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Design of a Fuzzy State Observer

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퍼지 상태 관측기의 설계

이현식 · 주상래 · 하윤수 · 진강규

Abstract

This paper presents a scheme for designing a fuzzy state observer for nonlinear systems. For this scheme, a Takagi-Sugeno type fuzzy model whose consequent part is of the state space form is obtained. It describes the locally linear input/output relationship of a system. The parameters of the fuzzy model are adjusted using a genetic algorithm. And then, the fuzzy full-order state observer is designed based on the fuzzy model. A simulation is carried out to demonstrate the effectiveness of the proposed scheme.

Keywords : fuzzy model, fuzzy state observer, genetic algorithms, nonlinear systems

1. INTRODUCTION

In many practical situations, state feedback control is one of useful schemes because there are the established design methodologies and the proved stabilities for the scheme. This scheme requires the feedback of all state variables for controller synthesis, but in practice, it may be impossible to measure all the state variables and, even if possible, some difficulties arise because of the limit of sensor numbers, the constraint on setting sensors and the cost. To solve this problem, one of the methods is to use a state observer based on the model of a system. However if nonlinearities and complexities of the system increase, obtaining the precise model may be difficult and, even if obtained, the parameters of the system may vary during operation. This causes the estimation errors between the system and the model, which can make the total system be unstable.

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In this paper, a methodology for designing a model-based fuzzy state observer is proposed to overcome such a problem. At first, the fuzzy model whose consequent part is described by a linear continuous-time model (denoted as subsystem) which represents locally input-output relationship of the system is obtained. Next, an asymptotically stable full-order state observer for each subsystem is designed and the fuzzy implication combines the observers for subsystems. Simulation studies on a two-tank system show the effectiveness of the proposed scheme.

2. FUZZY MODELLING

In this section, an overview of the fuzzy modelling technique proposed by the authors[1] is given to provide background knowledge of designing a fuzzy state observer.

2.1 Fuzzy Model

The proposed fuzzy model is a type of Takagi-Sugeno's[2-4] which is composed of a set of "if-then" rules. Each consequent part characterizes local input-output relationship of a system and is of the linear state space form as:

$$\boldsymbol{R}^i : \textit{\textbf{If}}\ v_1\ is\ F_1^i \text{ and } \cdots \text{ and } v_n\ is\ F_n^i,\ \textit{\textbf{then}}\ \dot{\boldsymbol{x}}^i = A^i \boldsymbol{x} + B^i u \ (1 \le i \le \ell) \quad \cdots\cdots (1)$$

$$y = C\boldsymbol{x} \quad \cdots\cdots (2)$$

where $\boldsymbol{R}^i$ is the ith fuzzy rule, $\boldsymbol{v}=[v_1, \cdots, v_n]^T \in R^n$ and u are the inputs of the fuzzy system, $\boldsymbol{x}^i=[x_1^i, \cdots, x_n^i]^T \in R^n$ is the state vector of each subsystem, $\boldsymbol{x}=[x_1, \cdots, x_n]^T \in R^n$ is the state vector of the fuzzy model and y is the output of the fuzzy model. A^i, B^i and C are matrices with proper dimension. In general, $\boldsymbol{v}$ can be of various forms. A special case is $\boldsymbol{v}= \boldsymbol{x}_p$, where $\boldsymbol{x}_p=[x_{p1}, \cdots, x_{pn}]^T \in R^n$ is the state vector of the system. It is assumed that (A^i, C) $(1 \le i \le \ell)$ is observable.

Given an input pair $(\boldsymbol{v}, \boldsymbol{u})$ of the fuzzy system, the truth value of the ith premise is calculated by

$$\rho^i = \prod_{j=1}^{n} F_j^i(x_{pj}) \quad \cdots\cdots (3)$$

where $F_j^i(x_{pj})$ is the membership grade of x_{pj} in F_j^i. The inferred output is then given by

$$\dot{\boldsymbol{x}} = \frac{\sum_{i=1}^{\ell} \rho^i (A^i \boldsymbol{x} + B^i u)}{\sum_{i=1}^{\ell} \rho^i} = \sum_{i=1}^{\ell} \xi^i (A^i \boldsymbol{x} + B^i u) \quad \cdots\cdots (4)$$

where $\xi^i = \sum_{j=1}^{\ell} \rho^j$ and it is further assumed that $\sum_{j=1}^{\ell} \rho^j > 0$ for all time.

