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Modification of hull form boundary
by the manipulation of area and centroid

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A curve based hull form variation with geometric constraints of area and centroid

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Abstract

To obtain the new shape from modifying an existing geometric shape is a common process of design encountered in automobile, aircraft, and shipbuilding industries. In particular, designing a new ship from a well-made existing ship, called hull form variation or variation, for short, has been a crucial process used in every design department of shipyards for prompt and efficient initial hull form creation. This process, however, is not only complicated but also unintuitive and thus requires an expert's skill and experience. An approach to performing the variation with the given geometric constraints of area and centroid is proposed. To modify an existing hull shape, a basic boundary curve of the shape is selected as a design variable. A parametric piecewise polynomial curve that satisfies new geometric requirements is generated and superimposed on the top of the selected boundary curve to yield the desired curve. The main process of the variation is performed in a linearized fashion that preserves the original shape as much as possible; thus, a new form is efficiently and promptly obtained. The proposed concept can be readily extended to similar modification processes of an existing geometric shape by adopting different geometric requirements.

Keywords: ship hull design, hull form variation, NURBS, curve superposition, curve minimization

Preface

This master thesis in Computer Aided Design is prepared in the 4th semester at Division of Naval Architecture and Ocean Systems Engineering, College of Ocean Science and Technology, Korea Maritime and Ocean University (KMOU), Autumn 2016.

This thesis is written as an introduction to a new numerical method in curve based ship hull form variation. These are a few traditional methods had published for a long time and are still useful today in ship’s software. Most of these techniques are based on a modified hull basic curves such as water lines, section lines. This thesis rather focuses on presenting the theory in a simple way with illustrative examples and basic formulas and evaluating the tolerance with a goal.

Readers with background from ship design and who wants to develop hull form variation may find this thesis.

This master thesis looks further into the basic geometrical quantity for determining the shape of goal’s curve. The NURBS, Particle Swarm Optimization, and Curve Energy Minimization have also been explored.

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Chapter 1. Introduction

The final key of this thesis is to present the new numerical method in curve based hull form design concept based on the basis curves such as waterlines, section curve, etc. This introduction presents some background information as a motivation for the work.

1.1 Hull form design concept

In general, in a design office, the starting point of a new project are the owner's demands such as dead weight, range, speed, etc. From them, the main dimensions and the geometrical form parameters can be found. Stability and resistance considerations give a preliminary definition of the mid-ship section, the flotation and the curve of sectional areas. Then the lines can be derived mainly from a series of parent designs, a single parent design or the geometrical hull form parameters.

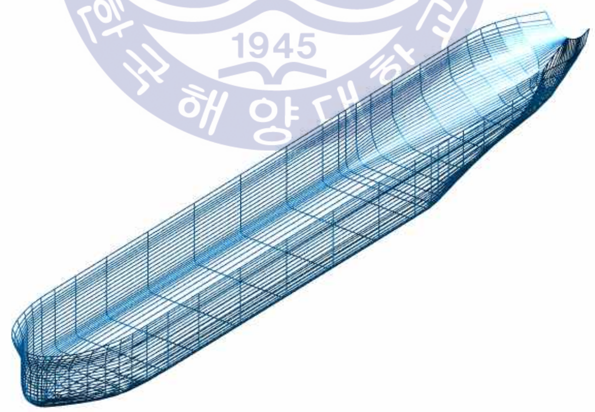

Fig. 1 Ship hull form design

The standard series approach consists in interpolating the desired new hull form within a systematic hull form series. The range of change of parameters is limited, so the forms that can be deduced from the series is restricted.

In the single parent design approach, the lines are obtained distorting existing forms. The good points of this method are its simplicity and its possibility to take advantage of an existing good design. However, changes must be moderate to avoid strong ones in the hydrodynamic behavior of the new hull. The procedures of distorting lines can be divided into three groups:

- Simple affine distortion, where the three principal dimensions are each multiplied by a standard ratio.

- Modified affine distortion where the simple affine distortion is applied in a modified, partial or compound form.

- Non-affine distortion, where the standard ratio can vary continuously in one or several directions.

In the form parameter approach, the lines are created according to specified data of the parameters that define the basic curves of the hull form. It has been the traditional procedure of designing the ship lines. In most of cases, because of the difficulty of creating new lines, the first sketch is, in some way although not systematically, derived from the shape of a built ship. Once the initial lines are obtained, an interactive process is started, drawing waterlines and sections, testing every time, whether they fit the contours, the mid-ship section and the curve of sectional areas. Other lines like longitudinal and diagonals are also checked. The last revision of the lines is made after the tests are carried out in a towing tank.

Nowadays, in ship hull form design, creating a new hull form is a time-consuming and challenging process. It also requires comprehensive knowledge of geometry as well as in the aesthetic point of view. Due to the humongous size and vast investment, the ship design process naturally becomes conservative, and every designer strives to create a new hull form that avoids abrupt and sometimes innovative design changes. To minimize the risk associated with a new creation, the ship designer has preferred a modification method, called the variation, to a new generation. The variation method yields a new hull form by modifying the existing one that was previously built and successfully verified on its shape and performance.

1.2 Motivation

'Hull form variation' term refers to the process of modifying a form from the mothership to become a new hull with features similar to her mother.

It is common to modify the existing shape of a curve following new design requirements. In shipbuilding design departments, in particular, this is the typical way to perform hull variation that generates a new hull form from the existing shape of a well-made or reference ship. Whenever the ship designer is given a task to design a new ship, he or she starts to search for a similar ship that was previously built. Next step is to confirm whether the existing ship found can be extended to satisfy the new design requirements. This variation approach has always been a handy but safe way as the performance, and other characteristics can be preserved or kept similarly up to a certain degree.

Lackenby (1950) established a solid way to perform the variation using shift functions. He extended the existing $1-C_P$ method to more flexibly. In the early stage of ship design, the prismatic coefficient C_P curve is known to be one of the most important elements that determine an initial hull form. The principle of the Lackenby's is the movement of both the longitudinal center of buoyancy LCB and the C_P. Several advantages associated with this method can be summarized as a rapid modification, dependency on empirical data including the knowledge of successful C_P curve, and direct linkage among some analysis modules. However, a simpler algorithm would be preferred for an immediate modification or variation.

Successive work to enhance the Lackenby's method has been continuously published. A series of research on hull form variation was published Alef and Collatz

(1976), Versluis (1977), Rabien (1979). Some used more geometric constraints such as sectional area curves or design waterlines Nowacki et al. (1977), Munchmeyer et al. (1979). Bole and Lee (2006) attempted to use the geometric properties as additional parameters in addition to usual numerical form parameters. To increase the performance of hull form variation, researchers began to adopt new technologies such as optimization and computational fluid dynamics (CFD) Gregory et al. (2010). A systematic approach to building a commercial package was introduced by Abt and Harries (2007). They claimed that they extended the classic Lackenby's method for more flexibility and higher quality.

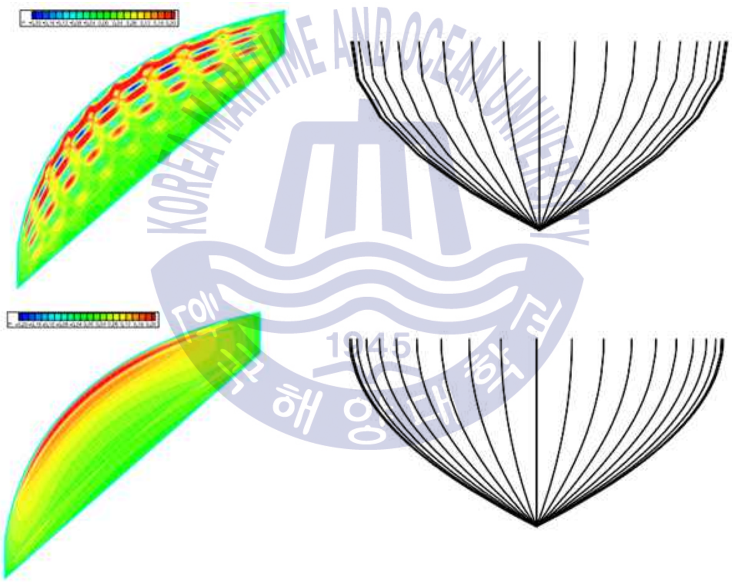

Fig. 2 Hull form variation with fairing curve's curvature.

Recently a new paradigm for ship design has been proposed. Unlike the traditional methods, an approach tries to lessen the stringent requirements imposed in determining some characteristic shapes of a ship. Some ship design engineers thought of generating a new hull form by considering the natural shape of major sections comprising a ship hull. Rather than clinging to an initial shape resulting from the preliminary design, which might have been too strict to generate a reasonable shape, they wanted to focus on other design factors that would enhance the entire performance of ship design. While this approach is against the traditional concept of hull form design or variation, it is worthy of consideration when it involves to the design of non-traditional vessels such as yachts and warships.

1.3 Clarification of study

From the ship's lines plan of the mother hull such as section curves, waterlines, etc., called original curve (*see Fig. 3*), the transformation of this curves to change the shape of the hull would be performed.

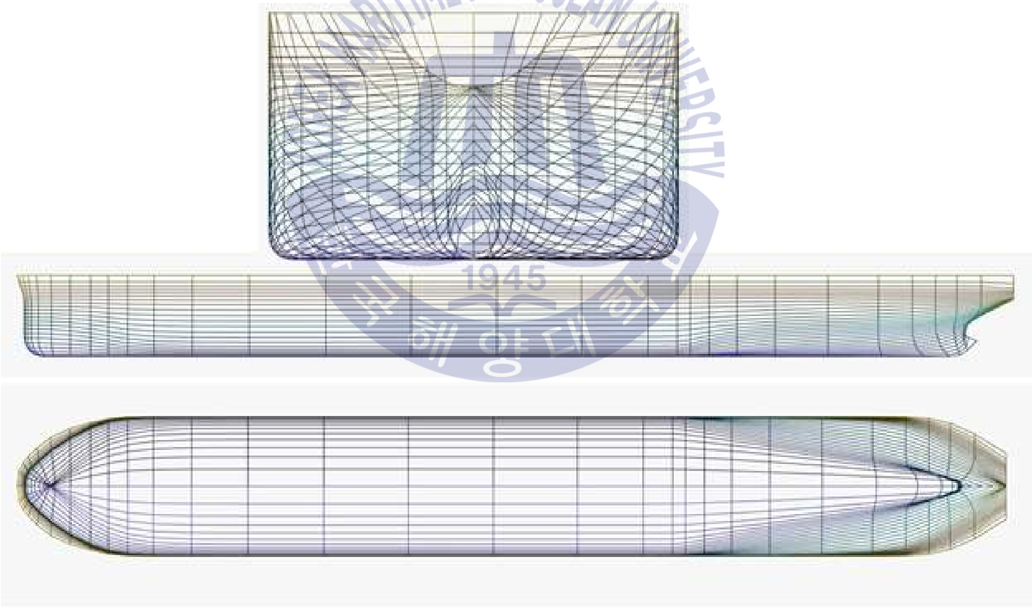

Fig. 3 Ship lines plan

Along with the actual requirements as the change in volume, the center of gravity position, the form factor, etc., designers will transform these requests into geometric constraints include area and centroid. The algorithm that is presented in this paper will use these constraints (area and centroid) to transform the original curves such as waterlines or section curves to obtain the new curves. These new curves satisfy the requirements including an area and a centroid that are set by the designer. Simultaneously, the advantages of the original curve will be preserved on the new curve. This means that the new hull form will retain the good characteristics of the mother hull form.

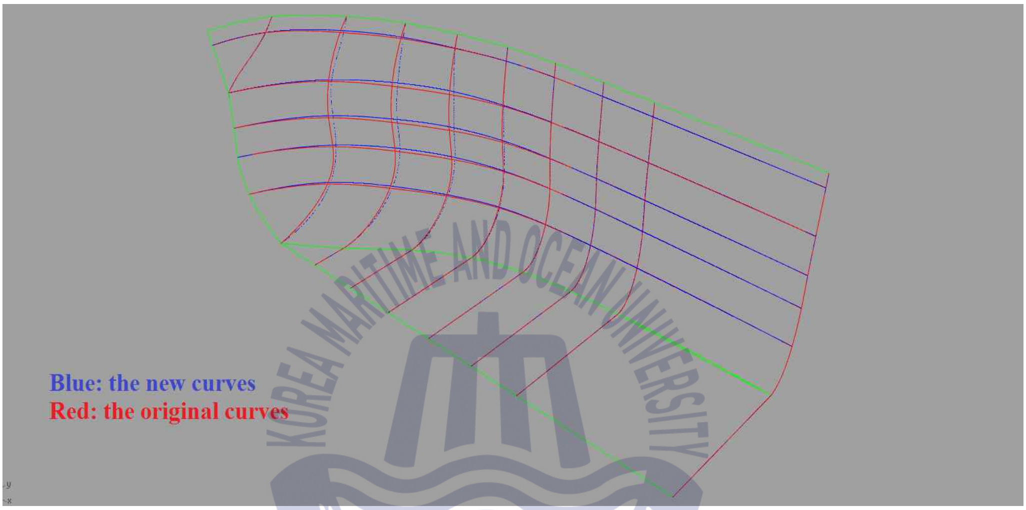

Fig. 4 The hull curve variation

1.4 Outline

The work of this thesis is to introduce a new numerical approach in the field of ship hull form design by transforming a hull form from existing one. Based on the basic curve such as waterline, sectional area curve, the section curve, etc., hull form variation approach is expected to preserve the good characteristics of the mother hull. The geometric constraints are also mentioned as a requirement from designers.

