It is well known that plastics generate heat in the fatigue process of high stress level and high

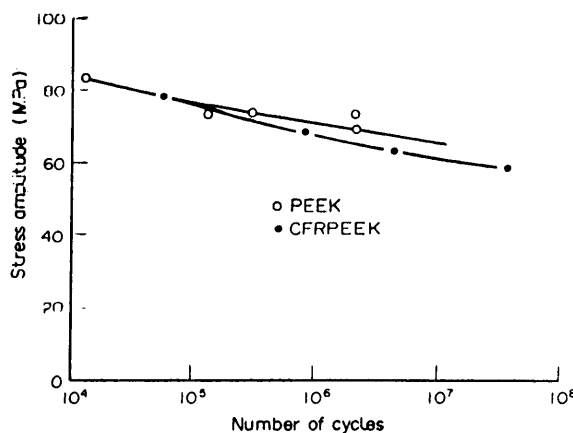


Fig. 7. $S - N$ curves of plain specimens.

Table 2. Mechanical properties

	σ_B (MPa)	E (MPa)	$\sigma_{0.2}$ (MPa)	ϕ (%)
PEEK	102	3500	77.4	27.5
CFRPEEK	132	8600	104.9	2.6

σ_B , ultimate tensile strength; E , Young's modulus; $\sigma_{0.2}$, 0.2% proof stress; ϕ , elongation.

strain rate [7, 14]. The measurement of temperature on the specimen was done during successive operations by using an infrared microscope at $\sigma_a = 83$ MPa, which is the highest stress amplitude in this study. Consequently, the temperature of the specimen did not increase. Therefore all specimens can be considered as having no elevation of the specimen temperature. As the distinct fatigue limit of these materials did not appear, the fatigue strength at 10^7 cycles was adopted.

Fatigue crack propagation curves of PEEK and CFRPEEK with a small hole of diameter 0.1 mm and depth of 0.1 mm are shown in Fig. 8. It is clear from this figure that the crack propagation rate of PEEK is very high; that is, the fatigue life of the plain specimen is controlled mainly by the behavior of a crack initiation.

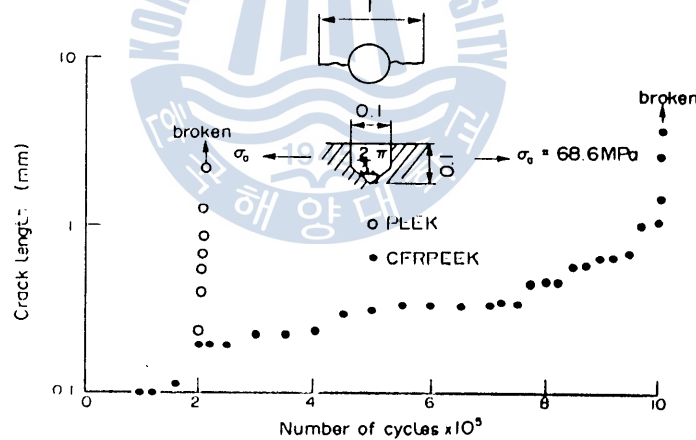


Fig. 8. Fatigue crack growth curves of PEEK and CFRPEEK with a small hole.

Figure 9 shows the change in the surface state of PEEK with a small hole of diameter 0.2 mm and depth of 0.1 mm, where the stress concentration factor is maximum at the specimen surface[15] due to the repetitions of the stress (about 10% higher than the fatigue strength at 10^7 cycles) whose

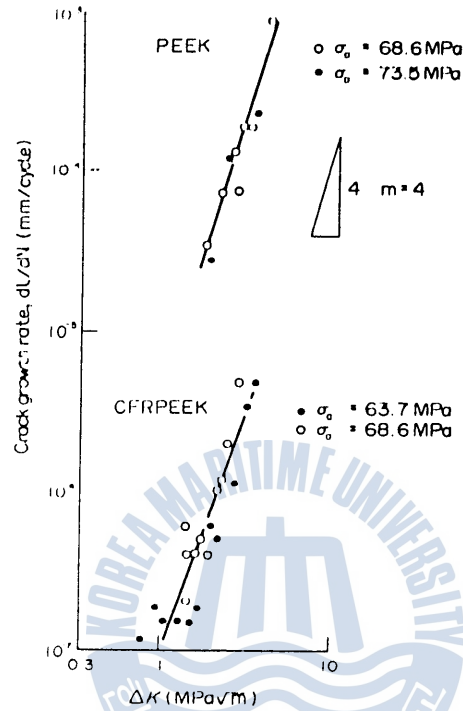


Fig. 11. Plot of dL/dN against ΔK .