2.2 Parameter Estimation Using a Genetic Algorithm

Once the structure of the fuzzy model has been defined, the next thing is to obtain the fuzzy partition of input space and the parameters of the state equation in the consequent part so that the dynamic characteristics of the fuzzy model is similar to that of the system. Since Gaussian membership functions(MFs) are considered for the "interior" fuzzy sets and sigmoid MFs for the "exterior" fuzzy sets as

$$F_j^i(x) = e^{\frac{-(x-(v_j^i)^2)}{2(\sigma_j^i)^2}} \quad \cdots\cdots (5a)$$

$$F_j^i(x) = \frac{1}{1+e^{-\mu_j^i(x-o_j^i)}}, \quad \cdots\cdots (5b)$$

ν_j^i, σ_j^i, μ_j^i, o_j^i $(1 \le i \le \ell,\ 1 \le j \le n)$ are the parameters of the premise part. The parameters of the consequent part are A^i and B^i $(1 \le i \le \ell)$ whose total elements are $n(n+1)\ell$.

The parameters of both parts are simultaneously adjusted to satisfy

$$\lim_{t \to \infty} [\boldsymbol{x}_p(t) - \boldsymbol{x}(t)] = \boldsymbol{0}. \quad \cdots\cdots (6)$$

As for assessing the performance of the model, the following objective function is adopted due to suitability to on-line estimation.

$$J = \int_{(k-W+1)T}^{kT} \| \boldsymbol{x}_p(t) - \boldsymbol{x}(t) \| \, dt \quad \cdots\cdots (7)$$

where W is the size of data window which is chosen depending on the desired parameter accuracy and the computational burden and T is the sampling time.

The adjustment of the parameters such that J is minimized introduces a multivariable optimization problem and its search space is likely to be multimodal. A genetic algorithm(GA)[5][6] is, therefore, used to search for the solution. The GA evaluates individuals in terms of their fitness and the fitness is computed from the objective function as

$$f = -J - \alpha \qquad (8)$$

where α is chosen to keep $f \geq 0$ for every generation and adjusted for keeping selection pressure[7]. Fig. 1 shows a parameter estimation approach of the fuzzy model based on a parallel configuration.

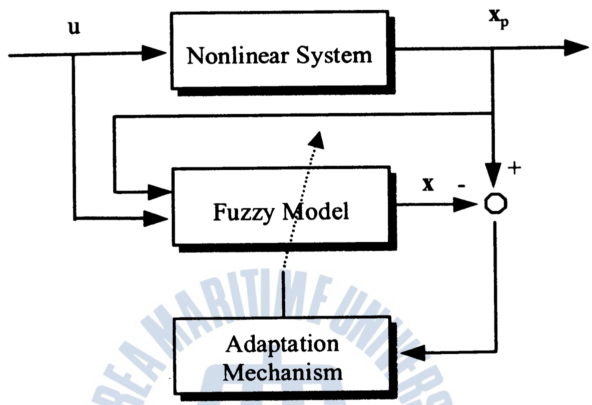


Fig. 1. Schematic diagram for fuzzy modelling.

3. DESIGN OF A MODEL-BASED FUZZY STATE OBSERVER

A fuzzy full-order state observer whose consequence is of Luenberger-type is designed in this section. Each observer in the consequence is designed based on the linear subsystem of its counterpart in the fuzzy model. The fuzzy state observer is of the following form:

$$\hat{R}^i : \textbf{\textit{If}}\ v_1\ \textit{is}\ F_1^i \text{ and } \cdots \text{ and } v_n\ \textit{is}\ F_n^i,\ \textbf{\textit{then}}\ \dot{\hat{x}}^i = A^i\hat{x} + B^iu + L^i(y - C\hat{x})\ (1 \leq i \leq \ell) \qquad (9)$$

where $\hat{x}$ is the estimate of x, $\dot{\hat{x}}^i$ is the estimate for the ith rule and L^i is the observer gain matrix for the ith rule. Therefore, $\dot{\hat{x}}$ is inferred as:

$$\dot{\hat{x}} = \frac{\sum_{i=1}^{\ell} \rho^i [A^i\hat{x} + B^iu + L^i(y - C\hat{x})]}{\sum_{i=1}^{\ell} \rho^i} = \sum_{i=1}^{\ell} \xi^i [A^i\hat{x} + B^iu + L^i(y - C\hat{x})] \qquad (10)$$

where the same truth value ρ^i as (4) is used again.