In **Chapter 2**, the traditional methods of modified hull form are briefly presented to help the readers visualize the designing of a new hull from an existing hull, also known as hull form variation. It should be emphasized that traditional methods were announced for a while but still useful today.

Chapter 3 presents the theoretical basis that is mentioned in this curve based hull form variation including the creating a curve by using NURBS technique, the optimized search method – Particle Swarm Optimization, and the minimizing energy curve.

The purpose of **Chapter 4** is to propose hull form variation approach based on the basic curves with geometric constraints of area and centroid. By applying the theoretical basis has been presented on the basic curves to figure out the new curves, which meet the design requirements that entered by the designers. By examining the distribution of curvature, the comparison of the new and original curve is also performed.

Finally, **Chapter 5** presents concluding remarks and discusses further work.

Chapter 2. Hull form’s geometry variation traditional method

Chapter 2 briefly review the hull modified method based on the geometry characteristics modification, known as the traditional method. These are mid-ship coefficient CM-vary, Lackenby’s method, and other methods. It should be emphasized that these traditional methods were announced for a while but still useful today.

There have been several different methods used for hull form variation. The parameters involved in the variation typically include geometric dimensions such as length, beam, draft, mid-ship area AM, and others. In addition, the variation considers the change of major shape coefficients that determine a hull form such as the prismatic coefficient CP, the block coefficient CB, the longitudinal center of buoyancy, and so forth. The specific notations used in the hull form definition are briefly depicted in Fig. 5.

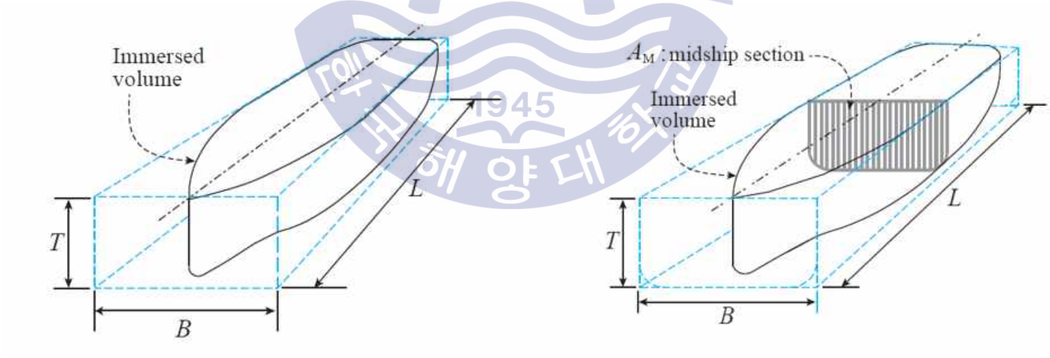

Fig. 5 Major coefficients that indicate how much the hull form (defined by the volume ∇) occupies within the bounding box (defined by the combination of length L, breadth B, and draft T). (Left) block coefficient $C_B = \nabla / (L.B.T)$), (Right) mid-ship coefficient $C_M = A_M / (B.T)$

2.1 Mid-ship coefficient C_M vary

A method is given by Hollister (1996) allows the mid-ship shape of a vessel to change independently of two factors that include the beam, and depth of each station. It uses a value called C_M-fact which varies the shape of each section diagonally in the direction of the bilge corner. It is defined as the intersection of the maximum beam and depth of the station. A C_M-fact value of 1.0 means that each section shape is rectangular (see Fig. 6).

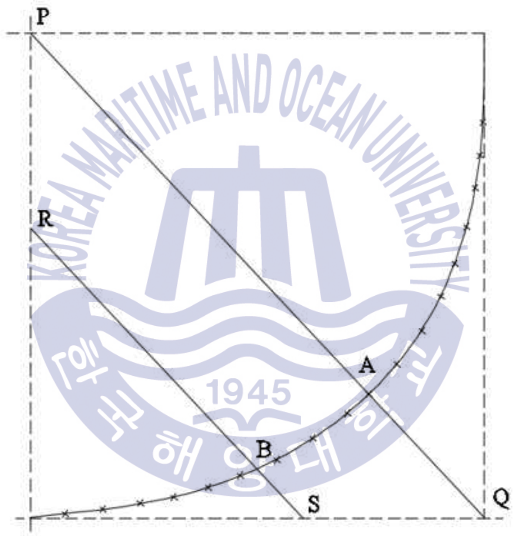

Fig. 6 Station showing C_M-fact diagonal

C_M-fact is related to the mid-ship coefficient (C_M) of a vessel. However, the C_M-fact is based on the overall maximum beam and depth of the vessel, rather than the waterline beam and the draft. Since all of the hull variations affect the whole hull shape, a C_M-type factor was established that also affects the whole section, rather than just the area below the waterline. This characteristic makes the factor independent of the draft and displacement.

The C_M-fact value for a particular station is the rate distance of the station from point P(0,0) to point Q(1,0). The C_M-fact for the hull is the largest C_M-fact of all of the stations and is the value used by the C_M-Vary routine to vary the shape of the hull.

C_M-Vary hull variation steps:

- Find the section with the largest C_M-fact value. This is the one C_M-fact value that is used for the entire boat.

- Given a new, target C_M-fact, determine the percent increase or decrease of the defining C_M-fact section along the diagonal (P-Q) variation line.

- Decrease this C_M-fact change percentage parabolically to zero at each end of the boat. This means that this hull variation tapers off to zero change at the ends of the boat.

- Apply the appropriately decreased C_M-fact changes the rate to each station. For each point (B), create a line (R-S) that parallel to the main station diagonal (P-Q). Then determining the intersections with the defining beam-depth station box (points R and S). Stretch or shrink each offset point along this line by the appropriate CM-fact modified factor. Point B move towards, or away from point S at the same rate as point A move towards or away from point Q. When the global value of C_M-fact approaches 1.0, the shape of each section approaches the rectangular shape of the defining beam-depth box.

2.2 Lackenby's method

This is an approach for hull variation developed by H. Lackenby, which allows the designer to modify any of the following variables, without affecting the L_{WL}, B_{WL} and the depth of the vessel.

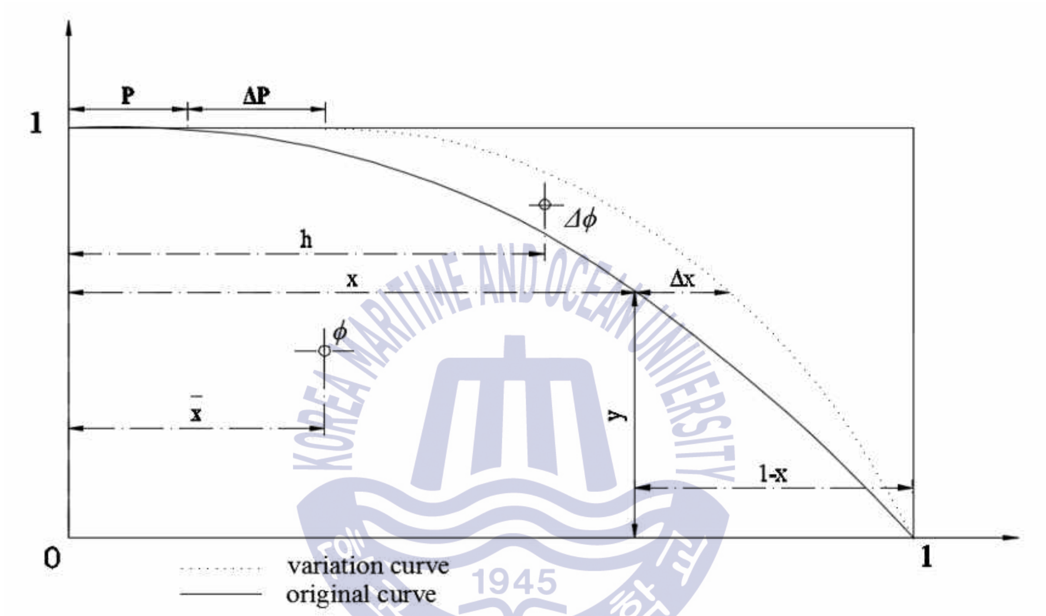

Fig. 7 Lackenby's sectional area curves variation

The Lackenby's method is a quadratic variation of the 'one minus prismatic' method whereby the parallel ship hull can be controlled independently of LCB and the prismatic coefficient C_P (see Fig. 7). Following this method, the sectional area curve is generated for the ship, and that curve is modified to reach the target values. The breadths and heights of the offsets in the station definition are maintained in this method. Only the longitudinal position of each station is shifted. It can generate a few iterations of this method to zero in on the desired values. The more stations involve, the more accurate it is and the fewer iterations it takes.

Although the Lackenby's hull variation technique allows the designer to vary and sets the most of these parameters, the displacement does change for a constant draft.

In this method, the author used the second order of polynomial to interpolating a new sectional curve from the original. Therefore, the result was limited.

2.3 Other methods

2.3.1 Block coefficient C_B method

The C_B (block coefficient) method had been popular in ship basic calculation software. This method allows the user to carry out minor modifications in the block coefficient of a model ship. The modification is done by translating ship sections in the longitudinal direction satisfying the constraint between new and original block coefficients.

2.3.2 'One minus Prismatic' method

Another traditional method was called the $1-C_P$ method, where $C_P = \nabla/(A_M \cdot L)$ stands for the prismatic coefficient. This method is similar to the C_B method, in that both use the section as a constraint. But, $1-C_P$ method uses the longitudinal center of buoyancy (LCB) instead of C_B.

2.3.3 Stretch

This method routine varies any or all of the three major dimensions of the ship (length, beam, and depth) by using a scale factor. This is a very simple hull variation approach that applied to all offsets of the station definition.

2.3.4 Balance

This technique doesn't actually change the shape of the station definition, but it does modify some of the major parameters. When the user selects a constant displacement hull variation as a constraint, this method will raise or sink down the hull to search for a new draft (T) which maintains a ship's displacement. This is performed with a searching technique which maintains zero trim while searching for the desired displacement. And if the draft changes, the L_{WL} and the B_{WL} will likely change. If a constant draft option is selected, this routine calculates the new displacement for the current hull shape.

Chapter 3. Theoretical basis

The content of **Chapter 3** is to present the relevant theoretical that support to the algorithm. The theory is presented in a general form. Therefore it can be applied in various fields, especially PSO optimal algorithm. NURBS function and curve energy minimization seem too familiar with the CAD/CAM system.

3.1 Ship lines – design variables

The goal of our problem is to generate a new shape from a set of constraints, with the following assumptions. The shape of the problem is described by a curve representation that includes parametric piecewise polynomials such as Bézier or NURBS. In addition, the shape consisting of a closed polygon is assumed to be planar. In ship design, this is a plausible assumption as the key shape of hull form is usually represented by a set of three orthogonal planar sections where each section contains profile, waterlines, and buttock lines, respectively (see Fig. 8). If the shape is not planar in other applications, it should be projected to an orthogonal plane.

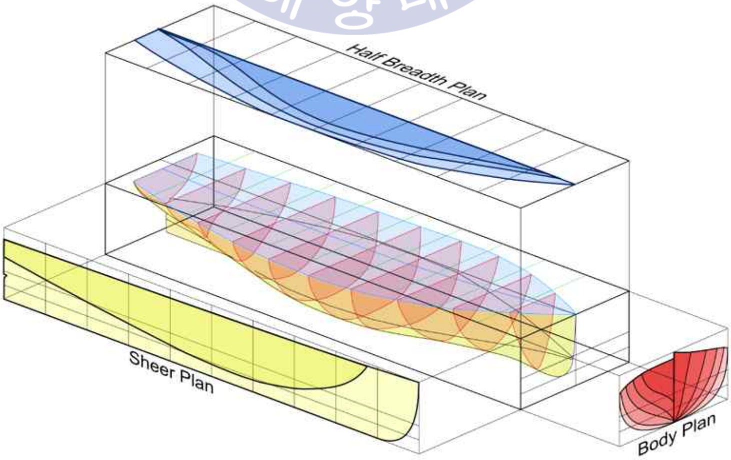

Fig. 8 Ship lines in 3D

A set of waterlines, the typical component of ship hull form, is shown in Fig. 9 Waterline curves are obtained by slicing a ship hull in parallel to the base. One or more waterline curves will be modified following the proposed algorithm.