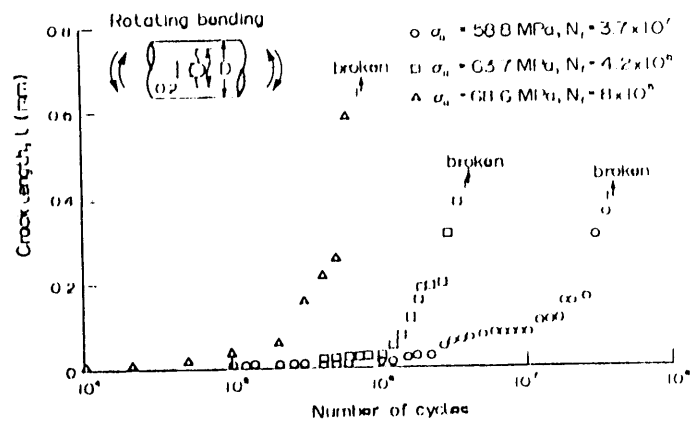


Fig. 12. The fatigue crack growth curves at $\sigma_0 = 58.8, 63.7$ and 68.8 MPa for CFRPEEK.

fatigue life is 3.7×10^4 cycles. This figure indicates that the fatigue crack initiation of PEEK is of the point-initiation type.

Figure 10 shows the crack initiation and the propagation processes of CFRPEEK due to the repetitions of the stress (about 3 % higher than the fatigue strength at 10^7 cycles) whose fatigue life is 4.2×10^7 cycles. The fatigue crack initiates from near the fiber end propagates to the circumferential direction after it grows to some extent along the fiber. The fibers do not break under this stress level. When a crack begins to propagate to the circumferential direction, the crack goes a step forward as surrounding fibers in most cases, and propagates with coalescence. This fact corresponds to the authors' previous result[16] of high cycle fatigue for short carbon-fiber reinforced polyamide 6.6, where the ductility of the matrix is similar to that of PEEK.

The relation between dl/dN and ΔK for PEEK and CFRPEEK is shown in Fig. 11. It is obvious that dl/dN is proportional to $(\Delta K)^m$ for PEEK, where ΔK is the range of the stress intensity factor and m is about 4. The crack growth rate for CFRPEEK is lower in double figures than that of PEEK. In the case of CFRPEEK there is scatter in the dl/dN versus ΔK plot during initial crack growth.

Fatigue crack propagation curve under $\sigma_a = 58.8, 63.7$ and 68.8 MPa for CFRPEEK are shown in Fig. 12. It was shown in Fig. 8 that the fatigue life of PEEK is determined mainly by the behavior of crack initiation. On the other hand, the fatigue life of CFRPEEK is controlled mainly by the behavior of crack propagation, as shown in Fig. 12. The major part of the fatigue life is consumed for the crack propagation to about 1 mm. Several cracks with similar size propagate until final fracture. This fact means that the life of crack propagation persists for the main part of the fatigue life and therefore the scatter of fatigue strength is comparatively small in CFRPEEK. As shown in Fig.8, the initiation of a fatigue crack for CFRPEEK is earlier than that for PEEK under the same stress amplitude because of the existence

of short carbon-fibers. This is due to the stress concentration at the fiber end.

It has been made clear from the above discussion that the short carbon-fibers give rise to a negative action against the fatigue crack initiation and a positive action against the fatigue crack propagations in this material.

Therefore, there is a need to evaluate the composite, because how the fatigue life will be increased or decreased by reinforcing the short carbon-fibers compared to that of the matrix resin is dependent on the loading and testing conditions.

4.3. *Rotating-bending fatigue of a notched specimen*

Figure 13 shows $S - N$ curves of the notched specimens for PEEK. The fatigue strengths are tabulated in Table 3. The surface states of the notch root after 10^7 cycles under stress repetitions at the fatigue limit are shown in Fig. 14. It was confirmed from the observation of the surface state of the notch root after 10^7 cycles that the crack is observed when the notch root radius is 0.1 and 0.05 mm.

Figure 15 shows the $S - N$ curves of notched specimens for CFRPEEK in comparison with the results of PEEK. A non-propagating crack with $\rho = 0.1$ mm and $\sigma_a = 34.3$ MPa at the notch root is shown in Fig. 16.

Hereafter, the limiting stress for macrocrack initiation, $\sigma_{\omega 1}$, is defined as the maximum nominal stress under which a macrocrack does not appear along the notch root, and the limiting stress for fracture in the range of macrocrack existing at 10^7 cycles, $\sigma_{\omega 2}$, is defined as the threshold nominal stress for crack propagation in the range where a non-propagating crack exists.

