1. Basic Construction for Finite Dimensional C^*-algebras

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Let $N \subset M$ be finite dimensional C^*-algebras with the same identity I. Then they are isomorphic to the multimatrix algebras of the form

$$N=\sum_{i=1}^{l}\oplus M_{k_i}(\mathbb{C}) \subset \sum_{j=1}^{m}\oplus M_{n_j}(\mathbb{C})=M,$$

where $M_n(\mathbb{C})$ denotes the algebra of $n \times n$ matrices over the complex field $\mathbb{C}$. Moreover, there is very convenient way of describing such pairs by so-called Bratteli diagrams. The Bratteli diagram of $N \subset M$ is a bipartite weighted multi-graph, expressed by the inclusion matrix $\Lambda(N, M)$ of $N \subset M$.

Our aim in this note is to apply the basic construction to a pair $N \subset M$. This idea originated from Jones's breakthrough work on subfactors of type II_1 factors in 1983. To this end we employ the standard representation (admitting a cyclic and separating vector), as is done in the type II_1 case.

Let τ be a normalizied faithful trace on M. If we denote by $L^2(M,\tau)$ the Hilbert space obtained by GNS construction with respect to the trace τ, then M acts standardly on $L^2(M,\tau)$. Since M is a finite dimensional C^*-algebra, we have $L^2(M,\tau)\cong M$ as vector spaces, but we prefer to use the symbol $L^2(M,\tau)$ to clarify the process of Jones basic construction.

Working with the standard representaion of M on $L^2(M,\tau)$, we see that there is a unique trace preserving conditional expectation $E: M \rightarrow N$ such that

$$\tau(xy)=\tau(E(x)y)$$

for all $x \in M$ and $y \in N$. Let e be the orthogonal projection in $L^2(M,\tau)$ given by $e(x)=E(x)$, for $x \in M$. Then $e \notin M$ and $JeJ=e$, where J denotes the antiunitary involution of $L^2(M,\tau)$.

The basic construction of $N \subset M$, denoted by $\langle M, e\rangle$, is the algebra generated by M and e on the Hilbert space $L^2(M,\tau)$. It turns out that the basic construction $\langle M, e\rangle=$

$\overline{span}$ { xey | $x, y \in M$ } satisfies the property $JNJ = \langle M, e \rangle$. This implies that whenever $f_1, \cdots, f_s$ are minimal central projections in N of sum I, then $Jf_1J, \cdots, Jf_sJ$ are minimal central projections in $\langle M, e \rangle$ with sum I. This means that $M_1 = \langle M, e \rangle$ is again a finite dimensional C^*-algebra and the inclusion matrix of $M \subset M_1$ is simply given by $\Lambda(M, M_1) = \Lambda(N, M)^T$, where $\Lambda(N, M)^T$ denotes the transpose matrix of $\Lambda(N, M)$. Hence we see that the Bratteli diagram of $M \subset M_1$ is simply the mirror image of that of $N \subset M$.

We are thus able to apply the same process to the pair $M \subset M_1$ to obtain their basic construction. Inductively, let $N = M_{-1}$, $M = M_0$, and $E_{n-1} : M_n \rightarrow M_{n-1}$ be a trace preserving conditional expectation with corresponding projection e_{n-1}. Then $M_{n+1} = \langle M_n, e_n \rangle$ is the basic construction of the inclusion $M_{n-1} \subset M_n$, for all $n \geq 0$, and hence we obtain a tower of finite dimensional C^*-algebras

$$N \subset M \subset M_1 \subset M_2 \subset M_3 \cdots.$$

This tower may have an easy graphical description in terms of Bratteli diagrams, when we begin with a trace τ of M satisfying certain additional property. Then it turns out that the algebra M_1 is independent on the initial choice of a trace τ. That is, when τ is a Markov trace for the inclusion $N \subset M$, the algebra M_1 has a trace which extends the trace of M in a proper way. Then the traces and conditional expectations for the algebras in the tower $N \subset M \subset M_1$ work properly, and the inclusion matrix of $M_1 \subset M_2$ is the transpose of $\Lambda(M, M_1) = \Lambda(N, M)^T$, which equals $\Lambda(N, M)$.

Consequently, iterating the basic construction under the Markov trace property, the algebras in the tower can be described easily by the Bratteli diagram in terms of $\Lambda(N, M)$ and $\Lambda(N, M)^T$, alternatively.

2. 병렬시스템 가동율의 어떤 베이지안 점 추청치들에 대하여

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N개의 부속품들에 대한 파손과 보수시간에 대한 분포가 지수분포에 따를 때, 파손에 대한 평균 (MTBF)이 각각 독립이고 이들 파손에 대한 평균 보수시간(MTBR)이 서로 독립으로 관찰되어 질때, 이들 요소들에 대한 것을 비교분석하여 Monte-earlo Simulation 하였다. 즉 병렬시스