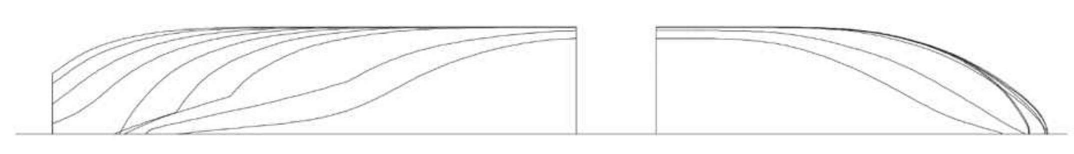

Fig. 9 A set of waterlines of a ship

3.2 Non-Uniform Rational B-Spline

Non-uniform rational B-spline or NURBS is a mathematical model that commonly used in computer graphics for generating and representing curves or surfaces. It offers an excellence flexibility and precision for processing both analytic and modeled shapes. NURBS are also commonly used in computer-aided design (CAD), manufacturing (CAM), and engineering (CAE) and are part of numerous industry-wide standards, such as an IGES, STEP, ACIS, or PHIGS. NURBS techniques are also found in most of the 3D modeling software packages and animation programs.

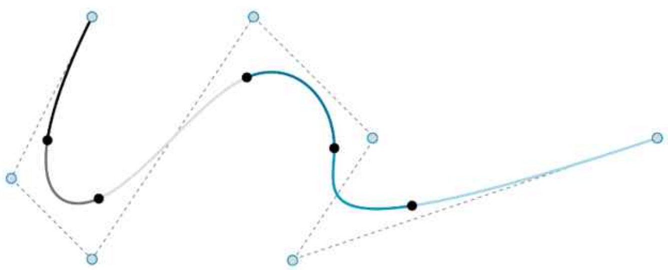

Fig. 10 NURBS curve segments

They can be efficiently processed by the computer programs and for easy human interaction. In general, editing NURBS curves is highly intuitive and predictable since control points are always either connected directly to the curve or act as if they were connected by a rubber band. Based on the type of user interface, editing can be performed via control points that are most obvious and common in Bezier curve editing, or via the other tools such as spline modeling or hierarchical editing.

The B-spline representation for curves is defined by Piegl and Tiller (1997), Eq. (1):

$$C(t) = \sum_{k=0}^{N} P_k R_k^n(t) \tag{1}$$

where t is the parametric variable; and $B_k^n(t)$' s are B-spline basis functions of degree n defined over the knot vector $T = \{t_0, t_1, \dots, t_n, \dots, t_{n+N+1}\}$. The basis functions are defined by the recursive form written in Eq. (2):

$$B_k^0(t) = \begin{cases} 1 & with\ t_k \le t \le t_{k+1} \\ 0 & otherwise \end{cases} \tag{2}$$

$$B_k^n(t) = \frac{t - t_k}{t_{n+k} - t_k} B_k^{n-1}(t) + \frac{t_{n+k+1} - t}{t_{n+k+1} - t_{k+1}} B_{k+1}^{n-1}(t)$$

The basis function of the rational form is associated with weights and defined by Eq. (3):

$$R_k^n = \frac{w_k B_k^n(t)}{\sum_{i=0}^{N} w_i B_i^n(t)} \tag{3}$$

3.2.1 Control point

The control points control and determine the curve' s shape. Generally, each point on the curve is represented by taking a weighted sum of a number of control points. The weight of each point varies corresponding to the governing parameter. For a curve of degree p, the weight of any control point is the only non-zero in p+1 intervals of the parameter space. Within those intervals, the weight changes according to a polynomial function (*basis functions*) of degree p. At the boundaries of the intervals, the basic functions go smoothly to zero, the smoothness being determined by the degree of the polynomial.

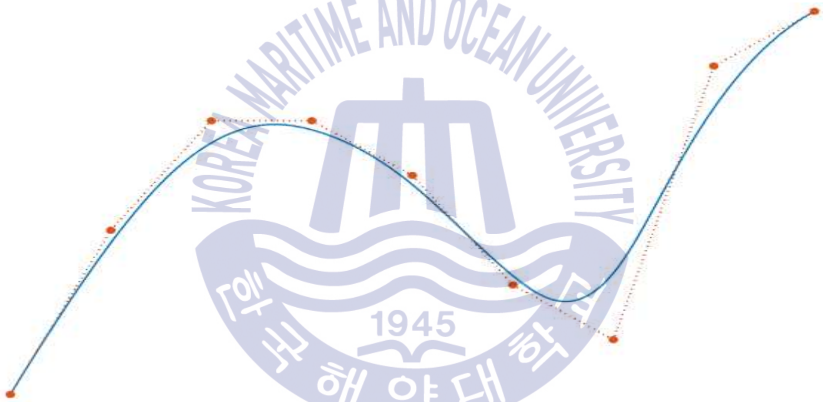

Fig. 11 NURBS curve with control points

In many applications, the fact that each control point affects only a certain segment of the entire curve, known as local support. In modeling, this property allows the changing of one part of a surface or curve while keeping other parts unchanged.

Adding more control points allows a better approximation to a given curve, even though curves can be represented exactly with a fewer number of control points. NURBS curves also need a scalar weight parameter for each control point. This allows for more control over the shape of the curve without using a large number of control points. In particular, it adds conic sections such as circles and ellipses to the set of curves that can be represented exactly. The term '*rational*' in NURBS refers to these weights.

3.3.2 Knot vector

The knot vector is a sequence of non-decrease parameter values that define where and how the control points affect the NURBS curve. The number of knots is always satisfied the condition: m = n + p + 1, where m is a number of knots, n equal number of control points, and p is a degree of curve. The knots in knot vector divides the parametric space into the intervals mentioned before, usually referred to as '*knot spans.*' Each time the parameter value enters to each new knot span, and a new control point is active, while an old control point is discarded. It follows that the values in the knot vector should be in non-decreasing order.

The knot vector usually begins with a knot that has multiplicity equal to the order. This makes sense since this activates the control points that have an influence on the first-knot span. Similarly, the knot vector also ends with a multiplicity knot. Curves with these knot vector will start and end at the first and last control point.

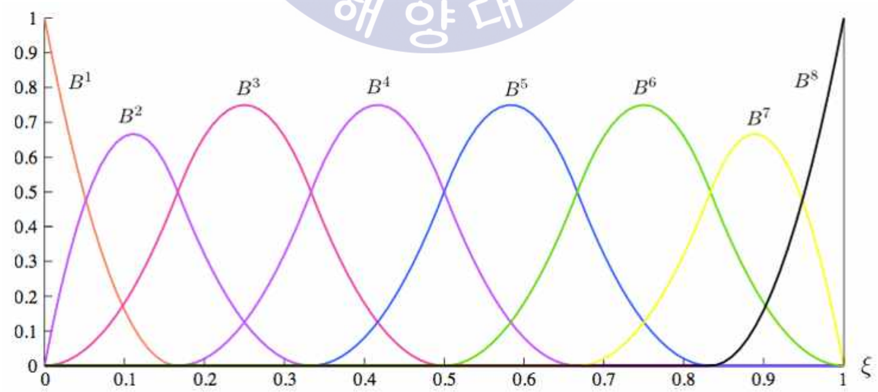

Fig. 12 Quadratic p = 2 B-spline basis constructed from the knot vector [0,0,0,1/6,1/3,1/2,2/3,5/6,1,1,1]

Moreover, the knot vector manipulates the input parameter in parametric space and the corresponding NURBS value on the curve in world space. Rendering an NURBS curve is usually done by stepping with a fixed stride through the parameter range. By changing the knot span lengths, more curve’s points can be used in a narrow region where the curvature is high.

3.2.3 Order

The order or degree of an NURBS curve defines the number of nearby control points that affect any given point on the curve. The parametric curve is represented mathematically by a polynomial of degree that one less than the order of the curve. Therefore, the second-order curves (which are represented by linear polynomials) are named linear curves, third-order curves are named quadratic curves, and fourth-order curves are named cubic curves. The number of control points must be greater than or equal to the curve’s degree.

Fig. 13 The different curves respect to orders

In practice, fourth-order or cubic curves are the ones most commonly used (relating to continuous property). Sometimes Fifth- and sixth-order curves are useful, especially in the case of handling a continuous higher order of derivatives. However, curves with higher orders are practically never used because it is unnecessary. Moreover, they lead to internal numerical problems and tend to require disproportionately large calculation times.

3.2.4 Local modification scheme

This property is the most important thing in geometry design. Changing the position of control point P_i only affects the curve $C(t)$ on the interval $[t_i, t_{i+p+1})$.This based on another important property of B-spline basis functions. Recall that $B_p^i(t)$ is non-zero on interval $[t_i, t_{i+p+1})$. If t is not in this interval, $B_p^i(t)P_i$ has no effect in computing $C(t)$ since $B_p^i(t)$ is zero. On the other hand, if t is in the indicated interval, $B_p^i(t)$ is non-zero. If P_i changes its position, $B_p^i(t)P_i$ is changed and consequently, $C(t)$ is changed.

This local modification scheme is very important and useful in curve design because designers can modify a curve in the narrow region without changing the whole shape of a curve in a global way.

3.3 Optimization techniques

3.3.1 Optimization in ship hull variation

The optimization methods have been widely used to solve many engineering problems including complex; nonlinear design problems occurred in ship design. These methods in general supply the most effective platform if used with good objective functions and proper constraints.

As for the hull form generation or modification, using an optimization technique implies the finding of a new shape by considering the geometric or functional requirements embedded in the formation. Depending on the characteristics of the problem, this method can be effective, but in most cases, the results suffer from sluggishness or divergence unless formulated carefully. Guaranteeing the global optimum is another critical issue associated with optimization.

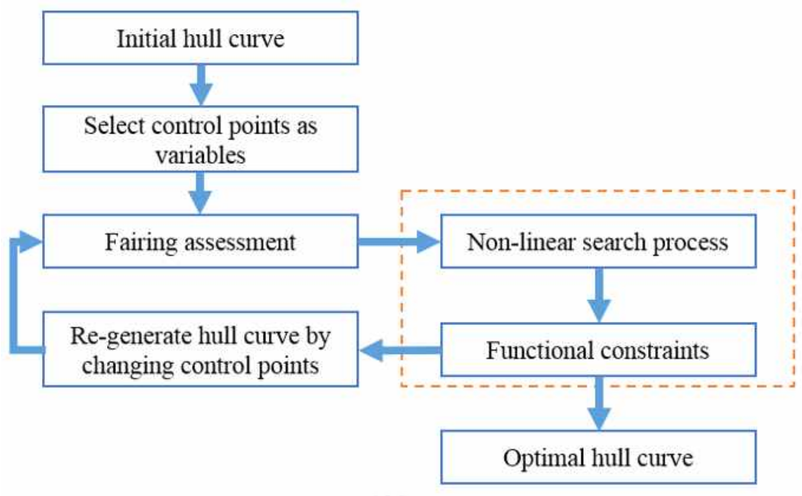

Fig. 14 Hull form optimization procedure

The optimization of a ship's hull form characteristics for improved fairness quality can be described as a multi-variable nonlinear constrained optimization problem:

$$\min f(X) \text{ subject to } g_i \geq 0 \quad i = 1,2,3,\ldots,m, \tag{4}$$

where $X = (x_1, x_2, \ldots, x_n)^T$ is the vector of design variables. Thus the aim is to find the value of that yields the best value of the object function, $f(X)$, within a design space defined by the constraints, $g_i(X)$. The structure of the hull form optimization procedure is illustrated in Fig. 14

3.3.2 Particle Swarm Optimization

Finding the objects in n-dimensional space are common to the mathematical problem. Unfortunately, most of those searches are nonlinear while the numerical methods typically yield bad results or big computational cost. Applying Heuristic search methods appears to be a good choice. And, Particle Swarm Optimization (PSO) is an appropriate proposal in this situation.