The relation between $\sigma_{\omega 1}$ or $\sigma_{\omega 2}$ and $1/\rho$ for PEEK is given in Fig. 17. If $\sigma_{\omega 2}$ for each notch depth $t = 0.2, 0.5, 2$ mm is constant, the notch root radius at the branch point, ρ_0 , can be assumed to be about 0.15 mm independent of t, d of the other geometry. This value is much lower than that of typical annealed carbon steel.

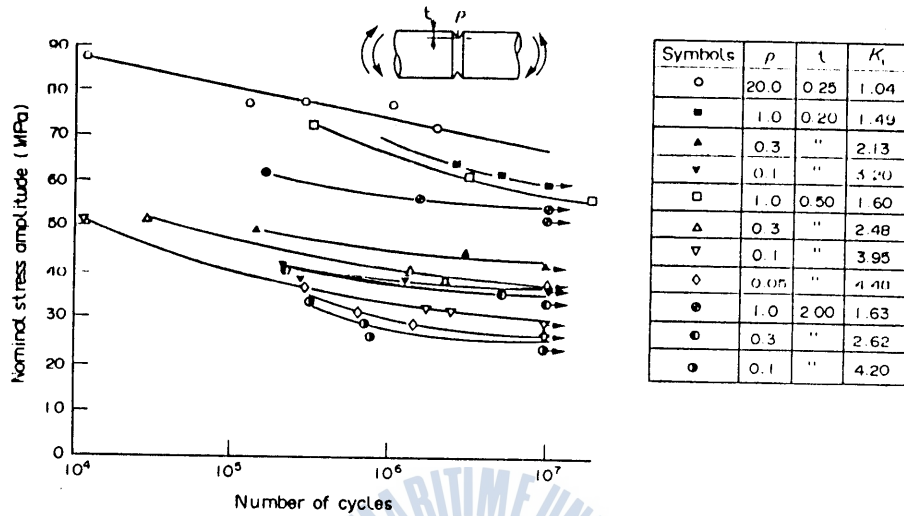


Fig. 13. S-N curves of the notched specimens for PEEK.

Table 3. Dimensions of notched specimens and fatigue limits of PEEK

t	ρ	Symbols	K_t	σ_{w1}	σ_{w2}	$K_t \sigma_{w1}$	$K_t \sigma_{w2}$
0.25	20	○	1.04	66.0	—	68.6	—
0.2	1.0	■	1.49	56.8	—	84.6	—
	0.3	▲	2.13	40.7	—	86.7	—
	0.1	▼	3.20	29.4	34.3	94.1	109.8
0.5	1.0	□	1.60	54.9	—	87.8	—
	0.3	△	2.48	35.3	—	87.5	—
	0.1	▽	3.95	24.5	26.9	96.8	106.3
	0.05	◇	4.48	22.1	26.5	99.0	118.8
2.0	1.0	⊕	1.63	52.4	—	85.4	—
	0.3	⊙	2.62	33.3	—	87.2	—
	0.1	⊗	4.20	22.1	24.5	92.8	102.9

t , notch depth (mm); ρ notch root radius (mm); K_t stress concentration factor; σ_{w1} , limiting stress for crack initiation (MPa); σ_{w2} , limiting stress for fracture in the range of non-propagating crack existing (MPa).

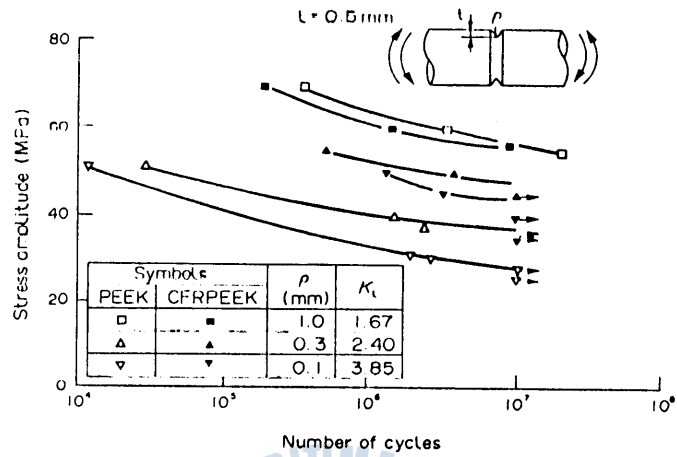


Fig. 15. S-N curves of the notched specimens for CFRPEEK in comparison with the results of PEEK.