In (10), if the matrix L^i is a stable one, $\hat{x}$ will approach x even for different initial conditions. To show this, defining the error vector as $\boldsymbol{e}=\boldsymbol{x}-\hat{\boldsymbol{x}}$ yields

$$\begin{aligned}\dot{\boldsymbol{e}} &= \dot{\boldsymbol{x}}-\dot{\hat{\boldsymbol{x}}} \\ &= \sum_{i=1}^{\ell}\xi^i(A^i\boldsymbol{x}+B^i u)-\sum_{i=1}^{\ell}\xi^i[A^i\hat{\boldsymbol{x}}+B^i u+L^i(y-C\hat{\boldsymbol{x}})]=\sum_{i=1}^{\ell}\xi^i\overline{A}^i\boldsymbol{e}\end{aligned} \quad \cdots\cdots (11)$$

where $\overline{A}^i=A^i-L^iC$

Theorem : The equilibrium of the error system is asymptotically stable if and only if given any positive definite, real, symmetric matrix Q, there exists a common symmetric positive definite matrix P such that

$$(\overline{A}^i)^T P+P\overline{A}^i=-Q \ (1\le i\le \ell)$$

For proof, refer to [8]. ■

Theorem explains that if L^i is chosen so that the above condition is satisfied, then $\hat{\boldsymbol{x}}(t)\rightarrow \boldsymbol{x}(t)$ as $t\rightarrow\infty$.

4. SIMULATION

To demonstrate the fuzzy state observer developed in the previous section, a set of simulation was carried out on a two-tank system shown in Fig. 2.

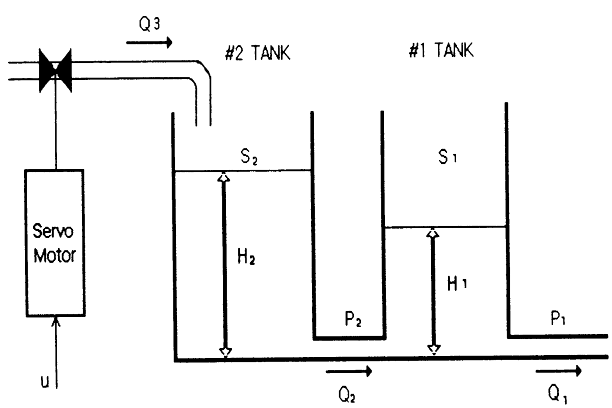


Fig. 2. A two-tank system.

$$A^1 = \begin{pmatrix} -0.2842 & 0.1421 & 0.0 \\ 0.2526 & -0.2526 & 88.4194 \\ 0.0 & 0.0 & -1.0 \end{pmatrix}, B^1 = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.001 \end{pmatrix},$$

$$A^2 = \begin{pmatrix} -0.2009 & 0.1005 & 0.0 \\ 0.1786 & -0.1786 & 88.4194 \\ 0.0 & 0.0 & -1.0 \end{pmatrix}, B^2 = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.001 \end{pmatrix},$$

$$A^3 = \begin{pmatrix} -0.164 & 0.082 & 0.0 \\ 0.1458 & -0.1458 & 88.4194 \\ 0.0 & 0.0 & -1.0 \end{pmatrix}, B^3 = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.001 \end{pmatrix}.$$

Fig. 3 shows the MFs of the fuzzy sets tuned by a genetic algorithm.