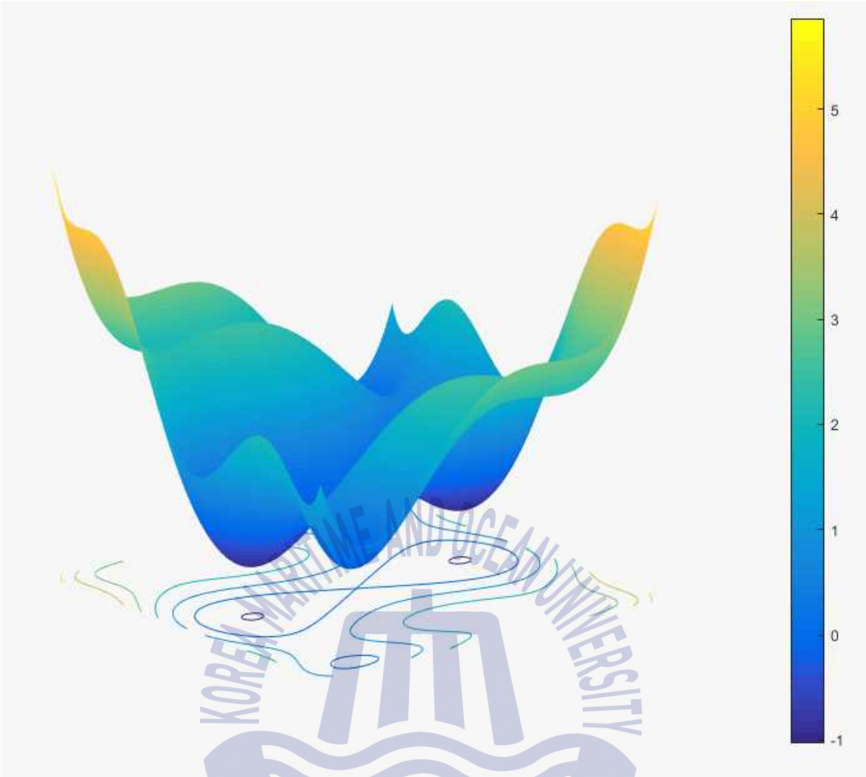

Fig. 15 Three-dimensional search space

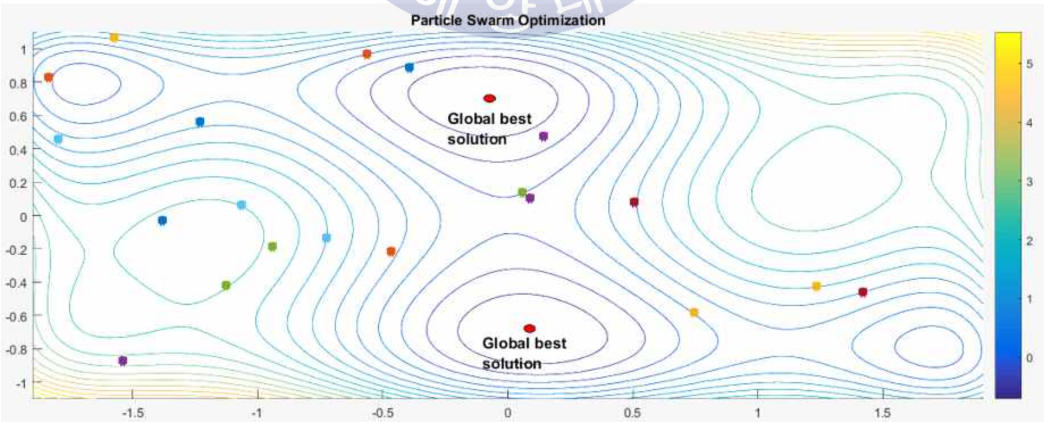

Fig. 16 The particles flying on search space

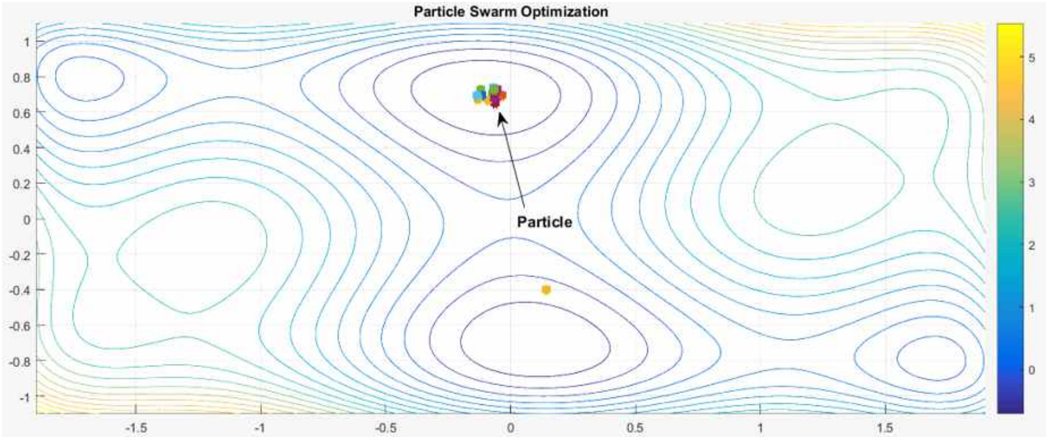

Fig. 17 Finding out the global best solution

PSO was the first introduced by Kenedy and Eberhart in (1995). It is developed from the swarm intelligence whose main idea is inspired by the movement behavior of bird and fish flock. The PSO method is initialized with a group of random particles or and then discover for optima by updating generations. At each iteration, a particle is updated by following two best values. The first one called P_{best} is the best solution (fitness) it has achieved so far. Another best value that is obtained so far by any particle in the population, called G_{best}, is the global best. After searching the two best values, the particle updates its position x' s, and direction v' s by adjusting the constants c_1 and c_2, as formulated in Eq. (5) and illustrated in Fig. 18

$$\begin{aligned} x_{k+1} &= x_k + v_{k+1} \\ v_{k+1} &= w_k x_k + r_1 c_1 (P_{Best} - x_k) + r_2 c_2 (G_{Best} - x_k) \end{aligned} \tag{5}$$

The inertia weight, w_k, controls the momentum of the particle by weighing the contribution of the previous velocity – controlling how much memory of the previous flight direction will influence the new velocity. Where c_1 and c_2, respectively, are learning factor for individual ability (cognitive), and social influence (group). r_1 and r_2 uniformly random numbers are distributed in the interval 0 and 1. The parameters c_1 and c_2 represent the weight of memory (position) of a particle towards memory (position) of the groups (swarm). So, multiply $c_1 r_1$ and $c_2 r_2$ to ensure that the particles will approach the optimum target about half of the difference.

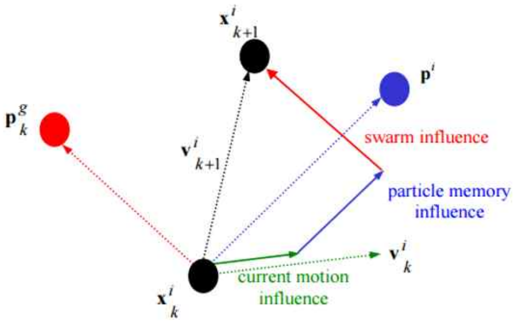

Fig. 18 The depiction of the velocity and position updates in Particle Swarm Optimization.

In the research of American Institute of Aeronautics and Astronautics (2004), they tried to comparing and finding a suitable optimization method, and there are two prominent candidates, PSO and GA (Genetic Algorithm). The results of the t-tests support the hypothesis that while both PSO and the GA obtain high-quality solutions, with quality indices of 99% or more with a 99% confidence level for most test problems, the computational effort required by PSO to arrive at such high-quality solutions is less than the effort required to arrive at the same high quality solutions by the GA. The results further show the computational efficiency superiority of PSO over the GA is statically proven with a 99% confidence level in 7 out of the 8 test problems investigated. Further analysis shows that the difference in computational effort between PSO and the GA is problem dependent. It appears that PSO outperforms the GA with a large differential in computational efficiency if unconstrained nonlinear problems with continuous design variables are solved, and less efficiency differential when we applied to constrained nonlinear problems with any kind of variables.

3.4 Curve energy minimization

In computer graphics and computer-aided design, the designer has always been searching for curves which are smooth both locally and globally. This is also true in our problem, as the curve needs to change without introducing unnecessary oscillation or distortion. Since the proposed algorithm allows the control points of a curve to move freely, an undesirable shape with high strain energy may occur. This situation must be avoided to ensure the smoothness of a resulting curve. The minimization of strain energy of a curve is applied to the variation for the purpose of obtaining a smooth curve.

3.4.1 Curve's curvature

Since the tangent vector or the velocity vector presents the direction of the curve, this means that the curvature is the rate at which the tangent line or velocity vector is turning. There are two refinements needed for this terminology. First, the rate at which the tangent vector of a curve is turning will depend on how fast it is moving along the curve. But curvature should be a geometric characteristic of the curve and will not be changed by the way one moves along it. Therefore we define curvature to be the absolute value of the rate at which the tangent vector is turning when one moves along the curve at a speed of one unit per second.

At first, remembering the determination of whether a curve is curving upwards or downwards ('concave up or concave down'), it may seem that curvature should be a signed quantity. However, the reality shows that this would be undesirable. As for looking at a circle, for instance, the top is concave down, and the bottom is concave up, but clearly user wants the curvature of a circle to be positive all the way round. Negative curvature simply does not make sense for curves.

The second problem with defining curvature to be the rate at which the tangent line is turning. In the plane, the situation is a simple definition. If φ is the angle between the tangent line and the x-axis, then one defines the curvature to be:

$$\kappa(s) = \left| \frac{d\varphi}{ds} \right| \tag{6}$$

where s is arc length.

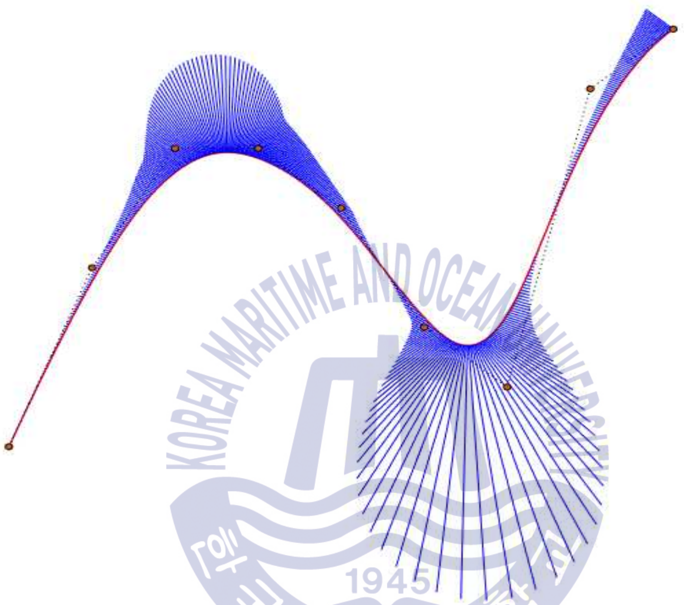

Fig. 19 Porcupine - the distribution of the curve's curvature

Here, $\frac{1}{|\kappa|}$ is called the radius of curvature. The osculating circle, when $\kappa \neq 0$, is the circle at the center of curvature with radius $\frac{1}{|\kappa|}$. It approximates the curve locally up to the second order.

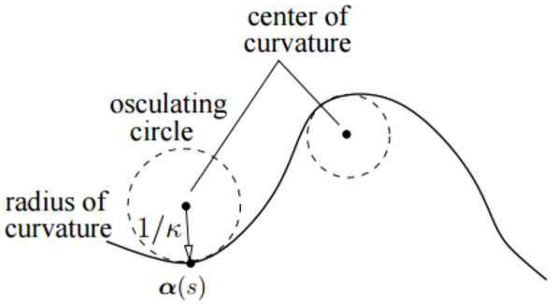

Fig. 20 Osculating circle

3.4.2 Curve's energy

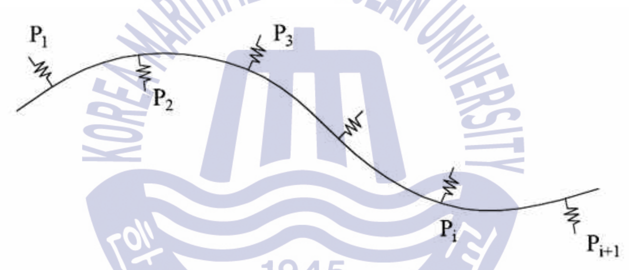

Fig. 21 Plastic curve

The minimum energy of the curve which passes through two specified points can be defined as follows:

$$E_{bend}(t) = \int \kappa^2 ds \tag{7}$$

Where κ is the curvature and s the arc distance, has a number of interesting applications.