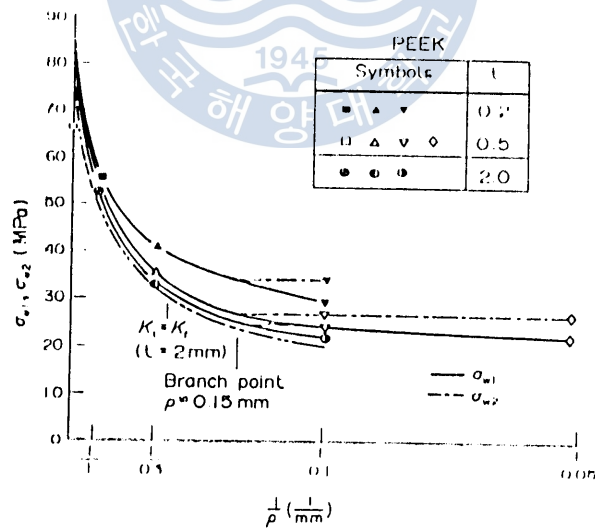


Fig. 17. Relation between σ_{w1} or σ_{w2} and $1/\rho$ for PEEK.

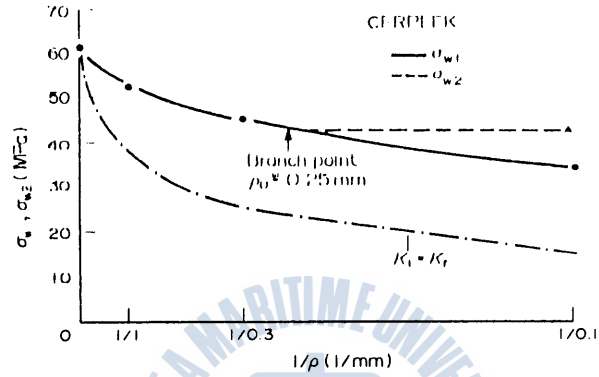


Fig. 19. Relation between σ_{w1} or σ_{w2} and $1/\rho$ for CFRPEEK.

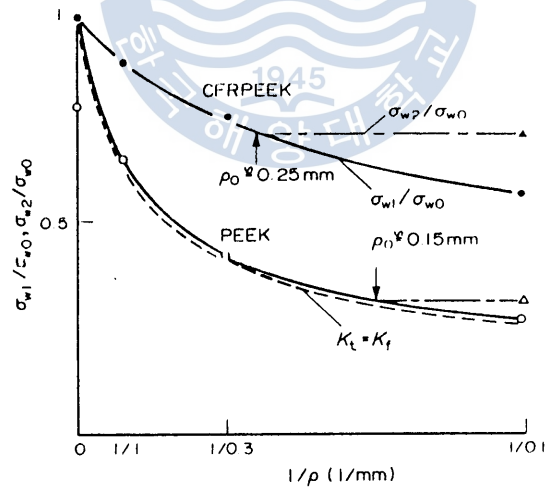


Fig. 20. Plot of σ_{w1}/σ_{w0} or σ_{w2}/σ_{w0} against $1/\rho$ for PEEK.

Yun Hae Kim

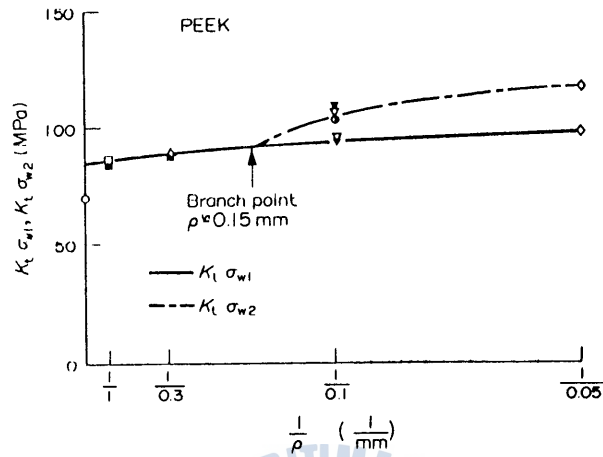


Fig. 21. Plot of $K_I \sigma_{w1}$ or $K_I \sigma_{w2}$ against $1/\rho$ for PEEK.