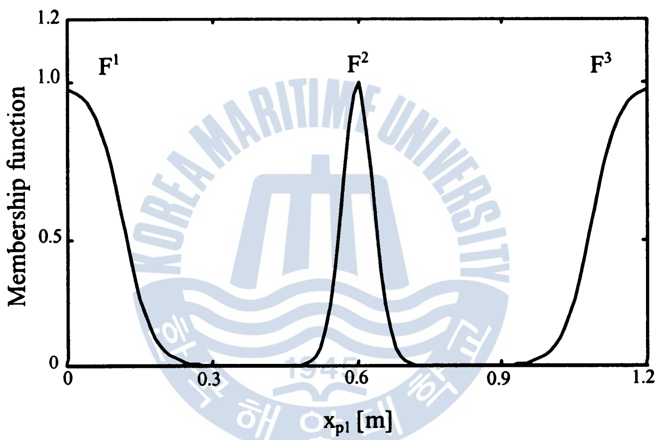


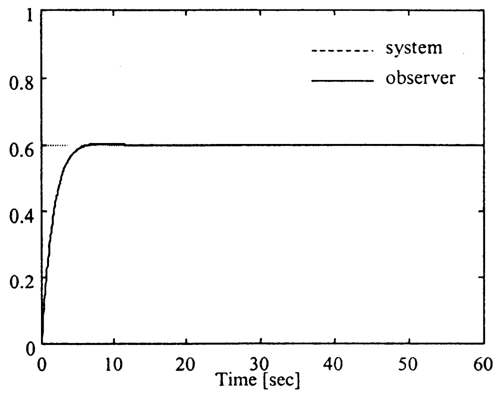
Fig. 3. MFs of the tuned fuzzy sets.

Then, the fuzzy state observer is given by

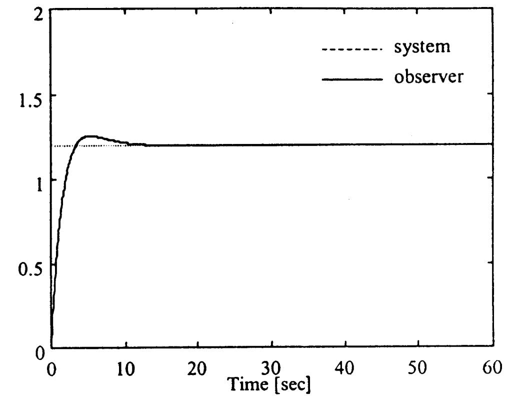
$$\hat{R}^i : \textit{If } x_{p1} \textit{ is } F^i, \textit{ then } \Delta\hat{\dot{x}}^i = A^i\Delta\hat{x} + B^i\Delta u^i + L^i(y_p - C\hat{x}) \ (1 \le i \le 3) \quad \cdots\cdots (19)$$

where $L^1 = [0.4632\ 0.9649\ 0]^T$, $L^2 = [0.6205\ 1.6049\ 0]^T$ and $L^3 = [0.6901\ 2.1626\ 0]^T$ are the results when $\overline{A}^i (1 \le i \le 3)$ have the poles at -1 and -0.5±j0.2.

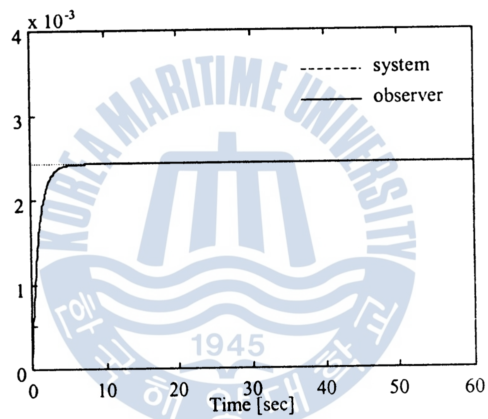
Fig. 4 shows the responses of the system and the fuzzy state observer. When the system was in the steady-state at $(\boldsymbol{x}_0^2, u_0^2)$, the observer was put into operation with zero initial state. As shown in the figure, the fuzzy state observer can successfully estimate the state of the system.



(a) x_{p1} and $\hat{x}_1$



(b) x_{p2} and $\hat{x}_2$



(c) x_{p3} and $\hat{x}_3$

Fig. 4. Responses of the system and the observer.

5. CONCLUSION

In this paper, a scheme for designing a model-based fuzzy state observer for nonlinear systems was presented. For this, the fuzzy model was constructed and its consequent parts were of the linear state space form, which describe the locally linear input-output relationship of the system. The parameters of the fuzzy model were adjusted using a genetic algorithm. Then, the fuzzy full-order state observer was designed, which was based on the estimated fuzzy model. The simulation results demonstrated the effectiveness of the proposed method.

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