The fairness of a curve is intimately related to the porcupine over the form, favoring gradual transitions and avoiding abrupt changes. The curvature $\kappa(t)$of planar curves $r(t) = [x(t), y(t)]$ have a positive or negative sign depending on whether on it curves to left or right. Thus, this signed curvature is highly desirable to detect inflection points as well as convex and concave regions of a curve.

$$\kappa(t)=\frac{\ddot{x}(t)\dot{y}(t)-\ddot{y}(t)\dot{x}(t)}{[\dot{x}(t)^2+\dot{y}(t)^2]^{3/2}} \tag{8}$$

For the evaluation of curves, curvature plots are employed. The curvature plot consists of segments normal to the curve emerging from a set of points on the curve. The whose lengths are proportional to the magnitude of curvature at the associated point. The characteristics of a curve are evidenced by the undulations of its curvature plot. If the curvature plot changes smoothly, the curve can be considered fair. Inflection points occur when curvature plot crosses the curve (sign change), flat regions produce zero curvature value, bulging tendencies produce locally increased, and flattening tendencies produce locally reduced curvature values.

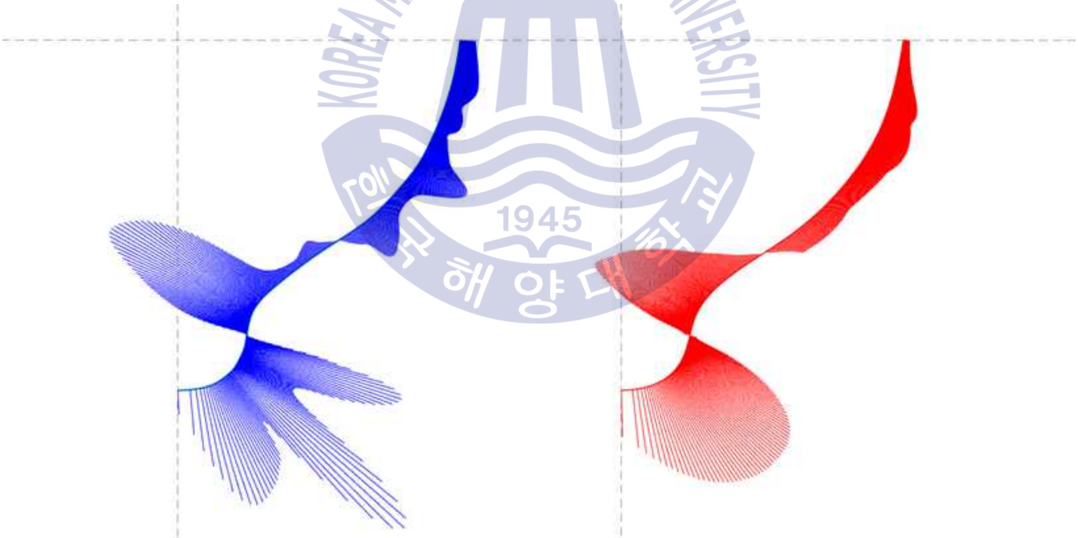

Fig. 22 Comparison curve energy

Left: Initial curve with high curve' s energy

Right: Fairness curve with lower curve' s energy

On Fig. 22, the initial curve on the left has many sharp changes in direction, although it is very hard to detect these unfair spots on the curve by only observing the curve shape. However, the curvature plot is so sensitive that these unfair spots can easily be detected. The curvature distribution of the initial curve is very uneven which is wiggling back and forth indicating unnecessary inflections on the curve. Therefore, this curve cannot be considered as a fair curve. After applying the curve energy minimization procedure to the initial curve, the curve on the right is generated which has a much smoother curvature plot although it deviates in shape very little.

Chapter 4

Curve based hull form variation with geometric constraints of area and centroid

This chapter provides information about the conditions, assumptions, and processes for implementing a basic curve transformation from the original one. The assessment of the errors and the distribution of the curvature of the new curve will be examined to make sure that the curve that is found to be consistent with design requirements.

4.1 Requirements for variation

The variation starts with the modification of the existing waterlines following the given requirements, showed in Fig. 29. Since the shape of a waterline possesses only geometric information, the given requirements for variation should be the factors concerning the geometric properties such as arc length, area, slope, curvature, and so forth.

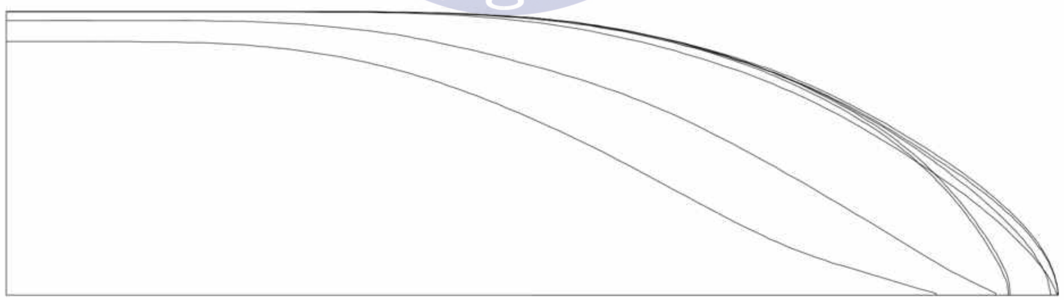

Fig. 23 The set of the fore part of waterlines

To pursue the goal, the problem is formulated as generating a new curve satisfying the given geometric requirements. The curve is defined by the boundary outline of the corresponding shape. Two geometric properties, the area and the centroid affecting the shape and thus performance of a ship are chosen as design parameters. These parameters have an important role in determining the hull shape in the design stage where no other detailed geometric information is available.

Since the area is selected as a design parameter, the curve must be closed by the other bounding curves. For example, the fore part of a waterline of a ship shown in Fig. 24, is bounded by x and y-axes to make the waterline closed.

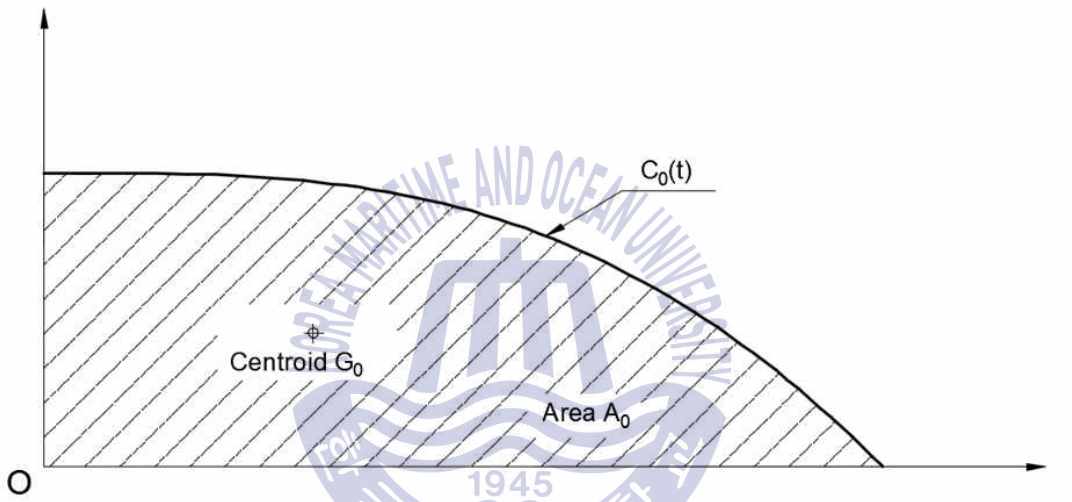

Fig. 24 The design parameters

Let be the planar curve that represents the part of the original waterline. The curve is described by the B-spline representation. The area A_0 and the centroid $G_0 = (x_0, y_0)$ can be automatically calculated following equation:

$$A_0 = \int_0^L dA \tag{9}$$

$$x_0 = \frac{\int x_i dA}{\int dA}; \; y_0 = \frac{\int y_i dA}{\int dA}$$

Then, we want to modify according to the new geometric requirements given by the user: area A_n and the centroid $G_n = (x_n, y_n)$. The mission is to find a new curve, say , that satisfies the new requirements, as depicted in Fig. 25.

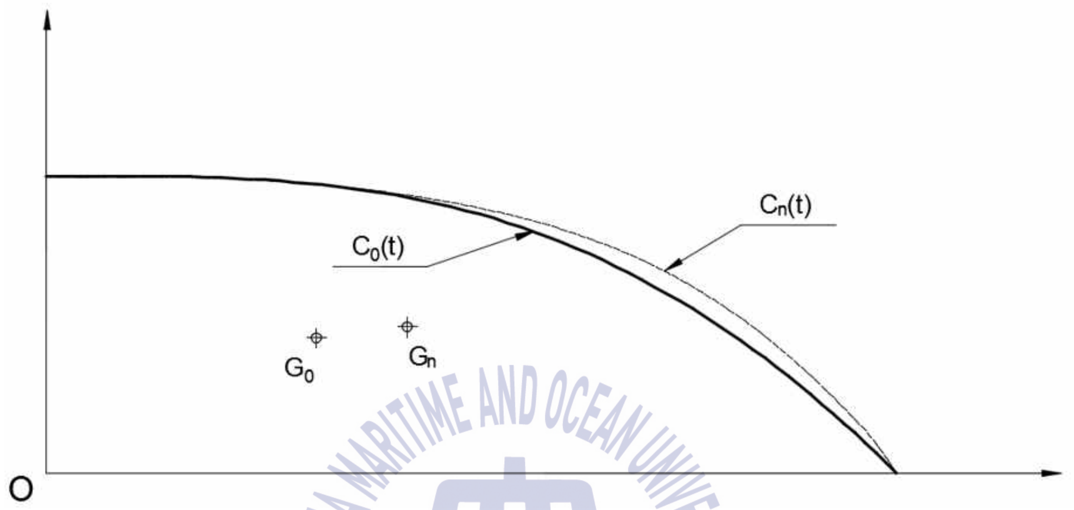

Fig. 25 Problem formulation for curve modification from $C_0(t)$ to $C_n(t)$

A new curve can be obtained in several ways, meaning that the uniqueness of resulting curves needs not to be guaranteed in this work. Therefore, there must be a certain condition in fulfilling the variation to yield a satisfactory result.

Performing the variation in this work requires a couple of design criteria. First, the new curve must be similar to the original with respect to its shape and smoothness. Since the most curves used in the hull variation are smooth locally and globally, the first criterion implies that the new curve must also be smooth. In fact, this requirement is crucial but natural, considering the definition of variation that takes advantage of the original shape. The second criterion requires the modification is recommended to spread over the whole curve as uniform as possible. This criterion is somewhat associated with the first one.

4.2 Overall process

With the problem formulation in mind, a complete algorithm, called curve based hull variation, is developed. The essential concept is to take advantage of curve superposition. Let starting with an original curve; an intermediate curve is created and superimposed to the original to yield a final curve that satisfies the given requirements.

The overall process can be regarded as performing a blending operation, as illustrated in Fig. 26. The blending operation is done by constructing an unknown intermediate curve.

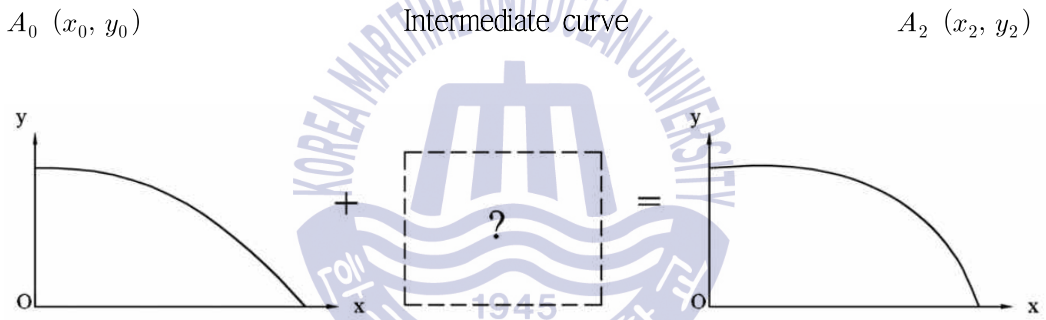

Fig. 26 Problem formulation as a blending operation

The left figure in Fig. 27 shows the original curve in solid. The final curve, yet not known, will be found by adding the deviation curve shown on the right, on the top of the original curve.