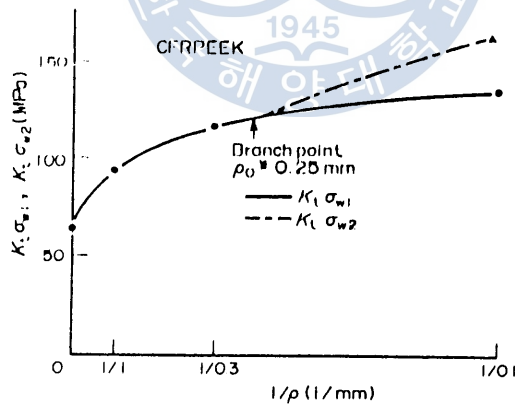


Fig. 22. Plot of $K_I \sigma_{w1}$ or $K_I \sigma_{w2}$ against $1/\rho$ for CFRPEEK.

The typical fracture appearance of the plain specimen with $\sigma_a = 73.5$ MPa and $N_f = 2 \times 10^4$ is shown in Fig. 18. It is clear from the fracture appearance near the fracture origin that defects strongly affect the fatigue life of the plain specimen. The fracture appearance where the fracture origin was a defect such as in Fig. 18a was also observed in the notched specimen where ρ was 1 mm except 0.3, 0.1 and 0.05 mm. On the basis of the estimated fatigue limit of the healthy plain specimen without defects (about 85 MPa), the value of σ_{ω_1} is considerably closer to the line of $K_t = K_f$, where K_t is the stress concentration factor and K_f is the notch factor. Considering the above discussion, it is found that the fatigue strength of PEEK is very sensitive to a notch.

Figure 19 shows the relation between σ_{ω_1} or σ_{ω_2} and $1/\rho$ for CFRPEEK. Assuming that σ_{ω_2} is constant independent of ρ in the case of $\rho < \rho_0$, ρ_0 is about 0.25 mm. The relations between $\sigma_{\omega_1}/\sigma_{\omega_0}$ or $\sigma_{\omega_2}/\sigma_{\omega_0}$ and $1/\rho$ for PEEK and CFRPEEK are given in Fig. 20. Here σ_{ω_0} means the fatigue limit of the plain specimen in the case of CFRPEEK, and is defined as the estimated fatigue limit of the plain specimen from the fatigue limit of the notched specimen in the case of PEEK.

The plots of $K_t\sigma_{\omega_1}$ or $K_t\sigma_{\omega_2}$ for PEEK and CFRPEEK against $1/\rho$ based on linear notch mechanics are shown in Figs 21 and 22. The effects of notch depths are clearly seen on the plot of σ_{ω_1} or σ_{ω_2} against $1/\rho$ for PEEK in Fig. 17. However, it is evident that in Fig. 21, rearranged by linear notch mechanics, all experimental data fall approximately on a unique characteristic curve independent of t , d and the other geometry. That is, the fatigue strength of an arbitrary notched specimen of this material will be estimated from the present results rearranged based on linear notch mechanics.

4.4. Fractography

Figures 23 and 24 show the fracture appearance near the center and the surface of the tensile specimens of CFRPEEK. On both fracture surfaces,

fracture of the fibers is observed, where the matrix resin does not adhere at the fiber end. The matrix resin in Fig. 24 is elongated and cut. On the contrary, it is fractured in a brittle manner in Fig. 23.

Figures 25 and 26 show the fracture appearance of a fatigue part and a part fractured in a brittle manner for the fatigue specimens. Part A in Fig. 25a is the part of crack initiation in Fig. 10. As shown in Fig. 25, the fibers in a fatigue part are covered with matrix resin and the fracture of fiber does not appear. However, the fibers in a part fractured in a brittle manner are broken. These results are nearly coincident with the previous report[16] for short carbon-fiber reinforced polyamide 6.6.

5. Conclusions

Rotating-bending fatigue tests for polyetheretherketone (PEEK) and short carbon-fiber reinforced polyetheretherketone (CFRPEEK) were carried out to investigate the fatigue characteristics of plain and notched specimens at room temperature. The fatigue mechanisms were clarified through successive surface observations using the plastic replica method. The results were discussed based on linear notch mechanics. The results obtained can be summarized as follows.

(1) In case of PEEK, the fatigue crack initiation is of the point-initiation type and the fatigue strength is very sensitive to a notch. In plain specimens, fracture usually starts from a defect near the surface and the crack propagation life is extremely short compared with the total life.

(2) In the case of CFRPEEK, the fatigue strength is much more insensitive to a notch than in PEEK. In general the fatigue crack initiates from near the fiber end and propagates to the circumferential direction after it grows to some extent along the fiber end. Short carbon-fibers give rise to a negative action against fatigue crack initiation and a positive action against fatigue crack propagation in CFRPEEK.

(3) The fatigue strength of an arbitrary notched specimen of these two materials will be estimated from each master curve rearranged based on "linear notch mechanics".

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