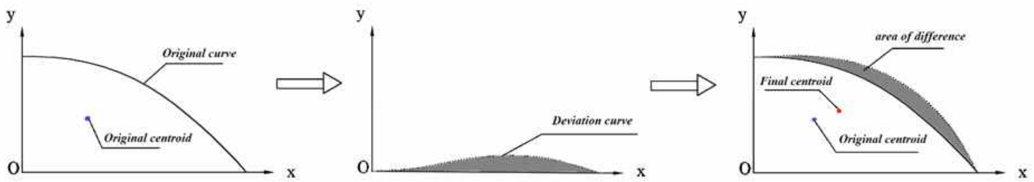

Fig. 27 Transformation of deviation portion on x-axis

Fig. 28 illustrates the overall process of variation. A geometric shape that has an area A_0 and centroid G_0 is defined by an original curve. The designer is asked to generate a new shape that will have the new requirements: area A_n and centroid G_n. The algorithm separates the design requirements in the first place, and an intermediate curve is constructed in an iterative manner. The intermediate curve is then superimposed to the original curve to yield a final shape. Each step will be discussed in detail.

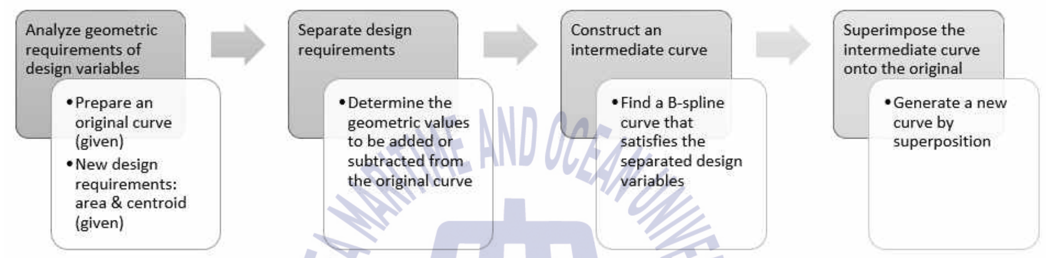

Fig. 28 Overall procedure of proposed variation algorithm

Before going into details, the terms used in this paper need to be clarified. The original or existing curve is the given curve that characterizes the shape of existing design, for example, a waterline or section curve of a ship. An intermediate or deviation curve is the one to be added to the original curve. Obtaining a good quality of intermediate curve is the key to the problem. The new or final curve refers to the curve obtained by adding the intermediate and original curves.

4.3 Characteristics of intermediate curve

The proposed algorithm takes advantage of an intermediate or deviation curve, to find a final curve. Thus, obtaining an appropriate deviation curve is essential in the proposed algorithm. The idea of adopting a deviation curve has been utilized in other applications. Stewart (1991) and Lu (1995) used this concept by proposing polynomial based shape functions for the manipulation of curves and surfaces.

Since the NURBS form is used to represent all curves, the strategy is to add two curves to obtain a new curve. Mathematically each curve is represented by:

$$C_i(t) = \sum_{k=0}^{N} P_k R_k^n(t) \tag{10}$$

where the script i is used to differentiating the curves, that is, the script O for the original, I for the intermediate, and F for the final curve.

Our strategy is to obtain $C_F(t)$ by superimposing $C_I(t)$ on top of $C_O(t)$, like Eq. (11):

$$\begin{aligned} C_F(t) &= C_O(t) + C_I(t) \\ &= \sum_{k=0}^{N} P_k^O R_k^n(t) + \sum_{k=0}^{N} P_k^I R_k^n(t) = \sum_{k=0}^{N} (P_k^O + P_k^I) R_k^n(t) \\ &= \sum_{k=0}^{N} P_k^F R_k^n(t) \end{aligned} \tag{11}$$

To make the above equation work mathematically, the following conditions or assumptions must be satisfied. Firstly, as for the basis functions, all the curves must share the same knot vector, and the same number of control points must be maintained. Next, the movement of control points is restricted in one direction in order to make Eq. (11) true. The latter condition implies the moving range of the final curve must be identical to those of the initial curve as well as the intermediate curve. This restriction is plausible because the purpose of variation is to make a minor tuning rather than a dramatic change. For major modification, it will be better to redesign from scratch.

Other important characteristics of the intermediate curve are smoothness and uniformly distributed shape. Smoothness means the intermediate curve must be locally and globally continuous and smooth. To achieve the smoothness, energy minimization is utilized. Uniformly distributed shape, also weakly connected to smoothness, is recommended to minimize the change from the original shape.

4.4 Curve based variation

4.4.1 Hull curve generation

Modeling a basic curve of the hull using NURBS techniques will be done in steps. The modeled curve need to ensure the accuracy of smoothness, an area and a centroid position with the minimum of the number of control points. The control points of the sample are given in vector form as follows:

```
>> CPx = [0 2.81 9.554 18.668 27.79 35.137 41.687 46.432 47.459];

>> CPy = [16.13 16.148 16.1 15.286 12.863 9.256 5.148 1.196 0];
```

The specific geometry of the curve as the area, the centroid, the curvature distribution, are calculated.

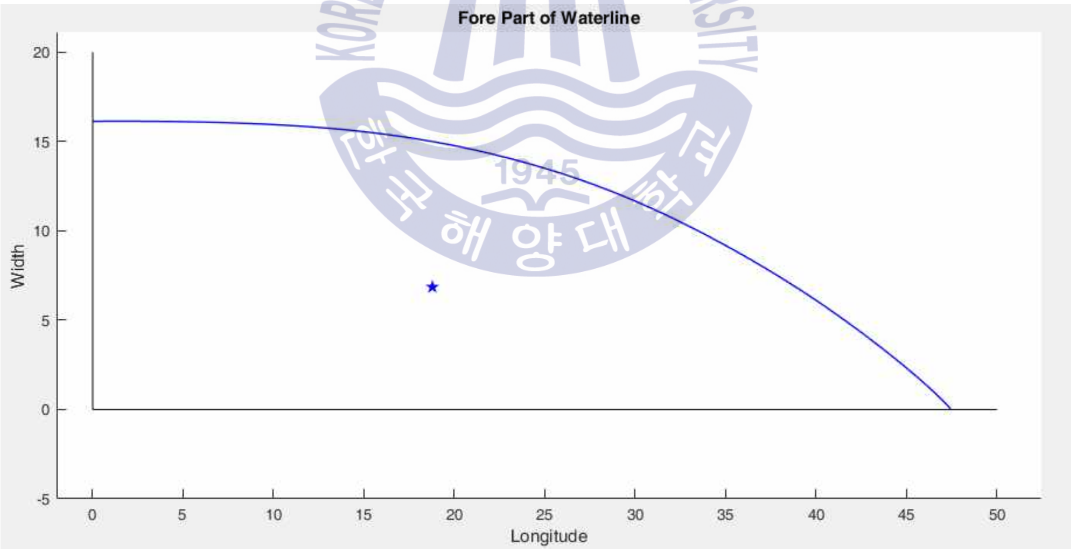

Fig. 29 The fore part of a waterline with centroid

The area and centroid that are delimited by the waterline and the axes are calculated by Eq. (9)

$$A_0 = \int_0^L dA = 564.9259$$

$$x_0 = \frac{\int x_i dA}{\int dA} = 18.7838$$

$$y_0 = \frac{\int y_i dA}{\int dA} = 6.8535$$

The distribution of waterline's curvature is also calculated to assess the energy level of this curve. That is depicted in Fig. 30

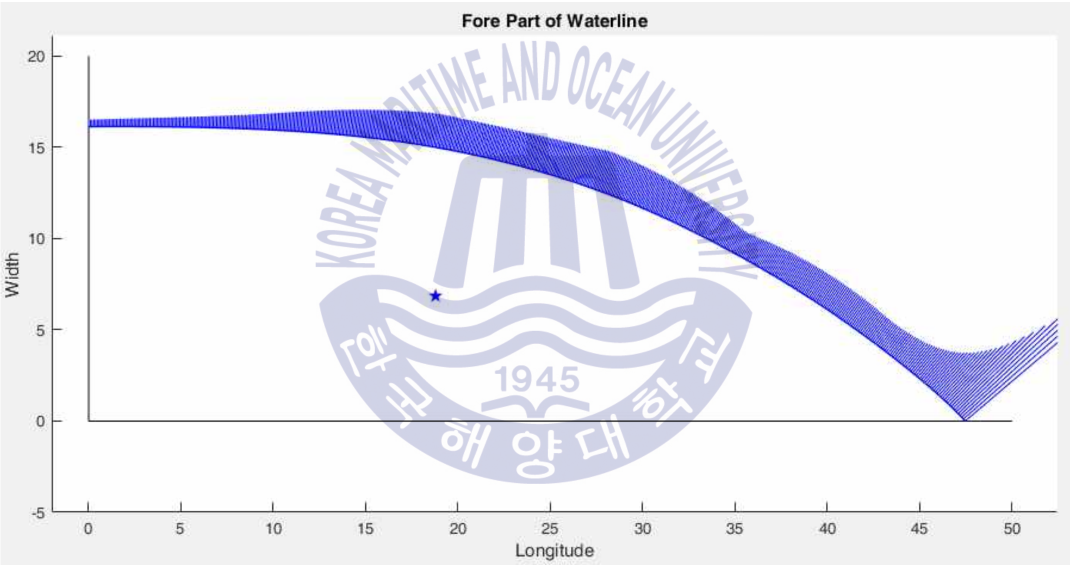

Fig. 30 Distribution of waterline's curvature – porcupine

To receive a new waterline shape, the designer must take into the new design parameter, called requirement parameters, including new area and new centroid position (*red star*). All of this requirement are completely received from the modification of the actual requirements as volume, LCG, or form factor.

New are: $A_{desired} = 575$

New centroid: $(x_{desired}, y_{desired}) = (19.02, 6.91)$

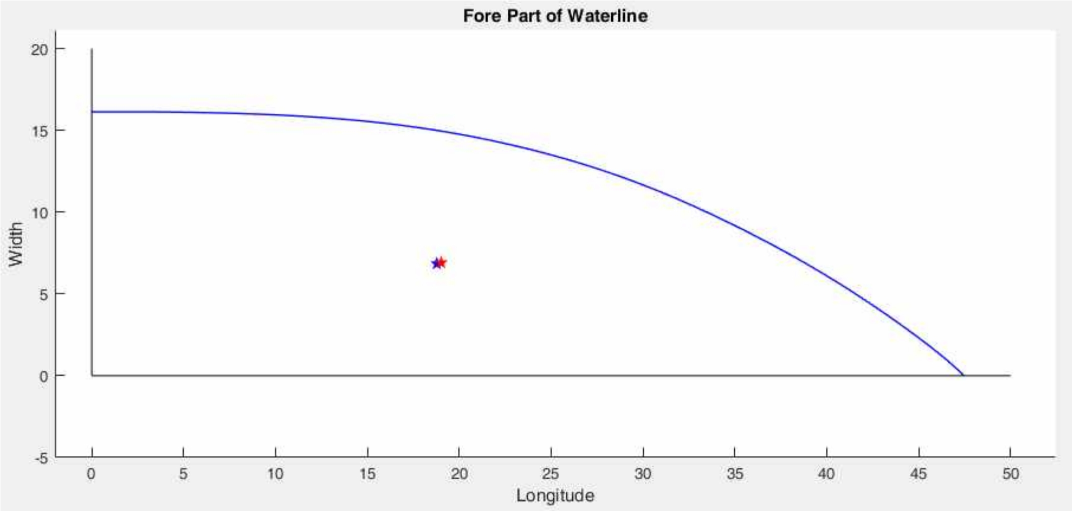

Fig. 31 The new requirement parameter of centroid

From the initial specifications and new requirement parameters, the relationships of the area, the centroid are calculated, in order to create an intermediate curve, called the deviation curve.

4.4.2 Geometric requirements for intermediate and deviation curves

A curve that has the same knot vector and the same number of control points is selected as an intermediate curve. According to the proposed problem formulation, the area, and the centroid is added to or subtracted from the original curve via the use of the intermediate curve. The differences of area and centroid between the given and new shapes need to be calculated. Calculations for those differences are done by the simple arithmetic. The subscripts 0, 1, and 2 indicate the geometric properties of the original, the intermediate, and the final curves, respectively.

For the area, the incremental area is simply obtained by subtracting the new area from the original one:

$$A_1 = A_2 - A_0 = 575 - 564.9259 = 10.0741 \tag{12}$$

The centroid itself is obtained by applying the first moment equation:

$$x_1 = \frac{A_2 x_2 - A_0 x_0}{A_2 - A_0} = \frac{575.19{,}02 - 564{,}9259.18{,}7838}{575 - 564{,}9259} = 32{,}4972 \tag{13}$$
$$y_1 = \frac{A_2 y_2 - A_0 y_0}{A_2 - A_0} = \frac{575.6{,}91 - 564{,}9259.6{,}8535}{575 - 564{,}9259} = 10{,}0766$$

In our problem, the modification is restricted along the y-axis, which means the intermediate curve is superimposed on the top of the original curve. Thus, in Eq. (11), only the y components of the control points of the intermediate curve are the design variables.

In Fig. 32, the original curve and the intermediate curve forming the shaded area (area A) in space are depicted. The curve named as the deviation curve is the transformed intermediate curve on the x-axis. It is crucial to transforming the shaded portion (area A) between the original and intermediate curves onto the x-axis, which means area A must be translated onto x-axis so that the area B under the deviation curve is identical to the area A.

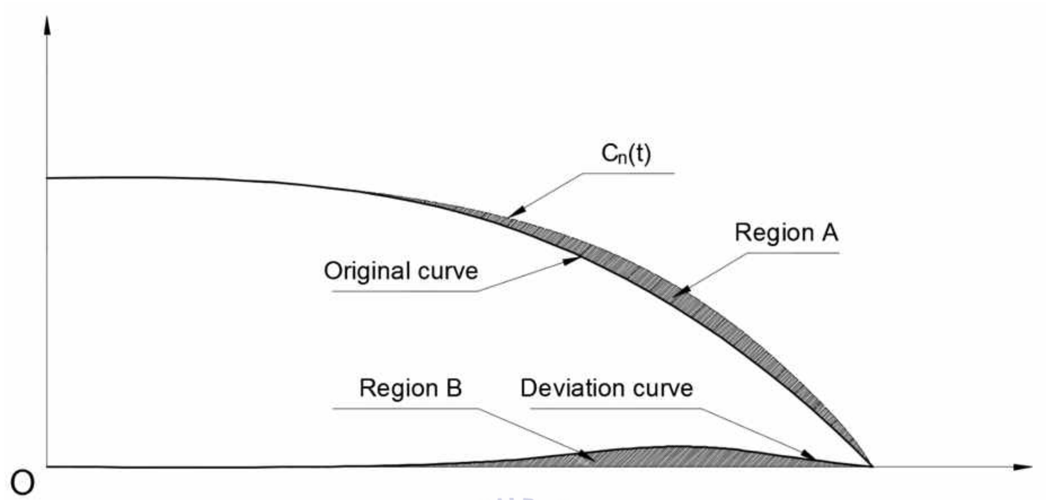

Fig. 32 Deviation versus original curve

Creating a curve that has new area requirement is simple. The centroid with respect to the y-axis remains unchanged, as the areas and the moment arms for the two areas A and B are identical. The centroid deviation with respect to the x-axis needs a geometric conversion of centroid between two trapezoidal shapes, depicted in Fig. 39

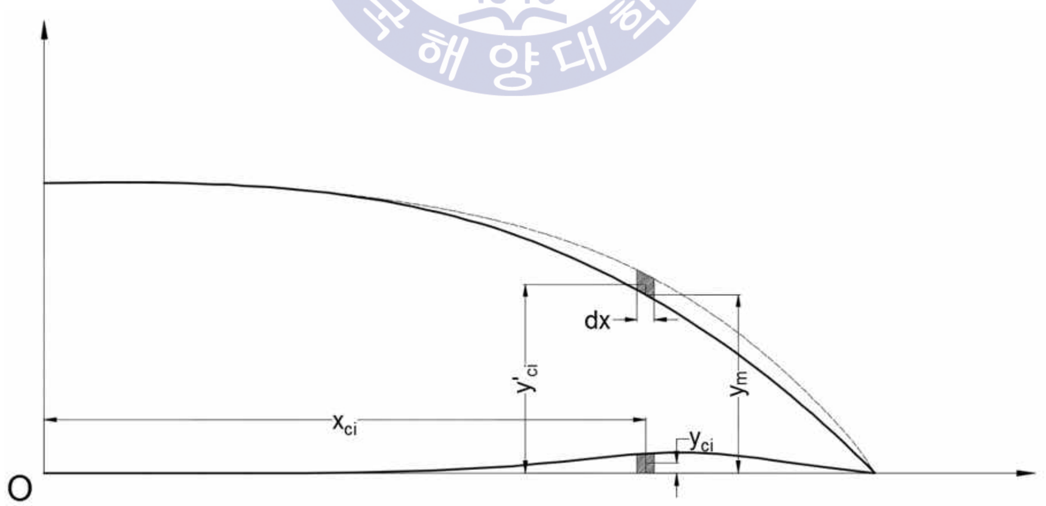

Fig. 33 Conversion of centroid in y direction between intermediate and deviation curve

Assuming that the disparity area is divided into small trapezoids, the relationship between two corresponding trapezoids can be found in Eq. (14):

$$y'_{ci} = y_{ci} + y_m \tag{14}$$

The centroid y_B of translated area (region B) and the original centroid y_A (region A) are computed as follows:

$$y_B = \frac{\int y_{ci} dA}{\int dA} \tag{15}$$

$$y_A = \frac{\int y'_{ci} dA}{\int dA} = \frac{\int (y_{ci} + y_m) dA}{\int dA} = \frac{\int y_{ci} dA}{\int dA} + \frac{\int y_m dA}{\int dA} = y_B + \frac{\int y_m dA}{\int dA} = 10{,}0766 \tag{16}$$

Then, the relation between two centroids is expressed as:

$$y_B = y_A - \frac{\int y_m dA}{\int dA} = 10{,}0766 - \frac{\int y_m dA}{\int dA} \tag{17}$$

and this relation will be used in finding a deviation curve. By balancing the two sides of the Eq. (17), the y component will be controlled by the optimal search algorithm.

4.4.3 Determination of deviation curve

To find the intermediate and thus deviation curves, the fitness function $f(C)$ is established as follows:

$$f(C(t)) = w_A f_{Area} + w_{Cx} f_{Centroid_x} + w_{Cy} f_{Centroid_y} + w_E f_{Energy} \tag{18}$$

The fitness function is the combination of geometric and energy terms that consider the change of area, the change of centroid, and the effect of curve strain energy.

The function will find a curve, $C(t)$, that minimizes the change of area, centroid, and strain energy. The weighting factors, w_i's, may be used at the user's discretion depending on his or her design intention.

The geometric terms in the fitness function are simply defined as the relative value between the desired and obtained values:

$$f_{Area} = (A_{obtained} - A_{desired})^2 = (A_{obtained} - 10{,}0741)^2 \tag{19}$$

$$f_{Centroid_x} = (x_{obtained} - x_{desired})^2 = (x_{obtained} - 32{,}4972)^2 \tag{20}$$

$$f_{Centroid_y} = (y_{obtained} - y_{desired})^2 = (y_{obtained} - 10{,}0766 + \frac{\int y_{m dA}}{\int dA})^2 \tag{21}$$

The energy term uses the strain energy of a curve:

$$f_{Energy} = \int \kappa^2(t)dt + \int \dot{\kappa}^2(t)dt \tag{22}$$

The design variables used in minimizing the fitness function are the control points of the intermediate B-spline curve. Since the movement of the intermediate curve is restricted along y direction, y components of control points are chosen. The number of design variables, therefore, is N+1y control points in Eq. (23):

$$(P_y)_i,\ i = 0, \ldots, N \quad \in \quad C(t) = \sum_{k=0}^{N} P_k R_k^n(t) \tag{23}$$

The particle swarm optimization performs the finding appropriate deviation curve based on the following procedure, showed in Fig. 40.

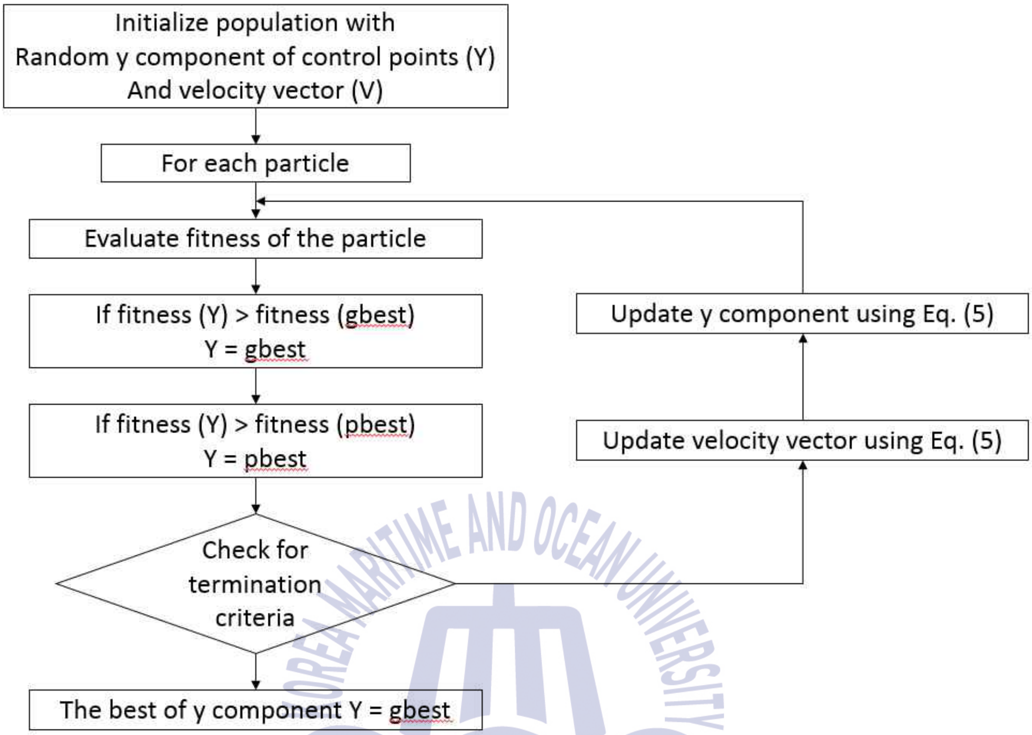

Fig. 34 Procedure of PSO for finding deviation curve

The control points of the deviation curve that have been found out by optimal search algorithm based on the above procedure, (see Fig. 34). However, following the initial assumption that the control points just move only in y direction. Therefore only y components of the control points are changed, while x components remain.

>> CPx_dev = [0 2.81 9.554 18.668 27.79 35.137 41.687 46.432 47.459];

>> CPy_dev = [0 0 0 0 0.522 0.544 0.348 0.062 0];

The distribution of the curvature of the deviation curve is also examined to ensure its smoothness.

4.4.4 Variation by superposition

Once the intermediate curve is found, the final step is to superpose the deviation curve onto the original curve. The superposition is simply done by adding the y components of control points of the original and deviation curves, as defined in the formulation:

$$C_{xF}(t) = C_{xO}(t) = C_{xI}(t) = \sum_{k=0}^{N} P_{xk}^{O} R_k^n(t) \tag{24}$$

$$C_{yF}(t) = C_{yO}(t) + C_{yI}(t) = \sum_{k=0}^{N} (P_{yk}^{O} + P_{yk}^{I}) R_k^n(t) = \sum_{k=0}^{N} P_{yk}^{F} R_k^n(t)$$

The control points of the new waterline that received from the superposition equation would be:

```
>> CPx_new = [0 2.81 9.554 18.668 27.79 35.137 41.687 46.432 47.459];
>> CPy_new = [16.13 16.148 16.1 15.286 13.385 9.8 5.496 1.259 0];
```

The new curve is showed in the Fig. 35

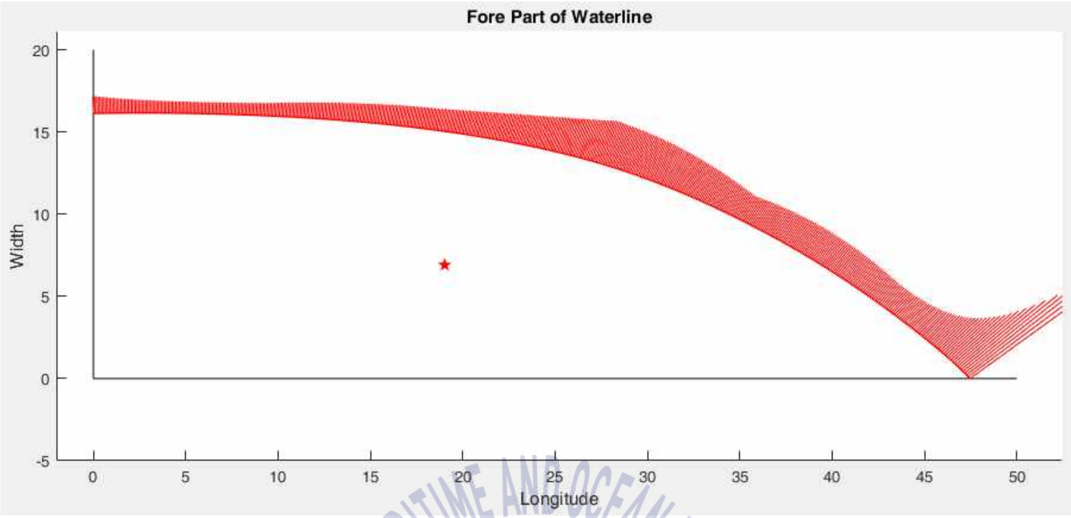

Fig. 35 New waterline with porcupine

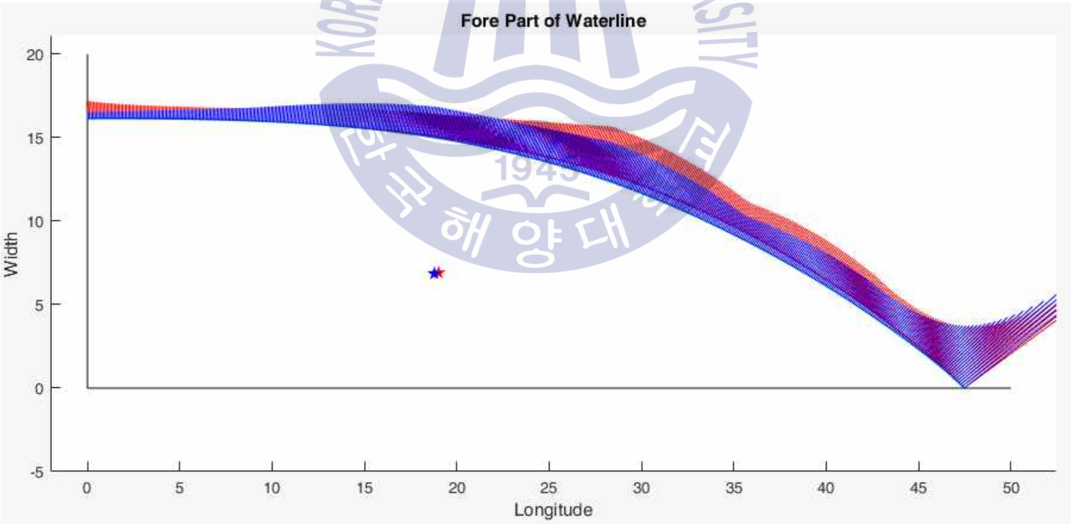

Fig. 36 Comparison between new and original waterline's curvature

The values of input, intermediate, and final requirements or results are listed in *Table 1*. The validity of the developed algorithm can be assured by their extremely low error rates.

Table 1. Error evaluation

Item No.	Description	Given by	Area	Centroid	Error rates (compared to Item No #)
0	Original waterline	user	564.9259	(18.7838, 6.8535)	
2	Requirements for new waterline	user	575.0000	(19.0200, 6.9100)	
1	Intermediate curve	theoretically calculated	10.0741	(32.4972, 10.0766)	
1.1	Intermediate curve	algorithm	10.0741	(32.4994, 10.0784)	A: 0% (#1) G: (0%, <0.018%)
1.2	Deviation curve	algorithm	10.0741	(32.4972, 0.2014)	Error is checked by the values of final curve
2.1	New waterline	algorithm (superposition)	575.000	(19.0241, 6.9109)	A: 0% (#2) G: (0%, < 0.02%)

4.5 Application

4.5.1 Stern section variation

The sections, what locate at engine room, are special shape because of hydrodynamics property. A typical stern section is depicted in Fig. 37. The result of stern section variation is evaluated in Table 2.

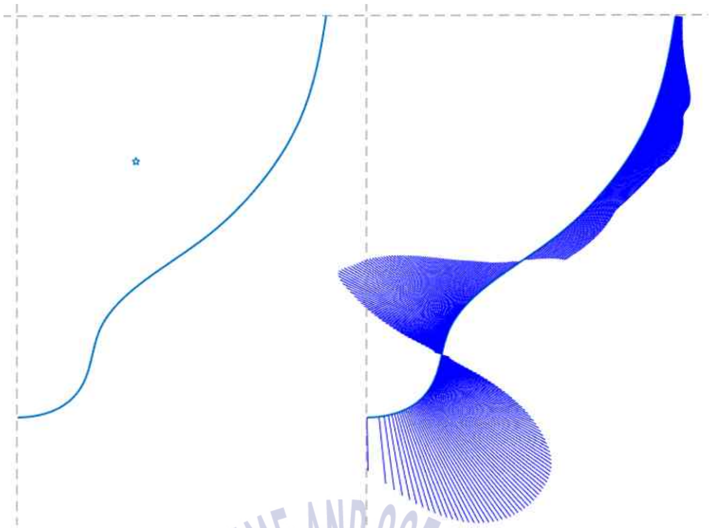

Fig. 37 Stern section variation

Table 2. Error evaluation – stern section variation

Item No.	Description	Given by	Area	Centroid	Error rates (compared to Item No #)
0	Original waterline	user	184.406	(6.766, 6.104)	
2	Requirements for new waterline	user	192.000	(6.800, 6.340)	
1	Intermediate curve	theoretically calculated	7.594	(7.6256, 12.0708)	
1.1	Intermediate curve	algorithm	7.5898	(7.6271, 12.0606)	A: 0.05% (#1) G: (<0.08%)
1.2	Deviation curve	algorithm	7.5898	(7.6271, 0.3034)	Error is checked by the values of final curve
2.1	New waterline	algorithm (superposition)	191.993	(6.7995, 6.342)	A: 0.006% (#2) G: (0%, < 0.03%)

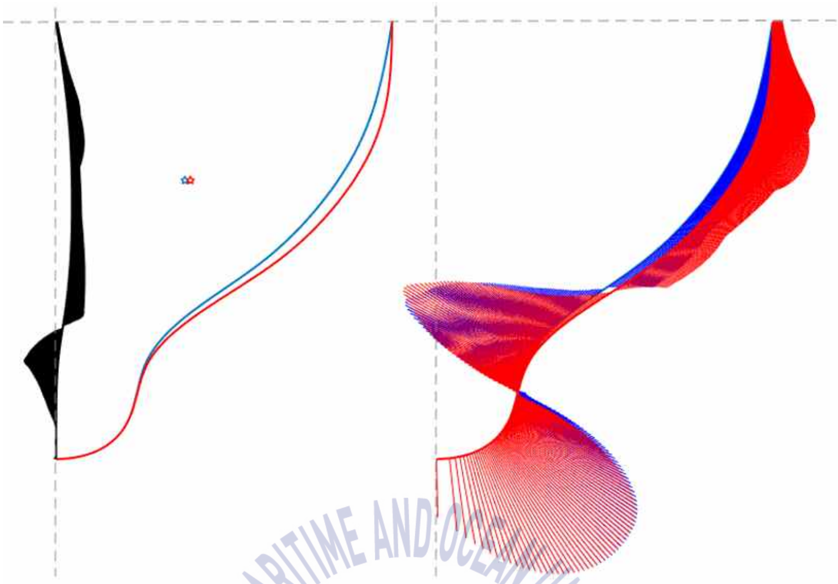

Fig. 38 Comparison between original and modified stern section

4.5.2 Bulbous bow section variation

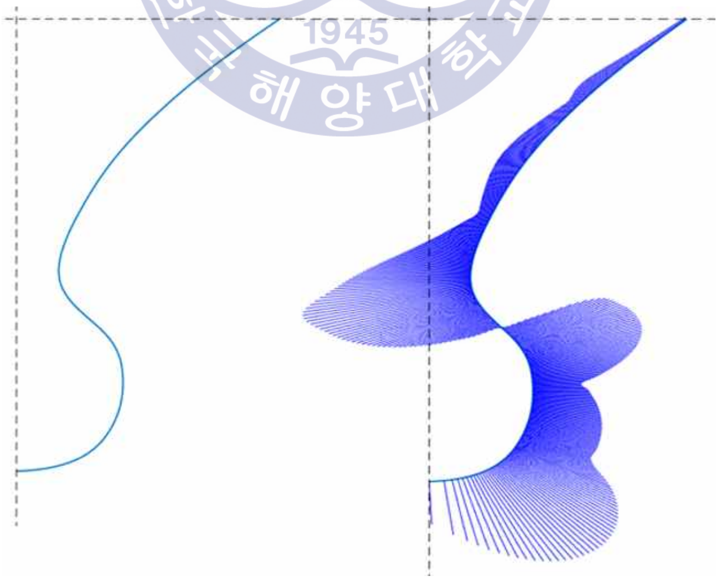

Fig. 39 The original section

Table 3. Error evaluation – Bulbous bow section variation

Item No.	Description	Given by	Area	Centroid	Error rates (compared to Item No #)
0	Original waterline	user	55.7596	(7.2954, 1.9519)	
2	Requirements for new waterline	user	62.0000	(7.3100, 2.0600)	
1	Intermediate curve	theoretically calculated	6.2404	(7.4405, 3.0259)	
1.1	Intermediate curve	algorithm	6.2404	(7.4405, 3.0280)	A: 0.00% (#1) G: (<0.07%)
1.2	Deviation curve	algorithm	6.2404	(7.4405, 0.2382)	Error is checked by the values of final curve
2.1	New waterline	algorithm (superposition)	61.9997	(7.3099, 2.0593)	A: 0.005% (#2) G: (0%, < 0.03%)

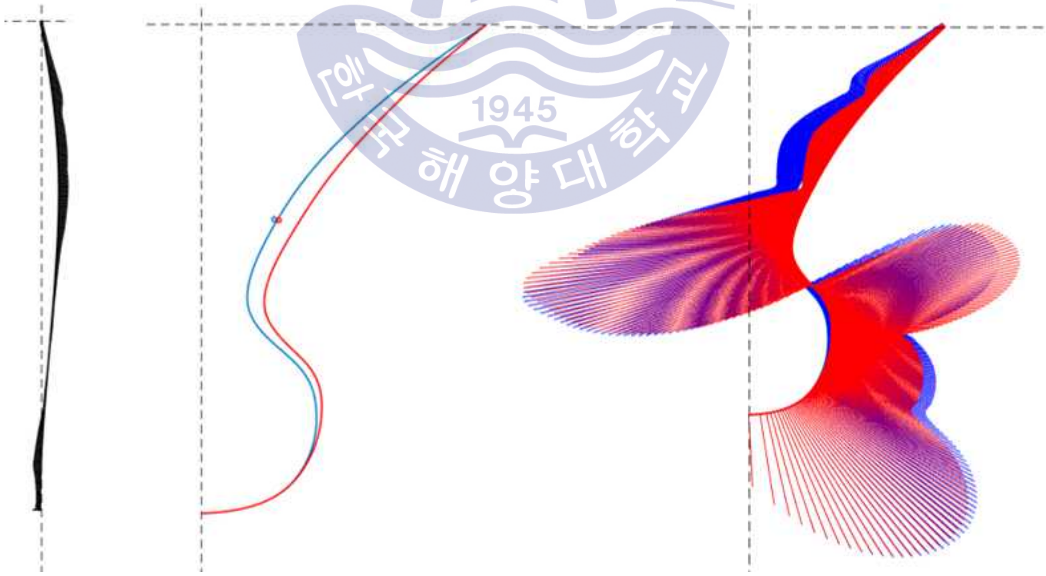

Fig. 40 Bulbous bow section variation

Chapter 5. Conclusions

A limitation observed in the proposed algorithm is the infeasibility of the shape of a new profile curve. The concept of variation is to modify the minor portion of the existing shape, assuming that the existing shape is satisfactory with respect to its functional requirements and smoothness. An excessive variation, however, may cause a wiggled and thus undesirable result. This is a truly possible case that can be commonly observed in real design practice as the undesirable shape can also satisfy the given requirements. Even though primary responsibility goes to the designer who sets up wrong requirements, this uncontrolled behavior should be avoided within an algorithm. The feasibility analysis is being worked by the authors.

A simple and straightforward, but efficient variation method of constructing a new shape from an existing one has been introduced. An intermediate curve was constructed to satisfy the given geometric requirements and later superimposed on the top of the existing curve. Conceptually, this process is similar to the one that uses shape functions. In this work, however, geometric requirements that facilitate a different hull form design have been incorporated into the problem formulation.

The proposed approach provides a concise and powerful design tool. Constructing and adding a simple curve form yields a new shape that is as smooth as the original shape. Since both the intermediate curve and consequently the final curve have the same knot vector and the same interval, the resulting variation always follows the original shape and curve structure. This is a great advantage in performing ship hull variations.

Even though the whole process is designed to work in a linear fashion, a nonlinear optimization step is used in finding the intermediate or deviation curve. This unavoidable process, however, is extremely simple and provides an opportunity in yielding an intermediate curve that has smooth and uniformly distributed shape, by considering the energy minimization concept.

The proposed approach generates satisfactory results and can readily be extended to more complicated design processes.

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